AN N-DIMENSIONAL FUNCTION - ONLY CODE FOR NON-LINEAR UNCONSTRAINED OPTIMIZATION

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1. *Introduction.* -

The present report documents a code, compiled in the two versions OTLSSS and OTLSSD, for minimizing n-dimensional functions.

This routine is to be inserted in a library which will be provided from the CNR, SOFMAT Project, to solve a wide range of mathematical and statistical problems arising in a variety of fields such as applied mathematics, physics, chemistry, engineering, biology, economics, manageral science, market research, governement, agricultural and medical research.

Such library will be available in FORTRAN language for minicomputers, namely for PDP 11/40. It will cater for both the novice and the experienced programmer, therefore the documentation of all routines must be comprehensive, detailed anc clear. Moreover the selection and the implementation of the algorithms and the choice of the test problems must reflect the aim of the library which tends to possess efficiency, usefulness, accuracy and reliability.

2. *Routine document.* -

The two codes OTLSSS and OTLSSD, written in FORTRAN language for the PDP 11/40 computer, are two versions of the same program respectively compiled in single and in double precision. This program has been developed to solve the problem of non-linear uncostrained optimisation having the following mathematical description

$$\min_{x \in R^n} F(x)$$

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The first three characters OTL refer to the field of unconstrained optimization, the fourth character S mentions the used Sutti's method, the fifth S indicates that this one is the second implementation of Sutti's method, and the final S and D distinguish the version in single precision from the version in double precision. OTLSSS and OTLSSD and the related subroutines differ only for same declarative stetements and for same library functions.

OTLSSS and OTLSSD read and print the following input parameters: dimension of the variable space, initial approximation of the minimizer, stopping tolerances, initial step lenght of the line search, maximum allowed number of function evaluations. Moreover these routines read the index of printing, then they call respectively the subroutines CNS and CNSD.

CNS and CNSD search for a minimum of a n-dimensional function by the Sutti's method, using function values only (1). This method is intended for quadratic, strictly convex and non-convex functions (1,2,3). It computes a sequence of points of descent by moving along sets of n linearly normalized independent directions. The initial set, consisting of the n coordinate axes, is modified in order to build mutually coniugate directions with respect to the hessian matrix of a quadratic objective function. CNS calls the subroutines SEARCH and CALFUN and CNSD calls SEARD and CALFUD. SEARCH and SEARD search for a minimum of an one-dimensional function by a method using function values only, which is based on quadratic interpolation (3). The method computes a point set bracketing the minimum of the objective function along the search direction and sets the position of the minimum in the vertex of the interpolating parabole. Safeguards to avoid spurious stationary points are provided. SEARCH and SEARD call respectively CALFUN and CALFUD.

CALFUN and CALFUD compute the function value in a required point. These subroutines must be supplied from the user.

The argument lists are the following:

```
SUBROUTINE CNS (XA,N,F,DIR,EPS,EPS1,EPS3,EPS4,IFMAX,XMU,IPRINT)
SUBROUTINE CNSD(XA,N,F,DIR,EPS,EPS1,EPS3,EPS4,IFMAX,XMU,IPRINT)
```

with

XA,real	n-dimensional vector containing, on entry, the user's estimate of the minimizer and, on exit, the computed minimizer;
N,integer	variable specifying the number n of independent variables: N must be assigned before entry;
F,real	variable containing function value in the current point, on exit F contains the estimated value of the minimum;
DIR,real	matrix of the search vectors: DIR is built in OTLSSS and in OTLSSD;
EPS,EPS1,EPS3, EPS4,real	variables containing the accuracies, to be assigned before entry: EPS and EPS1 must be to the relative accuracies to which the minimizer and the minimum are required, EPS4 and EPS3 scale EPS to the different accurancies EPS2 required in the line searches respetively along the 1-st, 2-nd,..., (n-k)-th direction and along the (n-k+1)-th,...,n-th direction. To make consistent these accuracies, EPS4 should be not smaller than 1 and not biger than 10, while EPS3 should be not smaller than 10^2 and not biger than 10^3, whenever EPS and EPS1 are set to 10^{-5};
IFMAX, integer	variable containing the maximum allowed number of function evaluations: IFMAX must be assigned before entry. It depends from the behaviour and from the dimension of the objective function and from the required accuracies: in the performed proofs IFMAX is set to 10^4;
XMU,real	variable containing the initial step lenght for the line search, to be assigned less or equal to 1 before entry;
IPRINT,integer	parameter controlling print as follows: for IPRINT=1 the current values and the final ones of the cycle index, of the iteration index, and of the minimizer and minimum approximations are printed; for IPRINT=0 only the final values are printed.

SUBROUTINE SEARCH (D,IFMAX1,EPS2,XO,N,FO,MU,X,IFUN)

SUBROUTINE SEARD (D,IFMAX1,EPS2,XO,N,FO,MU,X,IFUN)

with

D,real	n-dimensional vector to be computed before entry;
IFMAX1,integer	variable containing the difference between IFMAX and IFUN to be computed before entry;
EPS2,real	variable containing the accuracy to which the position of the one-dimensional minimum is required:EPS2 must be calculated before entry;
XO,real	variable containing the actual approximation of the minimizer;
FO,real	variable containing the function value in XO;
MU,real	variable containing the step lenght on entry;
X,real	variable containing the step lenght on exit;
IFUN,integer	variable containing the total number of function evaluations;

SUBROUTINE CALFUN (X,N,F,IFUN)

SUBROUTINE CALFUD (X,N,F,IFN)

with

X, real	variable containing the point at which the function value is required;
N,integer	variable specifying the number of independent variables;
F,real	variable containing the function value in X;
IFUN,integer	variable containing the total number of function evaluations.

The lenght of the codes, i.e. the total number of statements in OTLSSS and in OTLSSD are respectively 309 and 313. The size of the problems for which the codes has been designed is $n \leq 50$. The related required storage is of 9.132 words (9.132x16 bits) for OTLSSS and 14.986 words (14.986x16 bits) for OTLSSD. In the above sums none care is taken or of the subroutine CALFUN or of CALFUD.

The test problems solved by OTLSSS and OTLSSD on the PDP 11/40 of 32K words, at the Mathematical Institute, University of PARMA (ITALY), were the minimizations of the following functions:

1 - Extended Rosenbrock

2 - Extended Powell

3 - Oren's Quartic

4 - Penalty I

5 - More first function

6 - Trigonometric

7 - More second function

8 - Brown almost linear

9 - Mancino

10 - Watson

11 - Penalty II

12 - Chebyquad

For the mathematical description of the above functions with the related starting points $X_o = (X_{oi}), i = 1,\dots,n$, see ref.(4).

