Now let us go back to (2.23). In view of (2.25) we have

\[(2.27) \quad A e^{\lambda(\xi - \xi_o)} = \frac{e^{-u}}{\left[1-(1-e^{-u})^{\frac{1}{2}}\right]^2},\]

where

\[(2.28) \quad A = \frac{e^{-u_o}}{\left[1-(1-e^{-u_o})^{\frac{1}{2}}\right]^2}.\]

Finally, by means of simple calculations, (2.27) allows us to obtain the following expression of \(u\) in terms of \(\xi\):

\[(2.29) \quad u = \ell n \frac{\left[1 + 2Ae^{\lambda(\xi - \xi_o)}\right]^2}{4Ae^{\lambda(\xi - \xi_o)}\left[1+e^{\lambda(\xi - \xi_o)}\right]}.\]

3. SERIES REPRESENTATION OF \(\Psi(\alpha, \beta, \gamma; x)\) IN TERMS OF INCOMPLETE GAMMA FUNCTIONS.

Let us consider the binomial expansion

\[(3.1) \quad (1 - \frac{e^{-t}}{t^\beta})^\gamma = 1 + \frac{1}{\Gamma(-\gamma)} \sum_{n=1}^{\infty} \frac{\Gamma(n-\gamma)}{n!} \frac{e^{-nt}}{t^n^\beta},\]

where the series on the right is uniformly convergent for \(t \geq x\), \(x\) being any fixed number such that \(e^{-x} < x^\beta\).

We can thus write

\[(3.2) \quad \Psi(\alpha, \beta, \gamma; x) = -\frac{1}{\Gamma(-\gamma)} \sum_{n=1}^{\infty} \frac{\Gamma(n-\gamma)}{n!} \int_x^{\infty} dt \frac{x^{-n\beta-1}e^{-nt}}{t^n}.\]
Therefore, the use of the following integral representation for the incomplete $\Gamma$-function \[14\]

$$
\mu^\nu \Gamma(\nu, \mu x) = \int_0^\infty t^{\nu-1} e^{-\mu t} \, dt
$$

for $x > 0$ and $\text{Re} \mu > 0$, leads us to the expression

\[3.3\] \[ \psi(\alpha, \beta, \gamma; x) = -\frac{1}{\Gamma(-\gamma)} \sum_{n=1}^{\infty} \frac{\Gamma(n-\gamma)}{n!} \frac{\Gamma(n\beta-\alpha)}{\Gamma(n\beta)} \Gamma(\alpha-n\beta, nx). \]

Obviously, in the special case $\gamma = m$, where $m$ is a positive integer, the expression (3.3) reduces to a finite sum of incomplete $\Gamma$-functions, specifically:

\[3.4\] \[ \psi(\alpha, \beta, m; x) = -\sum_{n=1}^{m} (-1)^n \binom{m}{n} \frac{\Gamma(n\beta-\alpha)}{\Gamma(n\beta)} \Gamma(\alpha-n\beta, nx). \]

4. A RECURRENCE RELATION.

The following recurrence relation holds:

\[4.1\] \[ (1 - \frac{\gamma \beta}{\alpha}) \psi(\alpha, \beta, \gamma; x) = -\frac{1}{\alpha} x^\alpha \left[ 1 - (1 - \frac{e^{-x}}{x^\beta})^\gamma \right] + \]

+ $\frac{1}{\alpha} \left[ \psi(\alpha+1, \beta, \gamma; x) - \psi(\alpha+1, \beta, \gamma-1; x) - \beta \psi(\alpha, \beta, \gamma-1; x) \right],$

for $\alpha \neq 0$.

In fact, from (1.6) we can write

$$
\psi(\alpha, \beta, \gamma; x) = \int_x^\infty t^{\alpha-1} \left[ 1 - (1 - \frac{e^{-t}}{t^\beta})^\gamma \right] \, dt
$$

which yields