the decision time is bounded by a polynomial in $|x|$ and $\bar{k}$.

QED

Since no example is known of a problem which is strongly simple and not p-simple no application of theorem 4 can be provided which is different from the application given at the end of theorem 3.

As a conclusion of this paragraph we may observe that the results provided insofar have a twofold implication. On one side they can be used in order to characterize the computational complexity of one problem with respect to the given definitions, on the other side they establish conditions on the type of reductions that can be found among problems belonging to different classes, such as those discussed at the end of theorem 2 and theorem 3. As a further example we may observe that in the case of the reduction from PARTITION to MAX-CUT the existence of a much more succint reduction than the one given by Karp is ensured by noting that the first problem is strongly simple while the second is weakly rigid.

4. STRONG NP-COMPLETENESS AND ITS RELATION TO RIGIDITY

In the preceding paragraph we have seen that in some cases the characterization of a problem B that is not fully approximable comes out of the fact that we can reduce an NP-complete combinatorial problem A^c into a subset of B^c in which the measure is bounded by a polynomial.Garey and Johnson give another way of considering subsets of the set INPUT of a problem to study the different characteristics of NPCO problems. Their paper (1978) is an attempt to understand the different roles that numbers play in NPCO problems. Let

PROOF. If A is NP-complete in the strong sense there must exist a polynomial q such that the following set $Q = \{\langle x,k \rangle | \langle x,k \rangle \in A^c,\ MAX(x) \leq q(|x|)\}$ is NP-complete. Let us consider now the set

$Q' = \{\langle x,k \rangle | \langle x,k \rangle \in A^c, MAX(x) \leq q(|x|), \tilde{m}(x) \leq k \leq \tilde{m}(x) + p(MAX(x), |x|)\}$

As $Q \supseteq Q'$ in order to prove that $Q \equiv Q'$ it is sufficient to prove that

$Q - Q' \equiv \{\langle x,k \rangle | \langle x,k \rangle \in A^c, MAX(x) \leq q(|x|),\ k \geq \tilde{m}(x) + p(MAX(x), |x|)\}$

is the empty set. In fact given $\langle x,k \rangle$, with $k \geq \tilde{m}(x) + p(MAX(x), |x|)$, we have by hypothesis $k \geq \tilde{m}(x) + p(MAX(x), |x|) \geq m^*(x)$ and therefore $\langle x,k \rangle \notin A^c$. Let us consider now

$Q'' = \{\langle x,k \rangle | \langle x,k \rangle \in A^c,\ \tilde{m}(x) \leq k \leq \tilde{m}(x) + p(q(|x|, |x|)\}$

Clearly Q" is NP-complete and hence A is weakly rigid.

QED

5. CONCLUSIONS

In this paper we have shown that there exist close relations among different approaches to the classification of NP-complete optimization problems, giving also new results on the type of possible reductions among problems belonging to different classes. On the other side, it was proven that, violating some conditions, comparisons among different concepts do not hold any more.

Therefore we believe that, in the whole, our results are a useful contribution for a better understanding of properties of NPCO problems. We think that in order to provide meaningful characterizations of NOCO problems it is necessary to find the suitable level of abstraction because