The proofs have been performed for $n = 4,10$ and for $n = 4,8$ for the Extended Powell function. Moreover the following initial approximations of the minimizer were assumed: $X_o^1 = (X_{oi}), X_o^2 = (X_{oi} + \Delta_i)$ with $\Delta_i = 10^{-3}(1+|X_o|)$ and $X_o^3 = (10\ X_{oi})$, $i = 1,\dots,n$. The other input parameters were assigned as above described.

In the annexed listing 1 and 2, we present the executions of the programs OTLSSS and OTLSSD, with IPRINT = 0, for the sample problem

$$\min_{x_1,x_2} \ 100(x_2-x_1^2)^2 + (1-x_1)^2 \qquad\qquad x_o = (-1.2,1\ )$$

having analytical solution $x_{min} = (1,1)$, $F_{min} = 0$.

The annexed numerical tables 3 and 4 visualize the results obtained by OTLSSS and OTLSSD. The parameter NPROB is the number of the objective function in the above sequence, N is the size of the problem, XZERO indicates which starting vector is tested, CYC is the total number of the performed cycles, ITER the number of the iterations in the final cycle, IFUN the total number of function evaluations and F the computed minimum.

Listing 1

```
C OTLSSS
C QUESTO PROGRAMMA ESEGUE LA MINIMIZZAZIONE DI UNA FUNZIONE OBIETTIVO
C N-DIMENSIONALE NON VINCOLATA CON LA SECONDA IMPLEMENTAZIONE
C DEL METODO DI SUTTI(1975) IN SINGOLA PRECISIONE
C IPRINT=0 PRESENTA LA STAMPA DEI SOLI RISULTATI FINALI
C IPRINT=1 PRESENTA ANCHE LA STAMPA DI RISULTATI INTERMEDI
C N RAPPRESENTA LA  DIMENSIONE DEL DOMINIO DELLA FUNZIONE DA MINIMIZZARE
C IFMAX RAPPRESENTA IL MASSIMO NUMERO DI VALUTAZIONI
C DI FUNZIONE CONSENTITO
C XMU RAPPRESENTA IL PASSO INIZIALE PER LA RICERCA DI LINEA
C EPS E' LA PRECISIONE SUL PUNTO DI MINIMO
C EPS1 E' LA PRECISIONE SULLA FUNZIONE
C EPS3,EPS4 SONO I PARAMETRI PER DEDURRE LA PRECISIONE
C MONODIMENSIONALE DALLA PRECISIONE N-DIMENSIONALE
C X RAPPRESENTA IL PUNTO INIZIALE
C DIR RAPPRESENTA LA MATRICE DELLE DIREZIONI DI RICERCA
      DIMENSION X(50),DIR(50,50)
      READ(5,14)IFRINT
   14 FORMAT(I1)
  100 READ(5,2)N,IFMAX,XMU,EPS,EPS1
    2 FORMAT(2I5,5F10.5)
      READ(5,3)EPS3,EPS4
    3 FORMAT(F10.3,F10.3)
      READ(5,4)(X(I),I=1,N)
    4 FORMAT(10F7.3)
      WRITE(6,1)N
    1 FORMAT(1H1,3HN =,I3)
      WRITE(6,10)IFMAX,XMU
   10 FORMAT(/,1X,7HIFMAX =,I6,7X,5HXMU =,E14.7)
      WRITE(6,15)EPS,EPS1
   15 FORMAT(/,1X,5HEPS =,E13.6,5X,6HEPS1 =,E13.6)
      WRITE(6,13)EPS3,EPS4
   13 FORMAT(/,1X,5HEPS3=,E13.6,5X,6HEPS4 =,E13.6)
      WRITE(6,16)
   16 FORMAT(/,1X,2HX=)
      WRITE(6,17)(X(I),I=1,N)
   17 FORMAT(4(2X,E14.7))
      DO 8 I=1,50
      DO 8 J=1,50
    8 DIR(I,J)=0
      DO 9 I=1,50
    9 DIR(I,I)=1
      CALL CNS(X,N,F,DIR,EPS,EPS1,EPS3,EPS4,IFMAX,XMU,IPRINT)
   95 STOP
      END
```

```
      SUBROUTINE CNS(XA,N,F,DIR,EPS,EPS1,EPS3,EPS4,IFMAX,XMU,IPRINT)
C QUESTO SOTTOPROGRAMMA ESEGUE LA RICERCA N-DIMENSIONALE
C ALFHA RAPPRESENTA IL PASSO ARBITRARIO
C DIRL E' IL VETTORE POSTO NELLA L-ESIMA COLONNA DELLA MATRICE DIR
C IFMAX1 RAPPRESENTA IL NUMERO DI VALUTAZIONI DI FUNZIONE
C ANCORA DISPONIBILI
C I E' L'INDICE DI CICLO
C K E' L'INDICE DI ITERAZIONE CHE NON SUPERA N-1
      DIMENSION X(50),X0(50),D(50)
      DIMENSION XA(50),XB(50),XA1(50),DIR(50,50),DIRN(50)
      SC=XMU
      IFUN=0
      CALL CALFUN(XA,N,FA,IFUN)
      I=0
    1 I=I+1
      IF(IPRINT) 32,33,32
   32 CONTINUE
      WRITE(6,80)I
   80 FORMAT(/,/,/,/,/,1X,'CICLO I=',I3)
   33 CONTINUE
      K=0
    2 K=K+1
      IF(IPRINT) 34,35,34
   34 CONTINUE
      WRITE(6,88)K
   88 FORMAT(/,/,1X,'ITERAZIONE K=',I3)
   35 CONTINUE
      DO 3 II=1,N
    3 DIRN(II)=DIR(II,N)
      IFMAX1=IFMAX-IFUN
```

```
C MINIMIZZAZIONE LUNGO DIRL CON L=N
      IF(IPRINT)36,37,36
   36 CONTINUE
