mapped into a point closure. If $B = A$ we have nothing to prove. Now we suppose that $B \neq A$ and

$$(m) \qquad \left(\bigcup_{s\in S} f(s)\right) \cap (A-B) = \emptyset \quad ^{(3)} .$$

Then we consider the function f' that maps every $s \in S$ into the set $f(s) \cup A_s$, where $A_s = \{y \in A - B : y \leq s\}$. Clearly f' is an injective function and for every $s_1, s_2 \in S$ $s_1 \leq s_2$ iff $f(s_1) \cup A_{s_1} \subseteq f(s_2) \cup A_{s_2}$. Moreover if $s_1, \ldots, s_n$ are arbitrary elements of S then

$$\bigcup_{i=1}^{n} (f(s_i) \cup A_{s_i}) = \left(\bigcup_{i=1}^{n} f(s_i)\right) \cup \left(\bigcup_{i=1}^{n} A_{s_i}\right) .$$

Now let Z be the set of all upper bounds of $\{s_1, \ldots, s_n\}$ in $(S, \leq)$.

We want to prove that $\bigcap_{z\in Z} (f(z) \cup A_z) = \bigcup_{i=1}^{n} (f(s_i) \cup A_{s_i}) = \bigcup_{i=1}^{n} (f(s_i)) \cup \left(\bigcup_{i=1}^{n} A_i\right)$.

As a consequence of condition (m) $\bigcap_{z\in Z} (f(z) \cup A_z) = \left(\bigcap_{z\in Z} f(z)\right) \cup \left(\bigcap_{z\in Z} A_z\right)$;

moreover we already know that $\bigcap_{z\in Z} f(z) = \bigcup_{i=1}^{n} f(s_i)$ and $\bigcap_{z\in Z} A_z \supseteq \bigcup_{i=1}^{n} A_{s_i}$; then it is sufficient to prove that $\bigcap_{z\in Z} A_z \subseteq \bigcup_{i=1}^{n} A_{s_i}$.

Now if $x \in \bigcap_{z\in Z} A_z$ then x is a v-prime element of $(S, \leq)$ such that $x \leq z$ for every $z \in Z$. Moreover Z is the set of all upper bounds of $\{s_1, \ldots, s_n\}$, then as a consequence of the definition of v-prime element $x \leq s_i$ for some $i \in \{1, \ldots, n\}$, thus $x \in \bigcup_{i=1}^{n} A_{s_i}$ and hence $\bigcap_{z\in Z} A_z \subseteq \bigcup_{i=1}^{n} A_{s_i}$.

From this the enounced assertion follows.

REFERENCE

[1] D.DRAKE and W.J.THRON "On the representations of an abstract lattice as the family of closed sets of a topological space". Trans. of Amer. Math. Soc. 120(1965), 57-71.

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(3) The case $\left(\bigcup_{s\in S} f(s)\right) \cap (A-B) \neq \emptyset$ can easily be reconducted to condition (m).