# List of symbols

## Number sets and vector spaces

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathbb{N}$</td>
<td>set of natural numbers</td>
</tr>
<tr>
<td>$\mathbb{Z}$</td>
<td>set of integers</td>
</tr>
<tr>
<td>$\mathbb{Q}$</td>
<td>set of rational numbers</td>
</tr>
<tr>
<td>$\mathbb{R}$</td>
<td>set of real numbers</td>
</tr>
<tr>
<td>$\mathbb{C}$</td>
<td>set of complex numbers</td>
</tr>
<tr>
<td>$\mathbb{R}^n$</td>
<td>set of all real $n$-tuples</td>
</tr>
<tr>
<td>$\mathbb{S}^{n-1}$</td>
<td>unit sphere of $\mathbb{R}^n$</td>
</tr>
<tr>
<td>$\mathbb{R}_+^n$</td>
<td>set of all real $n$-tuples with non-negative coordinates</td>
</tr>
<tr>
<td>$\mathbb{C}^n$</td>
<td>set of all complex $n$-tuples</td>
</tr>
<tr>
<td>$a \land b, a \lor b$</td>
<td>minimum and maximum of $a$ and $b$</td>
</tr>
<tr>
<td>$</td>
<td>\alpha</td>
</tr>
<tr>
<td>Re $\lambda, \text{Im} \lambda$</td>
<td>real and imaginary part of $\lambda \in \mathbb{C}$</td>
</tr>
<tr>
<td>$#E$</td>
<td>cardinality of the set $E$</td>
</tr>
</tbody>
</table>

## Topological and metric space notation

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>$E$</td>
<td>topological closure of $E$</td>
</tr>
<tr>
<td>$\partial E$</td>
<td>topological boundary of $E$</td>
</tr>
<tr>
<td>$E^c$</td>
<td>the complementary set of $E$ in a domain $\Omega$ or in $\mathbb{R}^n$</td>
</tr>
<tr>
<td>$E \subset\subset F$</td>
<td>$E \subset F$, $E$ compact</td>
</tr>
<tr>
<td>$B(x_0, r)$</td>
<td>open ball with center $x_0$ and radius $r$</td>
</tr>
<tr>
<td>$B^+(0, r)$</td>
<td>$B(0, r) \cap \mathbb{R}_+^n$</td>
</tr>
<tr>
<td>$\mathcal{L}(X,Y)$</td>
<td>set of bounded and linear operators from $X$ to $Y$</td>
</tr>
<tr>
<td>$\mathcal{L}(X)$</td>
<td>dual space of the Banach space $X$</td>
</tr>
<tr>
<td>$X'$</td>
<td>dual space of the Banach space $X$</td>
</tr>
</tbody>
</table>
Matrix and linear algebra

- $I$: the identity matrix
- $\det B$: the determinant of the matrix $B$
- $e_i$: $i$-th vector of the canonical basis of $\mathbb{R}^n$
- $\text{Tr} B$: the trace of the matrix $B$
- $\|B\|_\infty$: the Euclidean norm of the matrix $B$, i.e.
  - $\left(\sum_{i,j=1}^n b_{ij}^2\right)^{1/2}$
- $\|B\|_{1,\infty}$:
  - $\left(\sum_{i,j,h=1}^n |D_h b_{ij}|^2\right)^{1/2}$
- $\|B\|_{2,\infty}$:
  - $\left(\sum_{i,j,h,k=1}^n |D_{hk} b_{ij}|^2\right)^{1/2}$
- $\langle \cdot, \cdot \rangle$ or $x \cdot y$: the Euclidean inner product between the vectors $x, y \in \mathbb{R}^n$