      WRITE(6,94)(XA(IB),IB=1,N)
   94 FORMAT(1X,2HX=,10E12.5)
      WRITE(6,89)FA
   89 FORMAT(1X,2HF=,E14.7)
      WRITE(6,93)
   93 FORMAT(1X,'MINIMIZZAZIONE MONODIMENSIONALE')
   37 CONTINUE
      EPS2=EPS*EPS3
      CALL SEARCH(DIRN,IFMAX1,EPS2,XA,N,FA,XMU,SC,IFUN)
      IF(IPRINT)38,39,38
   38 CONTINUE
      WRITE(6,94)(XA(IB),IB=1,N)
      WRITE(6,89) FA
   39 CONTINUE
      IF(IFUN.GT.IFMAX) GO TO 1001
      L=0
    4 L=L+1
C ESECUZIONE DEL PASSO ALPHA LUNGO DIRL L=1,...N-K
      IF(L.GT.(N-K)) GO TO 45
      DO 41 II=1,N
   41 DIRN(II)=DIR(II,L)
      ALPHA=AMAX1(ABS(SC)*0.5,0.0001)
      DO 42 II=1,N
   42 XA1(II)=XA(II)+ALPHA*DIRN(II)
      CALL CALFUN(XA1,N,FA1,IFUN)
      IF(IFUN.GT.IFMAX) GO TO 1001
      AMAX=AMAX1(EPS1,EPS1*ABS(FA))
      IF((FA-FA1).GT.AMAX*0.001) GO TO 43
      DO 44 II=1,N
      DIRN(II)=-DIRN(II)
   44 XA1(II)=XA(II)+ALPHA*DIRN(II)
      CALL CALFUN(XA1,N,FA1,IFUN)
      IF(IFUN.GE.IFMAX) GO TO 1001
      AMAX=AMAX1(EPS1,EPS1*ABS(FA))
      IF((FA-FA1).GT.AMAX*0.001) GO TO 43
      GO TO 4
   45 CONTINUE
C MINIMIZZAZIONE LUNGO DIRL CON L=1....N-K PER FALLIMENTO PASSO ALPH
      L=0
    5 L=L+1
      IF(L.GT.(N-K)) GO TO 1000
      DO 51 II=1,N
   51 DIRN(II)=DIR(II,L)
      DO 551 II=1,N
  551 XO(II)=XA(II)
      FO=FA
       IFMAX1=IFMAX-IFUN
      EPS2=EPS*EPS4
      CALL SEARCH(DIRN,IFMAX1,EPS2,XO,N,FO,XMU,SC,IFUN)
      IF(IPRINT) 31,433,31
   31 CONTINUE
      WRITE(6,95)
   95 FORMAT(1X,'MINIMIZZAZIONE MONODIMENSIONALE IN QUANTO FALLITA LA RI
     ACERCA DI UN PASSO ARBITRARIO DI DISCESA')
      WRITE(6,94)(XO(IB),IB=1,N)
      WRITE(6,89)FO
  433 CONTINUE
      IF(IFUN.GE.IFMAX) GO TO 1001
C ESECUZIONE DEI CRITERI D'ARRESTO
      IF(ABS(SC).GT.EPS) GO TO 113
      AMAX=AMAX1(EPS1,EPS1*ABS(FA))
      IF((FA-FO).GT.AMAX) GO TO 113
      GO TO 5
  113 CONTINUE
      DO 553 II=1,N
  553 XA1(II)=XO(II)
      FA1=FO
      GO TO 66
   43 CONTINUE
      IF(IPRINT)20,21,20
   20 CONTINUE
      WRITE(6,99)
   99 FORMAT(1X,'RICERCA RIUSCITA DI UN PASSO ARBITRARIO DI DISCESA')
      WRITE(6,94)(XA1(IB),IB=1,N)
      WRITE(6,89)FA1
   21 CONTINUE
   66 CONTINUE
C MINIMIZZAZIONI LUNGO DIRL CON L=N-K+1...N
      J=N-K
    6 J=J+1
      DO 61 II=1,N
   61 DIRN(II)=DIR(II,J)
      IFMAX1=IFMAX-IFUN
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```
      EPS2=EPS*EPS3
      CALL SEARCH(DIRN,IFMAX1,EPS2,XA1,N,FA1,XMU,SC,IFUN)
      IF(IPRINT)22,23,22
 22   CONTINUE
      WRITE(6,93)
      WRITE(6,94)(XA1(IB),IB=1,N)
      WRITE(6,89) FA1
 23   CONTINUE
      IF(IFUN.GE.IFMAX) GO TO 1001
      IF(J.EQ.N) GO TO 62
      GO TO 6
  62  CONTINUE
C CALCOLO DELLA NUOVA MATRICE DELLE DIREZIONI DI RICERCA
      XMOD=0.0
      DO 8 II=1,N
    8 XMOD=XMOD+(XA1(II)-XA(II))**2
      XMOD=SQRT(XMOD)
      DO 7 JJ=1,N
      IF(JJ.LT.L) GO TO 7
      IF(JJ.EQ.N) GO TO 71
      DO 72 II=1,N
   72 DIR(II,JJ)=DIR(II,JJ+1)
      GO TO 7
   71 DO 73 II=1,N
   73 DIR(II,JJ)=XA1(II)-XA(II)
    7 CONTINUE
      DO 81 II=1,N
      XA(II)=XA1(II)
   81 DIR(II,N)=DIR(II,N)/XMOD
      FA=FA1
  147 IF(K.EQ.(N-1)) GO TO 1
      GO TO 2
 1001 WRITE(6,104)
 104  FORMAT(1H1,14HIFMAX EXCEEDED)
      GO TO 1002
 1000 WRITE(6,100)IFUN,I,K
 100  FORMAT(/,/,1X,19HOPTIMUM FOUND AFTER,I5,15HFUNCTION CALLS,,
     1I5,7HCYCLES,,I5,11HSIMPLE IER,)
 1002 CONTINUE
      WRITE(6,101)
 101  FORMAT(/,1X,2HX=)
      WRITE(6,102) (XA(I),I=1,N)
  102 FORMAT(4(2X,E14.7))
      WRITE(6,103) FA
 103  FORMAT(/,1X,24HMINIMUM FUNCTION VALUE =,E14.7)
      RETURN
      END



      SUBROUTINE SEARCH(D,IFMAX1,EPS2,X0,N,F0,MU,X,IFUN)
C QUESTO SOTTOPROGRAMMA ESEGUE LA RICERCA MONODIMENSIONALE
