2) The graphs $G = \{u,v,w\}$ with the edges $u \to v$, $u \to w$, $v \to w$ and $G' = \{q,r,s,t\}$ with edges $q \to r$, $q \to s$, $q \to t$, $r \to s$, $r \to t$, $s \to t$ are examples of almost complete graphs. Moreover, the sets $\{u,v,w\}$, $\{q,r,s,t\}$ are examples of totally headed (i.e. totally tailed) subsets. Their compatible orders are, respectively, $u < v < w$, $q < r < s < t$.

3) In the graphs $G = \{f,g,h\}$ with the edges $f \to g$, $g \to h$, $h \to f$ and $G' = \{\ell,m,n,p\}$ with the edges $\ell \to m$, $\ell \to n$, $m \to n$, $m \to p$, $n \to \ell$, $n \to p$, $p \to \ell$, $p \to m$ the sets $\{f,g,h\}$ and $\{\ell,m,n,p\}$ are examples of non-headed minimal (i.e. non-tailed minimal) subsets.

2) *Singularities of a regular function.*

PROPOSITION 7. - *Let* S *be a topological space,* G *a finite directed graph,* $f : S \to G$ *an o-regular function from* S *to* G *and* $X = \{v_1, v_2, \ldots, v_n\}$ *a non-headed subset of* G $(n \geq 2)$. *Then it holds:*

$$v_1^f \cap \overline{v_2^f} \cap \ldots \cap \overline{v_n^f} = \phi;$$

$$\overline{v_1^f} \cap v_2^f \cap \ldots \cap \overline{v_n^f} = \phi$$

....

$$\overline{v_1^f} \cap \ldots \cap \overline{v_{n-1}^f} \cap v_n^f = \phi$$

Proof. - Since X is a non-headed subset, there is no vertex v_i, which is a predecessor of all the other n-1 vertices. Then, for every $i = 1, \ldots, n$ let w_i be a vertex such that $v_i \not\to w_i$. From o-regularity of f it is $v_i \cap \overline{w_i} = \phi$. Since w_i is one of the vertices $v_1, \ldots, v_{i-1}, v_{i+1}, \ldots, v_n$, it follows $\overline{v_1^f} \cap \ldots \cap \overline{v_{i-1}^f} \cap v_i^f \cap \overline{v_{i+1}^f} \cap \ldots \cap \overline{v_n^f} = \phi$.

DEFINITION 5. - *Let* S *be a topological space,* G *a finite directed*

graph, f : S → G *an o-regular (resp.* o^**-regular) function from* S *to* G *and* $X = \{v_1, v_2, \ldots, v_n\}$ *a* n-*tuple of vertices of* G *with* $n \geq 2$.
Then X *is called a* singularity *of* f *or a* singular set *of* f *if:*

i) X *is non-headed (resp. non-tailed);*

ii) $\overline{V_1^f} \cap \overline{V_2^f} \cap \ldots \cap \overline{V_n^f} \neq \phi$.

Moreover, X *is called a* proper singularity *of* f *if i) is replaced by:*

i') X *is non-headed minimal (i.e. non-tailed minimal).*

Finally, the closed set $\overline{V_1^f} \cap \overline{V_2^f} \cap \ldots \cap \overline{V_n^f}$ *is called the* support *of the singularity.*

PROPOSITION 8. - *If* $X = \{v_1, v_2, \ldots, v_n\}$ *is a singularity of* f, *then* X *has an empty intersection with the image of its support, i.e.* $f(\overline{V_1^f} \cap \ldots \cap \overline{V_n^f}) \cap X =$

Proof. - It follows from Proposition 7.

REMARK. - Since every non-headed (non-tailed) subset of G includes a non-headed minimal (non-tailed minimal) subset of G, every singularity includes a proper singularity, Hence, every singular couple is a proper singularity.

DEFINITION 6. - *Let* S *be a topological space,* G *a finite directed graph and* f : S → G *an o-regular (resp.* o^**-regular) function from* S *to* G. *The function* f *is called* completely o-regular (*resp.* completely o^*-regular) *or simply* c.o-regular (*resp.* c.o^*-regular),*if there are no singularities of* f.

We note that Definitions 5,6 can be extended to undirected graphs. Then it follows:

PROPOSITION 9. - *Let* S *be a topological space and* G *a finite undirected graph. Then a strongly regular function* (see [5] , Definition 3) f : S → G *from* S *to* G *is also c.regular.*

Proof. - By definition of strongly regular function there is no singular couple of vertices. Besides, by Proposition 6, there does not exist any non-headed minimal n-tuple with $n > 2$, then there are no proper singularities of f. Hence, by Remark to Proposition 8, f is c. regular.

DEFINITION 7. - *Let* S *be a topological space,* S' *a subspace of* S, G *a finite directed graph,* G' *a subgraph of* G *and* $f : S,S' \to G,G'$ *a function from the pair* S,S' *to the pair* G,G'. *The function* f *is called* completely o-regular (*resp.* completely o^*-regular) *or simply* c.o-regular (*resp.* c.o^*-regular), *if both* $f : S \to G$ *and its restriction* $f' : S' \to G'$ *are c.o-regular* (*resp.* c.o^**-regular*).

REMARK. - If S'' is a subspace of S', G'' a subgraph of G including G', $f : S,S' \to G,G'$ a c.o-regular (resp. c.o^*-regular) function, then also the functions $f : S,S'' \to G,G'$, $f : S,S' \to G,G''$ and $f : S,S'' \to G,G''$ are c.o-regular (resp. c.o^*-regular).

PROPOSITION 10. - *Every strongly regular function from a pair of topological spaces* S,S' *to a pair of finite undirected graphs* G,G' *is also c.regular.*

3) *The first normalization theorem.*

PROPOSITION 11. - *Let* S *be a normal topological space,* G *a finite directed graph,* $f : S \to G$ *an o-regular function from* S *to* G *and* $X=\{v_1,\dots,v_n\}$ *a singularity of* f. *Then there exists an o-regular function* g *from* S *to* G, *which is o-homotopic to* f *and such that:*