Uniqueness of the one-dimensional bounce problem as a generic property in $L^1([0,T];\mathbb{R})$.

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Sunto. - *Si prova l'esistenza di un sottoinsieme* M^*, *di seconda categoria in* $L^1([0,T];\mathbb{R})$, *tale che per ogni assegnata forza* f *in* M^* *il problema del rimbalzo unidimensionale ha unicità.*

Introduction.- In [1] we studied the problem of a material point moving on a straight line subject to a strength f depending on time and to a perfectly elastic bouncing law.

This problem consists in the following:

given $f \in L^1([0,T];\mathbb{R})$, $s < 0$ $b \in \mathbb{R}$ or $s = 0$, $b \leq 0$ (permissible data), find $u \leq 0$ such that it satisfies a lipschitz condition on $[0,T]$;

$$\int_0^T [u(t)\,\ddot{\phi}(t) - f(t)\phi(t)]\,dt \leq 0 \qquad \text{for every} \quad \phi \in C_0^\infty([0,T];\mathbb{R}^+);$$

for $u < 0$ one has $\displaystyle\int_0^T [u(t)\ddot{\phi}(t) - f(t)\phi(t)]\,dt = 0$ for every $\phi \in C_0^\infty([0,T];\mathbb{R})$;

for every $t \in]0,T[$ $\dot{u}^+(t)$ and $\dot{u}^-(t)$ exist; moreover $\dot{u}^+(0)$, $\dot{u}^-(T)$ exist and

$$\frac{1}{2}[\dot{u}^\pm(t)]^2 = \frac{1}{2}[\dot{u}^+(0)]^2 + \int_0^t f(\eta)\,\dot{u}(\eta)\,d\eta \qquad \text{for} \quad t \in [0,T];$$

$u(0) = s$, $\dot{u}^+(0) = b$ (energy conservation law).

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