C EPS2 E' LA PRECISIONE NELLA RICERCA MONODIMENSIONALE
C MU RAPPRESENTA IL PASSO INIZIALE PER LA RICERCA MONODIMENSIONALE
      DIMENSION X0(50),D(50),X1(50)
      REAL MU
      ITEST=3
      MU=AMAX1(ABS(X),1.5*EPS2)
      XSTEP=SIGN(MU,X)
      X=0.0
      F=F0
C CALCOLO DEI PUNTI DA INTERPOLARE
   51 GO TO 2000
 2100 GO TO(1,2,335,336),ITEST
  1   DO 70 I=1,N
   70 X1(I)=X0(I)+X*D(I)
      CALL CALFUN(X1,N,F,IFUN)
      GO TO 51
    2 CONTINUE
C CALCOLO DEL MINIMO MONODIMENSIONALE
      DO 377 I=1,N
  377 X1(I)=X0(I)+DV*D(I)
      CALL CALFUN(X1,N,FV,IFUN)
      IF(F-FV)50,50,59
   59 F=FV
      X=DV
   50 IF(F.LT.F0) GO TO 60
      F=F0
      X=0.0
  123 FORMAT (1X,I4)
      GO TO 2200
   60 DO 30 I=1,N
   30 X0(I)=X0(I)+X*D(I)
      F0=F
      GO TO 2200
```

```
335   CONTINUE
      GO TO 50
 336 WRITE(6,122)
 122 FORMAT(1H1,14HIFMAX EXCEEDED)
      GO TO 50
2000 CONTINUE
      GO TO(91,222,222),ITEST
 222 IS=6-ITEST
      ITEST=1
      IINC=1
      XINC=XSTEP+XSTEP
      MC=IS-3
      IF(MC) 4,4,15
   3 MC=MC+1
      IF(IFMAX1-MC) 12,15,15
C C CASO IN CUI E' STATO RAGGIUNTO IL MASSIMO NUMERO DI VALUTAZIONE
C DI FUNZIONE
  12 ITEST=4
  43 X=DB
     F=FB
     IF(FB-FC) 15,15,44
  44 X=DC
     F=FC
  15 GO TO 2100
  91 GO TO (5,6,7,8),IS
   8 IS=3
   4 DC=X
     FC=F
     X=X+XSTEP
     GO TO 3
C CONFRONTO TRA VALORE DELLA FUNZIONE NEL PUNTO INIZIALE E NEL
C PUNTO ATTUALE
   7 IF(FC-F) 9,10,11
  10 X=X+XINC
     XINC=XINC+XINC
     GO TO 3
   9 DB=X
     FB=F
     XINC=-XINC
     GO TO 13
  11 DB=DC
     FB=FC
     DC=X
     FC=F
  13 X=DC+DC-DB
     IS=2
     GO TO 3
   6 DA=DB
     DB=DC
     FA=FB
     FB=FC
  32 DC=X
     FC=F
     GO TO 14
   5 IF(FB-FC) 16,17,17
  17 IF(F-FB) 18,32,32
  18 FA=FB
     DA=DB
  19 FB=F
     DB=X
     GO TO 14
  16 IF(FA-FC) 21,21,20
  20 XINC=FA
     FA=FC
     FC=XINC
     XINC=DA
     DA=DC
     DC=XINC
  21 XINC=DC
     IF((DV-DB)*(DV-DC)) 32,22,22
  22 IF(F-FA) 23,24,24
  23 FC=FB
     DC=DB
     GO TO 19
  24 FA=F
     DA=X
  14 IF(FB-FC) 25,25,29
  25 IINC=2
     XINC=DC
     IF(FB-FC) 29,45,29
C CALCOLO DELL'ASCISSA DEL VERTICE DELLA PARABOLA INTERPOLANTE
  29 DV=(FA-FB)/(DA-DB)-(FA-FC)/(DA-DC)
     IF(DV*(DB-DC))33,33,37
  37 DV=0.5*(DB+DC-(FB-FC)/DV)
```

```
C ESECUZIONE DEL CRITERIO DI ARRESTO CON PRECISIONE ASSOLUTA O RELATIVA
  56   IF(ABS(DV-X)-ABS(EPS2))34,34,35
  35   IF(ABS(DV-X)-ABS(DV*EPS2)) 34,34,36
   34 ITEST=2
      GO TO 43
   36 IS=1
      X=DV
      IF((DA-DC)*(DC-DV))3,26,38
   38 IS=2
      GO TO (3,40),IINC
   33 IS=2
      GO TO (41,42),IINC
   41 X=DC
      GO TO 10
   40 IF(ABS(XINC-X)-ABS(X-DC)) 42,42,3
   42 X=0.5*(XINC+DC)
      IF((XINC-X)*(X-DC)) 26,26,3
   45 X=0.5*(DB+DC)
      IF((DB-X)*(X-DC)) 26,26,3
   26 ITEST=3
      GO TO 43
 2200 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CALFUN(X,N,F,IFUN)
C QUESTO SOTTOPROGRAMMA CALCOLA LA FUNZIONE DA MINIMIZZARE NEL PU
C ATTUALE
C X RAPPRESENTA IL PUNTO IN CUI VIENE CALCOLATA LA FUNZIONE
C N RAPPRESENTA LA DIMENSIONE DEL DOMINIO DELLA FUNZIONE DA MINIMIZZARE
C F RAPPRESENTA LA FUNZIONE OBIETTIVO
C IFUN RAPPRESENTA IL NUMERO DI VALUTAZIONI DI FUNZIONE ESEGUITE
      DIMENSION X(50)
      F=100.*(X(2)-X(1)*X(1))*(X(2)-X(1)*X(1))+(1-X(1))*(1-X(1))
      IFUN=IFUN+1
1000  CONTINUE
      RETURN
      END
```

```
N =  2

IFMAX = 10000        XMU = 0.5000000E 00

EPS = 0.100000E-04      EPS1 = 0.100000E-04

EPS3= 0.100000E 03      EPS4 = 0.100000E 01

X=
 -0.1200000E 01    0.1000000E 01


OPTIMUM FOUND AFTER  283FUNCTION CALLS,    12CYCLES,    1SIMPLE IER,

X=
  0.1000002E 01    0.1000005E 01

MINIMUM FUNCTION VALUE = 0.2684430E-10
```

