COMPLEMENTARY DISTRIBUTIONS AND PONTRJAGIN CLASSES

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SUMMARY.- *From a necessary condition for the existence of $k(>2)$ complementary distributions on a manifold we deduce connections between Pontrjagin classes of distributions and of transversal bundles.*

Let M be a riemannian smooth orientable manifold of dimension n (even or odd) and suppose that M has k complementary (smooth) distributions of oriented n_i-planes $(i=1,\dots,k)$; i. e. for every point $p \in M$ the tangent space $T_p(M)$ can be decomposed into the direct sum of the subspaces $T_p^1,\dots,T_p^k$ of $T_p(M)$ where $\dim T_p^i = n_i$ (and hence $n_1+\dots+n_k = n$).

Then one says that M admits an "almost product" or "almost multiproduct" structure.

In a paper of 1969 ([3]) one of the present authors showed that the vanishing of certain Pontrjagin classes is a necessary condition for the existence of k complementary distributions on M. After a review of these results we prove the following

<u>THEOREM A</u>

Let M be a riemannian smooth orientable manifold of dimension n. Furthermore let M have k complementary distributions $E_i \subset TM$ and let the bundle $Q_i = TM/E_i$ have fibre of dimension q_i with

$$q_i = n - n_i \qquad \text{and} \qquad n_1+\dots+n_k = n.$$

Finally let $p_r(E) \in H^{4r}(M;\mathbb{R})$ denote the r-th real Pontrjagin class of the bundle E. Then if

$$P_h(Q_i)P_s(E_i) = 0 \qquad \forall h,s \geq 1 \quad \text{and} \quad h+s = r$$

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one has

$$P_r(Q_i) = 0 \qquad 2r > \max(n_1,\dots,n_k).$$

Using Bott's "Vanishing Theorem" one can, in a certain sense invert the preceding result obtaining

THEOREM B

Let M *be a riemannian smooth orientable manifold of dimension* n. *Furthermore let* M *have* k *complementary distributions* $E_i \subset TM$ *and let the bundle* $Q_i = TM/E_i$ *have fibre of dimension* q_i *with*

$$q_i = n - n_i \qquad \text{and} \qquad n_1+\dots+n_k = n.$$

If Q_i *is integrable, then for* $2r > \max(n_1,\dots,n_k,4q_i)$

$$P_h(Q_i)p_s(E_i) = 0 \qquad \forall h,s \geq 1 \qquad \text{and} \qquad h+s = r$$

holds.

In order to give a self-contained presentation we recall the necessary preliminaries.

1. Preliminaries

Let M be a smooth (paracompact) manifold and let E be a vector $\mathbb{R}^q$-bundle on M. As is well known, the total (real) Pontrjagin class p(E) of E is defined by

$$p(E) = 1+p_1(E)+\dots+p_{\left[\frac{q}{2}\right]}(E) = \left[\det\left(I-\frac{1}{2\pi}\,\Omega\right)\right]$$

where Ω is the curvature of an arbitrary connection on E and $p_r(E)\in H^{4r}(M;\mathbb{R})$