

POINCARÉ RECURRENCE THEOREM FOR FINITELY ADDITIVE MEASURES

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SUMMARY.- In this paper we study the validity of Poincaré recurrence theorem for finitely additive measures.

§ 1.- DEFINITIONS AND PROBLEM

Let X be an arbitrary non empty point set, and $T : X \rightarrow X$ a transformation on X . If (X, \mathcal{A}, μ) is a charge space, i.e. , \mathcal{A} is a field of subsets of X and μ is a nonnegative charge (usually called finitely additive measure) the transformation T is called a measurable transformation if

$$(1.1) \quad \forall A \in \mathcal{A} : T^{-1}(A) \in \mathcal{A}$$

A measurable transformation T is said to be measure preserving if

$$(1.2) \quad \forall A \in \mathcal{A} : \mu(T^{-1}(A)) = \mu(A).$$

If T is a measure preserving transformation and $E \in \mathcal{A}$ then a point $x \in E$ is called recurrent if

$$\exists n \in \mathbb{N}^{(2)} \text{ such that } T^n x \in E$$

and x is called strongly recurrent if

$$T^n x \in E \quad \text{for infinitely many values of } n.$$

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(2) \mathbb{N} is the set $\{1, 2, 3, \dots\}$ of positive integers, $\mathbb{N}_0 = \{0, 1, 2, \dots\}$ and $\mathbb{Z} = \{\dots, -2, -1, 0, 1, 2, \dots\}$