Listing 2

```
C OTLSSD
C QUESTO PROGRAMMA ESEGUE LA MINIMIZZAZIONE DI UNA FUNZIONE OBIETTIVO
C N-DIMENSIONALE NON VINCOLATA CON LA SECONDA IMPLEMENTAZIONE
C DEL METODO DI SUTTI(1975) IN DOPPIA PRECISIONE
C IPRINT=0 PRESENTA LA STAMPA DEI SOLI RISULTATI FINALI
C IPRINT=1 PRESENTA ANCHE LA STAMPA DI RISULTATI INTERMEDI
C N RAPPRESENTA LA  DIMENSIONE DEL DOMINIO DELLA FUNZIONE DA MINIMIZZARE
C IFMAX RAPPRESENTA IL MASSIMO NUMERO DI VALUTAZIONI
C DI FUNZIONE CONSENTITO
C XMU RAPPRESENTA IL PASSO INIZIALE PER LA RICERCA DI LINEA
C EPS E' LA PRECISIONE SUL PUNTO DI MINIMO
C EPS1 E' LA PRECISIONE SULLA FUNZIONE
C EPS3,EPS4 SONO I PARAMETRI PER DEDURRE LA PRECISIONE MONODIMENSIONALE
C DALLA PRECISIONE N-DIMENSIONALE
C X RAPPRESENTA IL PUNTO INIZIALE
C DIR RAPPRESENTA LA MATRICE DELLE DIREZIONI DI RICERCA
      DIMENSION X(50),DIR(50,50)
      DOUBLE PRECISION X,F,DIR,XMU,EPS,EPS1
      READ(5,14)IPRINT
  14  FORMAT(I1)
 100  READ(5,2)N,IFMAX,XMU,EPS,EPS1
  2   FORMAT(2I5,3D15.5)
      READ(5,4)EPS3,EPS4
      READ(5,4)(X(I),I=1,N)
  4   FORMAT(4D15.5)
      WRITE(6,1)N
  1   FORMAT(1H1,3HN =,I3)
      WRITE(6,10)IFMAX,XMU
  10  FORMAT(/,1X,7HIFMAX =,I6,7X,5HXMU =,D14.7)
      WRITE(6,15)EPS,EPS1
  15  FORMAT(/,1X,5HEPS =,D13.6,5X,6HEPS1 =,D13.6)
      WRITE(6,13)EPS3,EPS4
  13  FORMAT(/,1X,5HEPS3=,D13.6,5X,6HEPS4 =,D13.6)
      WRITE(6,16)
  16  FORMAT(/,1X,2HX=)
      WRITE(6,17)(X(I),I=1,N)
  17  FORMAT(4(2X,D14.7))
      DO 8 I=1,50
      DO 8 J=1,50
    8 DIR(I,J)=0
      DO 9 I=1,50
    9 DIR(I,I)=1
      CALL CNSD(X,N,F,DIR,EPS,EPS1,EPS3,EPS4,IFMAX,XMU,IPRINT)
  95  STOP
      END
```

```
      SUBROUTINE CNSD(XA,N,F,DIR,EPS,EPS1,EPS3,EPS4,IFMAX,XMU,IPRINT)
C QUESTO SOTTOPROGRAMMA ESEGUE LA RICERCA N-DIMENSIONALE
C DIRL E' IL VETTORE POSTO NELLA L-ESIMA COLONNA DELLA MATRICE DIR
C ALPHA RAPPRESENTA IL PASSO ARBITRARIO
C IFMAX1 RAPPRESENTA IL NUMERO DI VALUTAZIONI DI FUNZIONE
C ANCORA DISPONIBILI
C I E' L'INDICE DI CICLO
C K E' L'INDICE DI ITERAZIONE CHE NON SUPERA N-1
      DIMENSION X(50),X0(50),D(50)
      DIMENSION XA(50),XB(50),XA1(50),DIR(50,50),DIRN(50)
      DOUBLE PRECISION XA,F,DIR,EPS,EPS1,XMU
      DOUBLE PRECISION X,X0,D,XB,XA1,DIRN,SC,EPS2,FA,ALPHA,FA1
      DOUBLE PRECISION AMAX,F0,XMOD
      SC=XMU
      IFUN=0
      CALL CALFUD(XA,N,FA,IFUN)
      I=0
    1 I=I+1
      IF(IPRINT) 32,33,32
  32  CONTINUE
      WRITE(6,80)I
  80  FORMAT(/,/,/,/,/,1X,'CICLO I=',I3)
  33  CONTINUE
      K=0
    2 K=K+1
      IF(IPRINT) 34,35,34
  34  CONTINUE
      WRITE(6,88)K
  88  FORMAT(/,/,1X,'ITERAZIONE K=',I3)
  35  CONTINUE
```

```
      DO 3 II=1,N
    3 DIRN(II)=DIR(II,N)
      IFMAX1=IFMAX-IFUN
C MINIMIZZAZIONE LUNGO DIRL CON L=N
      IF(IPRINT)36,37,36
   36 CONTINUE
      WRITE(6,94)(XA(IB),IB=1,N)
   94 FORMAT(1X,2HX=,4D15.5)
      WRITE(6,89)FA
   89 FORMAT(1X,2HF=,D30.20)
      WRITE(6,93)
   93 FORMAT(1X,'MINIMIZZAZIONE MONODIMENSIONALE')
   37 CONTINUE
      EPS2=EPS*EPS3
      CALL SEARD(DIRN,IFMAX1,EPS2,XA,N,FA,XMU,SC,IFUN)
      IF(IPRINT)38,39,38
   38 CONTINUE
      WRITE(6,94)(XA(IB),IB=1,N)
      WRITE(6,89) FA
   39 CONTINUE
      IF(IFUN.GT.IFMAX) GO TO 1001
      L=0
    4 L=L+1
      IF(L.GT.(N-K)) GO TO 45
C ESECUZIONE DEL PASSO ALPHA LUNGO DIRL L=1,...N-K
      DO 41 II=1,N
   41 DIRN(II)=DIR(II,L)
      ALPHA=DMAX1(DABS(SC)*0.5,1.D-4)
      DO 42 II=1,N
   42 XA1(II)=XA(II)+ALPHA*DIRN(II)
      CALL CALFUD(XA1,N,FA1,IFUN)
      IF(IFUN.GT.IFMAX) GO TO 1001
      AMAX=DMAX1(EPS1,EPS1*DABS(FA))
      IF((FA-FA1).GT.AMAX*0.001) GO TO 43
      DO 44 II=1,N
      DIRN(II)=-DIRN(II)
   44 XA1(II)=XA(II)+ALPHA*DIRN(II)
      CALL CALFUD(XA1,N,FA1,IFUN)
      IF(IFUN.GE.IFMAX) GO TO 1001
      AMAX=DMAX1(EPS1,EPS1*DABS(FA))
      IF((FA-FA1).GT.AMAX*0.001) GO TO 43
      GO TO 4
   45 CONTINUE
C MINIMIZZAZIONE LUNGO DIRL CON L=1....N-K PER FALLIMENTO PASSO ALPHA
      L=0
    5 L=L+1
      IF(L.GT.(N-K)) GO TO 1000
      DO 51 II=1,N
   51 DIRN(II)=DIR(II,L)
      DO 551 II=1,N
  551 XO(II)=XA(II)
      FO=FA
      IFMAX1=IFMAX-IFUN
      EPS2=EPS*EPS4
      CALL SEARD(DIRN,IFMAX1,EPS2,XO,N,FO,XMU,SC,IFUN)
      IF(IPRINT) 31,433,31
   31 CONTINUE
      WRITE(6,95)
   95 FORMAT(1X,'MINIMIZZAZIONE MONODIMENSIONALE IN QUANTO FALLITA LA RI
     ACERCA DI UN PASSO ARBITRARIO DI DISCESA')
      WRITE(6,94)(XO(IB),IB=1,N)
      WRITE(6,89)FO
  433 CONTINUE
      IF(IFUN.GE.IFMAX) GO TO 1001
C ESECUZIONE DEI CRITERI D'ARRESTO
      IF(DABS(SC).GT.EPS) GO TO 113
      AMAX=DMAX1(EPS1,EPS1*DABS(FA))
      IF((FA-FO).GT.AMAX) GO TO 113
      GO TO 5
  113 CONTINUE
      DO 553 II=1,N
  553 XA1(II)=XO(II)
      FA1=FO
      GO TO 66
   43 CONTINUE
      IF(IPRINT)20,21,20
   20 CONTINUE
      WRITE(6,99)
   99 FORMAT(1X,'RICERCA RIUSCITA DI UN PASSO ARBITRARIO DI DISCESA')
      WRITE(6,94)(XA1(IB),IB=1,N)
      WRITE(6,89)FA1
   21 CONTINUE
   66 CONTINUE
C MINIMIZZAZIONI LUNGO DIRL CON L=N-K+1...N
      J=N-K
    6 J=J+1
```

```
      EPS2=EPS*EPS3
      CALL SEARD(DIRN,IFMAX1.EPS2,XA1,N,FA1,XMU,SC,IFUN)
      IF(IPRINT)22,23,22
