U-proper set representation \(((f(S), \mathcal{S}, f)\) of \((S, \prec)\) is a point closure.

\section*{2. A BRIEF REVIEW OF PREORDERED SETS.}

Let \(S\) be a set and \(\prec\) a preorder relation for \(S\) (i.e. \(\prec\) exhibits the transitive and reflexive properties). All the most important notions about a poset can be extended to a preordered set (e.g. upper bound, lower bound, maximum, minimum, \(\sqcup\) u.b., \(\sqcap\) l.b., etc.); thus a right tail of a preordered set \((S, \prec)\) will be every \(Y \subseteq S\) such that \(\forall x, y \in S : x \prec Y\) and \(x \prec y \implies y \in Y\).

We observe that if \(Y_1 \subseteq S\) then the set \(r(Y_1) = \{x \in S : x\) is an upper bound of \(Y_1\}\) is a right tail of \((S, \prec)\); in particular the principal filter \(r(y) = r(\{y\})\) generated by \(y \in S\) is a right tail of \((S, \prec)\). Moreover

\begin{itemize}
  \item[(1i)] \(r(X) = \bigcap_{x \in X} r(x)\) and if \(X\) is a right tail then \(X = \bigcup_{x \in X} r(x)\).
  \item[(ii)] \(r(S) = \emptyset\) and \(r(\emptyset) = S\).
\end{itemize}

Now let \(\mathcal{E}\) be a subset of \(\mathcal{P}(S)\) (the power set of \(S\)), \(x\) an element of \(S\) and \(\mathcal{E}_x = \{X \in \mathcal{E} : x \in X\}\). Then we define, for every \(x, y \in S\)

\begin{itemize}
  \item[(j)] \(x \prec y(\mathcal{E}) \iff \mathcal{E}_x \subseteq \mathcal{E}_y\).
\end{itemize}

Clearly the defined relation is a preorder relation. Moreover if \(\mathcal{E}'\) is the set of set complements of the elements of \(\mathcal{E}\) it follows, since

\begin{itemize}
  \item[(jj)] \(x \prec y(\mathcal{E}) \iff y \prec x(\mathcal{E}')\).
\end{itemize}

\(\square\)

We shall prove that there exists at least a \(U\)-proper set representation of \((S, \prec)\).
One can easily verify that every element of $\mathcal{C}$ is a right tail of $S$ preordered with respect to relation defined in (j). On the other hand if $(S, \leq)$ is a preordered set and $\mathcal{Q}$ is the set of the right tails of $(S, \leq)$ then for every $x, y \in S$ the following properties hold

\[(e) \quad x \leq y \iff x \leq y(\mathcal{Q}) \text{ according to (j), and } x \leq y \iff x \leq y(\mathcal{Q}_0),\]

where $\mathcal{Q}_0$ is the set of the principal filters of $(S, \leq)$ generated by an element of $S$. And hence, as a consequence of (jj), the following property holds:

\[(ee) \quad x \leq y \iff y \leq x(\mathcal{Q}_0),\]

where $\mathcal{Q}_0$ is the set of all set complements in $S$ of the elements of $\mathcal{Q}_0$.

Now we can give the following

**THEOREM 1.** Let $\mathcal{C} \subseteq \mathcal{P}(S)$ be, $Y \subseteq \mathcal{C}$ and $y \in Y$. Then $Y$ is a point closure in $\mathcal{C}$ with respect to $y$ iff $y$ is maximum in $Y$ with respect to relation defined in (j).

**PROOF.** In fact: $y$ is maximum in $Y \iff \forall x \in Y: y \subseteq x \iff \forall x \in Y, \forall z \in C_y: z \subseteq x \iff \forall z \in C_y: z \subseteq Y$.

Q.E.D.

**N. 3. A CHARACTERIZATION OF V-PRIME AND STRONGLY V-PRIME ELEMENTS OF A POSET.**

Henceforth let $(S, \leq)$ be a poset. Then we can consider the function $g: S \to \mathcal{P}(S)$ mapping an element $x \in S$ into the set $g(x) = \{y \in S: x \leq y\} = S - r(x)$. Clearly $f$ is an injective function; moreover $\forall x, y \in S \quad x \leq y \iff g(x) \subseteq g(y)$.