Function spaces: let $f : X \to Y$

- $f \restriction E$ or $f|_E$: restriction of $f$ to $E \subset X$
- $\chi_E$: characteristic function of the set $E$
- $u_t$: partial derivative with respect to $t$
- $D_i$: partial derivative with respect to $x_i$
- $D_{ij}$: $D_i D_j$
- $Du$: space gradient of a real-valued function $u$
- $D^2 u$: Hessian matrix of a real-valued function $u$
- $\Delta u$: $\text{Tr}(D^2 u)$
- $C(X, Y)$: space of continuous functions from $X$ into $Y$
- $C(\Omega)$: space of continuous functions valued in $\mathbb{R}$ or $\mathbb{C}$
- $C_c(\Omega)$: functions in $C(\Omega)$ with compact support in $\Omega$
- $C_0(\Omega)$: closure in the sup norm of $C_c(\Omega)$
- $\text{UC}_b(\Omega)$: space of the uniformly continuous and bounded functions on $\Omega$
- $C^{k,\alpha}(\Omega)$: space of $k$-times differentiable functions with $D^m f$ for $|m| \leq k$ bounded and continuous up to the boundary
- $C^{\alpha}(\Omega)$: space of $\alpha$-Hölder continuous functions, $\alpha \in (0, 1)$
- $C_0^{k,\alpha}(\Omega)$: space of $f \in C^k(\Omega)$ with $D^m f \in C^{\alpha}(\Omega)$ for $|m| \leq k$ and $\alpha \in (0, 1)$
- $S(\mathbb{R}^n)$: Schwartz space of rapidly decreasing functions
- $[u]_{C^{\alpha}(\Omega)}$: the seminorm $\sup_{x,y \in \Omega} \frac{|u(x) - u(y)|}{|x-y|^\alpha}$
- $\| \cdot \|_{L^\infty(\Omega)}$: sup norm
- $\| \cdot \|_{L^p(\Omega)}$: usual Lesbegue space
- $\| \cdot \|_{W^{k,p}(\Omega)}$: usual Sobolev space
- $[L^p(\Omega), \| \cdot \|_{L^p(\Omega)}])$: space of functions belonging to $W^{k,p}(\Omega')$ for every $\Omega' \subset \subset \Omega$
- $W^{k,p}_0(\Omega)$: closure of $C_c^\infty(\Omega)$ in $W^{k,p}(\Omega)$
- $W^{-m,p}_0(\Omega)$: dual space of $W^{m,p}_0(\Omega)$ with $\frac{1}{p} + \frac{1}{p'} = 1$
- $BV(\Omega)$: functions with bounded variation in $\Omega$
### Operators

- $\mathcal{A}$: linear operator
- $\mathcal{A}^*$: formal adjoint operator of $\mathcal{A}$
- $\mathcal{A}$: realization of $\mathcal{A}$ in a Banach space $X$
- $D(\mathcal{A})$: the domain of $\mathcal{A}$
- $\rho(\mathcal{A})$: resolvent set of the linear operator $\mathcal{A}$
- $\sigma(\mathcal{A})$: spectrum of the linear operator $\mathcal{A}$
- $I$: identity operator
- $[\mathcal{A}, \mathcal{B}]$: the operator $\mathcal{A}\mathcal{B} - \mathcal{B}\mathcal{A}$ defined in $D(\mathcal{A}\mathcal{B}) \cap D(\mathcal{B}\mathcal{A})$

### Measure theory and BV functions

- $\mathcal{B}(X)$: $\sigma$- algebra of Borel subsets of a topological space $X$
- $[\mathcal{M}(X)]^m$: the $\mathbb{R}^n$-valued finite Radon measures on $X$
- $\mathcal{M}^+(X)$: the space of positive finite measures on $X$
- $\mathcal{L}^n$: Lebesgue measure in $\mathbb{R}^n$
- $\omega_n$: Lebesgue measure of $B(0, 1)$ in $\mathbb{R}^n$
- $\mathcal{H}^k$: $k$-dimensional Hausdorff measure
- $|E|$ or $\mathcal{L}^n(E)$: the Lebesgue measure of the set $E$
- $|\mu|$: total variation of the measure $\mu$
- $\mu|_E$: restriction of the measure $\mu$ to the set $E$
- $Du$: distributional derivative of $u$
- $\mathcal{P}(E, \Omega)$: perimeter of $E$ in $\Omega$
- $\mathcal{P}(E)$: perimeter of $E$ in $\mathbb{R}^n$
- $\nu_E$: generalized inner normal to $E$
- $E^t$: set of points of density $t$ of $E$
- $\mathcal{F}E, \partial^* E$: reduced and essential boundary of $E$