 22   CONTINUE
      WRITE(6,93)
      WRITE(6,94)(XA1(IB),IB=1,N)
      WRITE(6,89) FA1
 23   CONTINUE
      IF(IFUN.GE.IFMAX) GO TO 1001
      IF(J.EQ.N) GO TO 62
      GO TO 6
  62 CONTINUE
C CALCOLO DELLA NUOVA MATRICE DELLE DIREZIONI DI RICERCA
      XMOD=0.0
      DO 8 II=1,N
    8 XMOD=XMOD+(XA1(II)-XA(II))**2
      XMOD=DSQRT(XMOD)
      DO 7 JJ=1,N
      IF(JJ.LT.L) GO TO 7
      IF(JJ.EQ.N) GO TO 71
      DO 72 II=1,N
   72 DIR(II,JJ)=DIR(II,JJ+1)
      GO TO 7
   71 DO 73 II=1,N
   73 DIR(II,JJ)=XA1(II)-XA(II)
    7 CONTINUE
      DO 81 II=1,N
      XA(II)=XA1(II)
   81 DIR(II,N)=DIR(II,N)/XMOD
      FA=FA1
  147 IF(K.EQ.(N-1)) GO TO 1
      GO TO 2
 1001 WRITE(6,104)
 104  FORMAT(1H1,14HIFMAX EXCEEDED)
      GO TO 1002
 1000 WRITE(6,100)IFUN,I,K
 100  FORMAT(/,/,1X,19HOPTIMUM FOUND AFTER,I5,15HFUNCTION CALLS,,
     1I5,7HCYCLES,,I5,11HSIMPLE IER,)
 1002 CONTINUE
      WRITE(6,101)
 101  FORMAT(/,1X,2HX=)
      WRITE(6,102) (XA(I),I=1,N)
 102  FORMAT(4(2X,D14.7))
      WRITE(6,103) FA
 103  FORMAT(/,1X,24HMINIMUM FUNCTION VALUE =,D14 7)
      RETURN
      END
```

```
      SUBROUTINE SEARD(D,IFMAX1,EPS2,X0,N,F0,MU,X,IFUN)
C QUESTO SOTTOPROGRAMMA ESEGUE LA RICERCA MONODIMENSIONALE
C MU RAPPRESENTA IL PASSO INIZIALE PER LA RICERCA MONODIMENSIONALE
C EPS2 E' LA PRECISIONE NELLA RICERCA MONODIMENSIONALE
      DIMENSION X0(50),D(50),X1(50)
      DOUBLE PRECISION D,EPS2,X0,F0,MU,X,F,X1,DV,FV,DB,FB
      DOUBLE PRECISION FC,DC,XSTEP,XINC,DA,FA
      ITEST=3
      MU=DMAX1(DABS(X),1.5*EPS2)
      XSTEP=DSIGN(MU,X)
      X=0.0
      F=F0
C CALCOLO DEI PUNTI DA INTERPOLARE
   51 GO TO 2000
 2100 GO TO(1,2,335,336),ITEST
  1   DO 70 I=1,N
   70 X1(I)=X0(I)+X*D(I)
      CALL CALFUD(X1,N,F,IFUN)
      GO TO 51
  2   CONTINUE
C CALCOLO DEL MINIMO MONODIMENSIONALE
      DO 377 I=1,N
  377 X1(I)=X0(I)+DV*D(I)
      CALL CALFUD(X1,N,FV,IFUN)
      IF(F-FV)50,50,59
   59 F=FV
      X=DV
   50 IF(F.LT.F0) GO TO 60
```

```
      F=FO
      X=0.0
  123 FORMAT (1X,I4)
      GO TO 2200
   60 DO 30 I=1,N
   30 XO(I)=XO(I)+X*D(I)
      FO=F
      GO TO 2200
 335  CONTINUE
      GO TO 50
  336 WRITE(6,122)
  122 FORMAT(1H1,14HIFMAX EXCEEDED)
      GO TO 50
 2000 CONTINUE
      GO TO(91,222,222),ITEST
  222 IS=6-ITEST
      ITEST=1
      IINC=1
      XINC=XSTEP+XSTEP
      MC=IS-3
      IF(MC) 4,4,15
    3 MC=MC+1
      IF(IFMAX1-MC) 12,15,15
C C CASO IN CUI E' STATO RAGGIUNTO IL MASSIMO NUMERO DI VALUTAZIONE
C DI FUNZIONE
   12 ITEST=4
   43 X=DB
      F=FB
      IF(FB-FC) 15,15,44
   44 X=DC
      F=FC
   15 GO TO 2100
   91 GO TO (5,6,7,8),IS
    8 IS=3
    4 DC=X
      FC=F
      X=X+XSTEP
      GO TO 3
C CONFRONTO TRA VALORE DELLA FUNZIONE NEL PUNTO INIZIALE E NEL
C PUNTO ATTUALE
    7 IF(FC-F) 9,10,11
   10 X=X+XINC
      XINC=XINC+XINC
      GO TO 3
    9 DB=X
      FB=F
      XINC=-XINC
      GO TO 13
   11 DB=DC
      FB=FC
      DC=X
      FC=F
   13 X=DC+DC-DB
      IS=2
      GO TO 3
    6 DA=DB
      DB=DC
      FA=FB
      FB=FC
   32 DC=X
      FC=F
      GO TO 14
    5 IF(FB-FC) 16,17,17
   17 IF(F-FB) 18,32,32
   18 FA=FB
      DA=DB
   19 FB=F
      DB=X
      GO TO 14
   16 IF(FA-FC) 21,21,20
   20 XINC=FA
      FA=FC
      FC=XINC
      XINC=DA
      DA=DC
      DC=XINC
   21 XINC=DC
      IF((DV-DB)*(DV-DC)) 32,22,22
   22 IF(F-FA) 23,24,24
   23 FC=FB
      DC=DB
      GO TO 19
   24 FA=F
      DA=X
   14 IF(FB-FC) 25,25,29
   25 IINC=2
      XINC=DC
```

```
      IF(FB-FC) 29,45,29
C CALCOLO DELL'ASCISSA DEL VERTICE DELLA PARABOLA INTERPOLANTE
   29 DV=(FA-FB)/(DA-DB)-(FA-FC)/(DA-DC)
      IF(DV*(DB-DC))33,33,37
   37 DV=0.5*(DB+DC-(FB-FC)/DV)
C ESECUZIONE DEL CRITERIO DI ARRESTO CON PRECISIONE ASSOLUTA O RELATIVA
  56  IF(DABS(DV-X)-DABS(EPS2))34,34,35
  35  IF(DABS(DV-X)-DABS(DV*EPS2))34,34,36
   34 ITEST=2
      GO TO 43
   36 IS=1
      X=DV
      IF((DA-DC)*(DC-DV))3,26,38
   38 IS=2
      GO TO (3,40),IINC
   33 IS=2
      GO TO (41,42),IINC
   41 X=DC
      GO TO 10
  40  IF(DABS(XINC-X)-DABS(X-DC)) 42,42,3
   42 X=0.5*(XINC+DC)
      IF((XINC-X)*(X-DC)) 26,26,3
   45 X=0.5*(DB+DC)
      IF((DB-X)*(X-DC)) 26,26,3
   26 ITEST=3
      GO TO 43
 2200 CONTINUE
      RETURN
      END
```

```
      SUBROUTINE CALFUD(X,N,F,IFUN)
C QUESTO SOTTOPROGRAMMA CALCOLA LA FUNZIONE DA MINIMIZZARE NEL PUNTO
C ATTUALE
C X RAPPRESENTA IL PUNTO IN CUI VIENE CALCOLATA LA FUNZIONE
C N RAPPRESENTA LA DIMENSIONE DEL DOMINIO DELLA FUNZIONE DA MINIMIZZARE
C F RAPPRESENTALA FUNZIONE OBIETTIVO
C IFUN RAPPRESENTA IL NUMERO DI VALUTAZIONI DI FUNZIONE ESEGUITE
      DOUBLE PRECISION X(50),F
      IFUN=IFUN+1
      F=100.*(X(2)-X(1)*X(1))*(X(2)-X(1)*X(1))+(1-X(1))*(1-X(1))
1000  CONTINUE
      RETURN
      END
```

```
 N =  2

 IFMAX = 10000        XMU = 0.5000000D 00

 EPS = 0.100000D-04      EPS1 = 0.100000D-04

 EPS3= 0.100000D 03      EPS4 = 0.100000D 01

 X=
  -0.1200000D 01    0.1000000D 01


 OPTIMUM FOUND AFTER  268FUNCTION CALLS,    12CYCLES,    1SIMPLE IER,

 X=
   0.1000002D 01    0.1000005D 01

 MINIMUM FUNCTION VALUE = 0.2554003D-10
```

