1. Introduction.

In this note we are examining again the model proposed by S. Paveri-Fontana in [5] and studied in various papers, in particular [1] and [2].

The problem of evolution, connected with such a model is

$$
(1)\quad \begin{cases}
\left(\frac{\partial}{\partial t} + v\frac{\partial}{\partial x}\right)u + \frac{\partial}{\partial v}\left(\frac{w-v}{T}u\right) = F(u) & x\varepsilon R; t>0;\ v,w\varepsilon(v_1,v_2) = V \\
 & (0\le v_1<v_2<+\infty) \\
u(x,v,w;0) = u_o(x,v,w) & x\varepsilon R; v,w\varepsilon \bar{V} \\
u(x,v,w;t) = 0 & t\ge 0; x\varepsilon R; v,w\notin \bar{V}
\end{cases}
$$

where, if $f = f(x,v,w)$,

$$(2)\ F(f) = q[(J_1f)\cdot(J_2f) - f\,J_3J_1f] \qquad q \text{ constant in } [0,1]$$

$$J_1f = \int_{v_1}^{v_2} f(x,v,w')\,dw'$$

$$J_2f = \int_{v}^{v_2} (v'-v)f(x,v',w)\,dv'$$

$$J_3f = \int_{v'}^{v} (v-v')f(x,v',w)\,dv' .$$

The meaning of the symbols can be found in [5], [1] and [2]. In [2], the problem (1) is studied when u belongs to the space of the uniformly continuous and bounded functions $X = U.C.B.(R^3)$ and the existence and uniqueness of the local (in time) strict solution is proved. Noted that $u = u(x,v,w;t)$ is a car density and that

$$\int_{-\infty}^{+\infty} dx \int_{v_1}^{v_2} dv \int_{v_1}^{v_2} u(x,v,w;t)dt$$

gives the total number of cars on the motorway at the time t, the most natural space to study the problem (1) is $L^1(R^3)$. In [1], mollifying the non-linear part of the equation, i.e. F, we obtained the existence and uniqueness of the global strictsolution. Mollifyng, in our case, means replacing F with

(3) $F_\varepsilon(f)(x,v,w) = q[K_\varepsilon(J_1 f)\cdot(J_2 f) - f\, K_\varepsilon J_3 J_1 f]$

where

(4) $(K_\varepsilon f)(x,v,w) = \int_x^{+\infty} k_\varepsilon(x'-x) f(x',v,w)\,dx'$

and

(5) $k_\varepsilon \in L^\infty(0,+\infty)$; $k_\varepsilon(y) \geq 0$; $k_\varepsilon(y) = 0$ if $y \notin (0,\varepsilon)$; $\int_0^\infty k_\varepsilon(y)dy = 1$.

The aim of this work is to study the original problem, i.e.(1) , in L^1 and to find the connexion between the solution $u(t)$ of (1) and the solution $u_\varepsilon(t)$ of the mollified problem.

Precisely we prove that if $u_o \in L^1 \cap L^\infty$ then (1) has a unique local "mild" solution, i.e. the integral version of (1) has a unique local solution. If $[0,\bar{t}]$ is the existence time interval of such solution $u(t)$, we have

$$\lim_{\varepsilon\to o+} ||u_\varepsilon(t) - u(t)|| = 0$$

uniformly respect to t in $[0,\bar{t}]$. $||\cdot||$ is the usual norm in L^1.

We shall use the well-known results of linear semigroup theory for which we refer to [4] Chapter 9. For the results on the non linear evolution equations (in particular for semi-linear ones) we refer to [3], [6] and [8].

2. THE ABSTRACT PROBLEM.

Denote $X = \{f = f(x,v,w);\ f \in L^1(R^2 x\bar{V})\}$ and $X_o = \{f; f\in X,\ f(x,v,x) = 0$ a.e. if $v \notin V\}$ X_o is a closed subspace of X and we use it to get the third relation in (1).

Define

$$(6)\qquad \begin{cases} A_1 f = v\, f_x - \frac{w-v}{T} f_v + \frac{1}{T} f \\ D(A_1) = \{f\in X_o;\ \exists f_x, f_v,\ v\, f_x + \frac{w-v}{T} f_v \in X_o\} \end{cases}$$