PROOF. If A is NP-complete in the strong sense there must exist a polynomial q such that the following set $Q=\{\langle x,k\rangle \mid \langle x,k\rangle \in A^c,\ MAX(x) \leq q(|x|)\}$ is NP-complete. Let us consider now the set

$$Q' = \{\langle x,k\rangle \mid \langle x,k\rangle \in A^c, MAX(x) \leq q(|x|), \tilde{m}(x) \leq k \leq \tilde{m}(x)+p(MAX(x),|x|)\}$$

As $Q \supseteq Q'$ in order to prove that $Q \equiv Q'$ it is sufficient to prove that

$$Q-Q' \equiv \{\langle x,k\rangle \mid \langle x,k\rangle \in A^c, MAX(x) \leq q(|x|),\ k \geq \tilde{m}(x)+p(MAX(x),|x|)\}$$

is the empty set. In fact given $\langle x,k\rangle$, with $k \geq \tilde{m}(x)+p(MAX(x),|x|)$, we have by hypothesis $k \geq \tilde{m}(x)+p(MAX(x),|x|) \geq m^*(x)$ and therefore $\langle x,k\rangle \notin A^c$. Let us consider now

$$Q'' = \{\langle x,k\rangle \mid \langle x,k\rangle \in A^c,\ \tilde{m}(x) \leq k \leq \tilde{m}(x)+p(q(|x|,|x|)\}$$

Clearly Q" is NP-complete and hence A is weakly rigid.

QED

5. CONCLUSIONS

In this paper we have shown that there exist close relations among different approaches to the classification of NP-complete optimization problems, giving also new results on the type of possible reductions among problems belonging to different classes. On the other side, it was proven that, violating some conditions, comparisons among different concepts do not hold any more.

Therefore we believe that, in the whole, our results are a useful contribution for a better understanding of properties of NPCO problems. We think that in order to provide meaningful characterizations of NOCO problems it is necessary to find the suitable level of abstraction because if a too general point of view is taken NPCO problems appear to be hardly distinguishable while if too many details are taken into consideration it is difficult to grasp similarities among different problems. The results stated in this paper are, as we feel, at the right level. For the same reason we would like to broaden our considerations and results to other approaches which stand at the same level of abstraction. In Ausiello, D'Atri, Protasi (1977) a distinction was introduced between convex and non convex problems (a problem is said to be convex if, for every integer k between the worst and the best solution, there is, at least, an approximate solution of measure k). It is interesting to observe that many examples show that the property of being non convex is related to the approximation properties of the problems.