Table 3

NPROB	N	XZERO	CYC	ITER	IFUN	TEMP	F
1	4	1	8	3	500	8	0.6853185E-11
1	4	2	9	3	539	9	0.1694644E-10
1	4	3	16	3	992	12	0.4928680E-10
2	4	1	6	1	335	9	0.4614324E-10
2	4	2	6	3	325	9	0.1663981E-08
2	4	3	5	2	289	9	0.4886158E-04
3	4	1	4	1	200	8	0.1043268E-22
3	4	2	4	1	197	7	0.3698358E-22
3	4	3	2	3	158	7	0.9999406E-13
4	4	1	6	3	502	10	0.2911109E-04
4	4	2	13	3	943	14	0.2416260E-04
4	4	3	10	3	783	13	0.2552328E-04
5	4	1	4	1	248	7	0.0000000E 00
5	4	2	4	1	242	8	0.1776357E-13
5	4	3	5	1	280	9	0.1705303E-12
6	4	1	3	3	175	11	0.4571443E-10
6	4	2	4	1	209	11	0.1149114E-10
6	4	3	4	2	207	11	0.3028162E-03
7	4	1	1	1	46	6	0.2666667E 01
7	4	2	1	1	89	6	0.2666667E 01
7	4	3	1	1	68	7	0.2666667E 01
8	4	1	5	1	253	9	0.5435652E-12
8	4	2	4	2	204	8	0.4867218E-12
8	4	3	4	1	229	8	0.6373639E 00
9	4	1	3	2	137	15	0.5829702E-07
9	4	2	3	3	178	20	0.3902016E-08
9	4	3	4	1	206	20	0.1164722E-09
10	4	1	4	3	197	33	0.6958773E-01
10	4	2	4	2	193	32	0.6958733E-01
10	4	3	4	3	197	33	0.6958773E-01
11	4	1	3	3	190	11	0.9537200E-05
11	4	2	2	3	128	10	0.9547088E-05
11	4	3	3	2	239	13	0.9715007E-03
12	4	1	4	1	218	10	0.1008360E-11
12	4	2	4	1	210	11	0.5739853E-13
12	4	3	6	2	523	15	0.1447365E-09

