strictsolution. Mollifyng, in our case, means replacing F with

(3) $F_\varepsilon(f)(x,v,w) = q[K_\varepsilon(J_1 f)\cdot(J_2 f) - f\, K_\varepsilon J_3 J_1 f]$

where

(4) $(K_\varepsilon f\,)(x,v,w) = \int_x^{+\infty} k_\varepsilon(x'-x) f(x',v,w)\,dx'$

and

(5) $k_\varepsilon \in L^\infty(0,+\infty);\ k_\varepsilon(y) \geq 0;\ k_\varepsilon(y) = 0$ if $y \notin (0,\varepsilon);\ \int_0^\infty k_\varepsilon(y)dy = 1$.

The aim of this work is to study the original problem, i.e.(1) , in L^1 and to find the connexion between the solution $u(t)$ of (1) and the solution $u_\varepsilon(t)$ of the mollified problem.

Precisely we prove that if $u_o \in L^1 \cap L^\infty$ then (1) has a unique local "mild" solution, i.e. the integral version of (1) has a unique local solution. If $[0,\bar{t}]$ is the existence time interval of such solution $u(t)$, we have

$$\lim_{\varepsilon\to 0+} ||u_\varepsilon(t) - u(t)|| = 0$$

uniformly respect to t in $[0,\bar{t}]$. $||\cdot||$ is the usual norm in L^1.

We shall use the well-known results of linear semigroup theory for which we refer to [4] Chapter 9. For the results on the non linear evolution equations (in particular for semi-linear ones) we refer to [3],[6] and [8].

2. THE ABSTRACT PROBLEM.

Denote $X = \{f = f(x,v,w);\ f \in L^1(R^2 x\bar{V})\}$ and $X_o = \{f; f\in X,\ f(x,v,x) = 0$ a.e. if $v \notin V\}$ X_o is a closed subspace of X and we use it to get the third relation in (1).

Define

$$(6)\qquad \begin{cases} A_1 f = v\, f_x - \frac{w-v}{T} f_v + \frac{1}{T} f \\ D(A_1) = \{f \in X_o;\ \exists f_x, f_v,\ v\, f_x + \frac{w-v}{T} f_v \in X_o\} \end{cases}$$

where $f_x = \frac{\partial f}{\partial x}$, $f_v = \frac{\partial f}{\partial v}$ are distributional derivatives.

If we consider the linear homogeneous problem connected with (1) and use the method of characteristics, we have

(7) $u(x,v,w;t) = \exp\frac{t}{T} u_\circ (\bar{x}(t), \bar{v}(t),w)$

where

$$\bar{x}(t) = \bar{x}(x,v,w;t) = x-wt+(w-v)T(\exp\frac{t}{T} - 1)$$

$$\bar{v}(t) = \bar{v}(x,w;t) = w -(w-v) \exp\frac{t}{T} .$$

If we denote

(8) $[Z(t)f] (x,v,w) = \exp\frac{t}{T} f(\bar{x}(t),\bar{v}(t),w) \qquad t\in R$

then we have as in [1] .

Lemma (1) . (a) $\{Z(t);\ t\in R\} \subset \mathcal{B}(X)$; (b) $||Z(t)f||=||f||$ for $f\in X$; (c) $\{Z(t);\ t\in R\}$ is a group.

If $Z_\circ(t)$ is the restriction of $Z(t)$ to the subspace $X_\circ$, $Z_\circ(t)$ maps $X_\circ$ into itself for $t \geq 0$ and we have

Lemma (2). (a) $\{Z_\circ(t);\ t \geq 0\} \subset \mathcal{B}(X_\circ)$ and is a semigroup (b) $||Z_\circ(t)f|| =||f||$, for $f\in X_\circ$; (c) $Z_\circ(t)$ is strougly continuous in t for $t\geq 0$

If we denote by $A_\circ$ the infinitesimal generator of $Z_\circ(t)$ ([4]) Chapeter 9) it is easy to prove that A_1 is the restriction of $A_\circ$ to the set $D(A_1) \subset D(A_\circ)$ and that $Z_\circ(t)[D(A_1)] \subset D(A_1)$ (see [1]).

The natural domain of F is

$$D(F) = \{f : f \in X_\circ \ ,\ F(f) \in X_\circ$$

and because this is not the whole $X_\circ$ it is useful to introduce the following sets

$$X_\infty = L^\infty(R^2 \times \bar{V}) \quad \text{and}$$

$$s(r) = \{f : f \in X_\circ \cap X_\infty \; ; \; ||f||_\infty \leq r\}$$

where r is a positive constant and

$$||f||_\infty = \text{ess sup} \, \{|f(x,v,x)| \; : \; (x,v,w) \in R^2 \times \bar{V}\}.$$

We have

<u>Lemma</u> (3). (a) $X_\circ \cap X_\infty \subset D(F)$; (b) $||F(f)|| \leq q \, d||f|| \; ||f||_\infty$ if $f \in X_\circ \cap X_\infty$, where $d = (v_2 - v_1)^3$; (c) $s(r)$ is closed in $X_\circ$.

<u>PROOF</u>.

(a),(b): If $f \in X_\circ \cap X_\infty$ and $v \notin V$ then $F(f)(x,v,w) = 0$ a.e.

(c) If we suppose that $f_n \in s(r)$, $||f_n - f \to 0$ as $n \to \infty$, but $f \notin s(r)$, then we obtain a contradiction $\blacksquare$

<u>Remark</u> (1). It is useful to introduce $s(r)$ because $X_\circ \cap X_\infty$ is not closed in $X_\circ$.

With the preceding notation, the problem (1) assumes the abstract form

$$\text{(9)} \qquad \frac{du}{dt} = A_\circ u(t) + F(u(t)) \quad t>0; \; \lim_{t \to 0+} u(t) = u_\circ \in D(A_\circ)$$

where $u : [0,+\infty) \to X_\circ$ and $\frac{d}{dt}$ is a strong derivative . The integral version of the problem (9) is

$$\text{(10)} \qquad u(t) = u_1(t) + \int_0^t Z_\circ(t-s) F(u(s)) \, ds \qquad t > 0$$

where

$$\text{(11)} \qquad u_1(t) = Z_\circ(t) u_\circ$$

and from (b) of Lemma (2) $||u_1(t)|| = ||u_\circ||$

Every solution of (9) is also a solution of (10), but the converse is not generally true. For this reason every solution of (10) is said to be a "mild" solution of (9) (see [3]).