Introduction

The tangent and cotangent spaces of a bundle are well known. But their affine structure is not sufficiently analised. We study deeply this structure, showing its fundamental role in many classical operations, suggesting new points of view, which we want to use in further works on Analytical Mechanics and Field Theory.

Let $\eta \equiv (E,p,M)$ be a bundle. We show that the tangent (cotangent) space $TE(T^*E)$ has an affine structure on the horizontal space $hTE \equiv E \times_M TM$, (vertical space, $\nu T^*E \equiv T^*E/hT^*E$) besides the vector structure on E(§1,2). Specializing the previous results to $E \equiv TM$ and $E \equiv T^*M$, we get a systematical table of canonical structures, in particular we see the exchange and symplectic isomophisms as pull-back maps (§3).
We give an intrinsical definition and we find an explicit intrinsical expression of the Lie derivative of a tensor, by means of the tangent functor (§4).

We define a pseudo-connection on η identifying each affine fiber of TE with its vector space, or, equivalently, choicing a "zero" on each affine fiber. Then we get immediately an affine structure on the space of all connections.We define the linear connections, requiring the previous identifications to be bilinear on the horizontal tangent space.
We get a functional construction of the tensor product of two linear connections and of the dual of a linear connection. In the case $E \equiv TM$ we can explain the classical notions by the previous results (§5).

In the following all manifolds and maps are C^∞. We leave to the reader the coordinate expression of formulas and the proof of some proposition.