NPROB	N	XZERO	CYC	ITER	IFUN	TEMP	F
1	10	1	31	9	7879	97	0.1547769E-01
1	10	2	8	1	2420	33	0.1331713E 00
1	10	3	21	8	5610	131	0.1971989E 00
2	8	1	10	7	1814	28	0.3519635E-08
2	8	2	9	7	1597	25	0.2412440E-05
2	8	3	12	5	2665	38	0.1382726E 01
3	10	1	2	6	1286	22	0.1671677E-16
3	10	2	7	4	1887	33	0.2008595E-18
3	10	3	3	9	1323	23	0.8578724E-18
4	10	1	7	7	2830	47	0.7836072E-04
4	10	2	7	9	2347	42	0.8022473E-04
4	10	3	1	9	1073	21	0.1569622E-02
5	10	1	14	1	3538	68	0.5694076E-09
5	10	2	12	5	2991	59	0.5836753E-09
5	10	3	19	7	4790	89	0.3911815E-08
6	10	1	3	9	722	43	0.3328136E-05
6	10	2	6	5	1464	78	0.3849585E-07
6	10	3	4	7	1021	57	0.3792897E-07
7	10	1	1	1	129	9	0.4142857E 01
7	10	2	1	2	257	12	0.4142857E 01
7	10	3	1	2	138	10	0.4142858E 01
8	10	1	5	3	1171	29	0.8728691E-02
8	10	2	9	8	2196	49	0.3500761E-02
8	10	3	20	2	515	102	0.9329068E 01
9	10	1	3	9	1110	520	0.9229598E-06
9	10	2	3	1	773	364	0.1649823E-08
9	10	3	5	1	1276	598	0.7796643E-09
10	10	1	3	2	696	237	0.7461312E-04
10	10	2	3	8	776	263	0.6785664E-04
10	10	3	3	2	696	237	0.7461312E-04
11	10	1	5	2	2033	159	0.2944994E-03
11	10	2	4	9	1581	118	0.2945766E-03
11	10	3	3	3	1634	130	0.2979358E-03
12	10	1	5	8	1209	84	0.4784756E-0[illegible]
12	10	2	5	3	1151	81	0.4831340E-0[illegible]
12	10	3	7	9	2984	195	0.2152653E-0[illegible]

Table 4

NPROB	N	XZERO	CYC	ITER	IFUN	TEMP	F
1	4	1	8	1	438	18	0.1885337D-14
1	4	2	7	3	406	18	0.7893525D-11
1	4	3	15	2	810	30	0.5298247D-10
2	4	1	7	3	390	19	0.4175356D-13
2	4	2	8	3	470	22	0.2444547D-13
2	4	3	5	3	300	16	0.6097524D-04
3	4	1	4	1	199	11	0.9424900D-23
3	4	2	4	1	199	12	0.4550525D-22
3	4	3	3	1	143	11	0.3774113D-14
4	4	1	6	3	375	20	0.2909931D-04
4	4	2	6	3	375	20	0.2909966D-04
4	4	3	11	3	776	37	0.2509131D-04
5	4	1	4	1	193	14	0.2232696D-21
5	4	2	4	1	193	14	0.4521384D-21
5	4	3	4	1	211	15	0.1526355D-14
6	4	1	4	1	205	52	0.2328290D-15
6	4	2	4	1	205	51	0.3531229D-15
6	4	3	4	3	219	54	0.3028241D-03
7	4	1	1	1	29	7	0.2666667D 01
7	4	2	1	1	50	8	0.2666667D 01
7	4	3	1	1	33	8	0.2666667D 01
8	4	1	4	1	175	12	0.3627790D-11
8	4	2	4	1	194	13	0.4590915D-11
8	4	3	4	1	197	14	0.6373640D 00
9	4	1	3	1	128	105	0.6961723D-08
9	4	2	3	1	126	104	0.4653792D-07
9	4	3	4	1	180	146	0.3501550D-12
10	4	1	4	3	227	156	0.6958772D-01
10	4	2	4	3	227	160	0.6958772D-01
10	4	3	4	3	227	156	0.6958772D-01
11	4	1	2	3	87	27	0.9720844D-05
11	4	2	2	3	106	41	0.9499948D-05
11	4	3	3	2	152	44	0.9714704D-05
12	4	1	4	1	191	21	0.6416760D-13
12	4	2	4	1	188	28	0.2190830D-13
12	4	3	7	1	449	39	0.1317098D-12
NPROB	N	XZERO	CYC	ITER	IFUN	TEMP	F
1	10	1	14	[illegible]	3837	222	0.1749111D 00
1	10	2	9	1	2435	153	0.3328696D-01
1	10	3	10	9	2813	163	0.2779195D 00
2	8	1	9	7	1693	105	0.1821409D-08
2	8	2	10	7	1887	118	0.3475085D-07
2	8	3	19	7	3558	234	0.9311145D-10
3	10	1	3	2	696	45	0.1536921D-20
3	10	2	3	2	696	45	0.1546087D-20
3	10	3	2	5	718	41	0.3342667D-13
4	10	1	10	9	3142	239	0.8964973D-04
4	10	2	8	7	2480	190	0.7415362D-04
4	10	3	6	7	1858	144	0.1021991D-03
5	10	1	5	1	1245	109	0.1364837D-13
5	10	2	5	8	1420	111	0.8588377D-10
5	10	3	6	3	1633	124	0.7062060D-10
6	10	1	5	8	1340	723	0.4473892D-06
6	10	2	5	6	1282	693	0.3214896D-10
6	10	3	4	8	1092	591	0.7681483D-10
7	10	1	1	1	124	25	0.4142857D 01
7	10	2	1	1	217	39	0.4142857D 01
7	10	3	1	1	211	36	0.4142857D 01
8	10	1	9	6	2278	180	0.1773174D-09
8	10	2	10	1	2451	194	0.8389859D-12
8	10	3	11	2	3179	252	0.3898880D 01
9	10	1	3	1	570	3241	0.1106292D-16
9	10	2	2	1	298	1697	0.1166062D-04
9	10	3	4	9	1072	6099	0.4612646D-06
10	10	1	2	9	552	679	0.9520944D-04
10	10	2	2	9	555	951	0.7608388D-04
10	10	3	2	9	552	679	0.9520944D-04
11	10	1	10	9	3447	2694	0.2945644D-03
11	10	2	8	8	2666	1863	0.2943835D-03
11	10	3	9	8	2974	2052	0.2952027D-03
12	10	1	9	1	2169	602	0.4772714D-02
12	10	2	5	8	1294	366	0.4777713D-02
12	10	3	12	5	4379	1186	0.6718247D-01

R E F E R E N C E S

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