1. INTRODUCTION

Various results on the properties of NP-complete optimization problems and on the characterization of these problems either with respect to their approximation properties or with respect to their combinatorial structure have been presented in the literature.

In particular we have considered the approaches given by Paz and Moran (1977) and by Garey and Johnson (1978) because of the interest of their results.

Paz and Moran introduce a classification of NP complete optimization problems based on the fact that considering only those inputs of the problem whose measure is bounded by an integer, it is possible to divide all the problems in different classes as regards their computational complexity (rigid, simple and p-simple problems). Furthermore these classes are then related to the approximability properties of the problem.

Under a different approach Garey and Johnson give another characterization which is based on the concept of strong NP-complete problem (limiting ourselves to those inputs,whose "value" is bounded by a polynomial in the length of the input, we still obtain an NP-complete problem) and of pseudopolynomial algorithm (an algorithm which is polynomial in the length of the input and in the magnitude of the greatest number occurring in the istance). Also in this case very interesting relations among these concepts and approximation properties are stated.

These papers, are, without any doubt, very interesting and new results of remarkable importance have been captured. Nevertheless, it seems to lack an attempt of organizing all these results in a unified framework as general as possible.

Furthermore any effort of comparison among different approaches has not been attempted.

The aim of our paper is therefore a first step in this direction. Starting from the observation that, intuitively, there is a similarity among some of the consequences of Paz and Moran, Garey and Johnson approaches, we have introduced a formal framework in which it is possible to establish clear connections among different concepts of the two approaches, at least under restricted but reasonable hypotheses. So, for istance, we have established under what conditions a pseudopolynomial problem is p-simple and viceversa. Beside this, our point of view allowed to derive some new consequences both concerning the classification of problems and the characterization of reductions that exist among different problems. We have stated what conditions must be verified to have a polynomial reduction from a rigid problem to a simple problem, from a p-simple problem to a p-simple problem and so on. Finally it seems interesting to us that some of these results can be interpreted as a formalization of facts that are used in practice when studying the solution of a particular problem, such as, for example, the fact that a problem with polynomially bounded objective function cannot be fully approximated.

In particular, in §.2 we give the basic terminology and notation. In §.3 we very briefly summarize the Paz and Moran approach with a slightly different formulation, giving new results such as those above stated concerning the characterization of reductions among problems belonging to different classes. In §.4 after recalling the main definitions and results of the Garey and Johnson approach, we establish under what conditions the results of these two approaches can be compared, eventually exhibiting some examples which show that violating the conditions, the two approaches lead to different conclusions in the classification of NP-complete problems.

2. BASIC CONCEPTS AND TERMINOLOGY

In order to establish a formal ground for the study of the properties of optimization problems we first give an abstract notion of optimization problem which is broad enough to include most common problems of this kind. Following the literature (Johnson (1973), we consider an *NP-optimization problem* to be characterized by a polynomially decidable set INPUT of instances, a polynomially decidable set OUTPUT of possible solutions, a mapping SOL:INPUT → P(OUTPUT) which, given any instance x of the problem, in polynomial time nondeterministically provides the *approximate solutions* of x and a mapping m:OUTPUT → Z (where Z is the set of relative integers) which in polynomial time provides the *measure* of an approximate solution (if A is a maximization problem) or its opposite (if A is a minimization problem). Note that in this way we allow a uniform approach to both maximization and minimization problems.

Since we are interested in studying those optimization problems which are "associated" to NP-complete recognition problems we restrict ourselves to considering a particular class of NP-complete problems:

DEFINITION 1. Let A be an NP optimization problem. The *combinatorial problem associated* to A is the set