So we have the following

THEOREM 1. For the algebra $S(\cdot,\times)$ the following properties hold

1) $S = G_1 \otimes G_2$ and $g^2 = 1$ $\forall g \in G_2$.

2) $g_1 g_2 \times h_1 h_2 = g_1 h_2$ $(g_1, h_1 \in G_1 \,,\; g_2, h_2 \in G_2)$.

Conversely we can prove

THEOREM 2. Let the group $S(\cdot)$ be the direct product of two subgroup G_1, G_2 such that $g^2 = 1$ for every $g \in G_2$. A semigroup operation "$\times$" with an idempotent element exists in S such that

$$\forall\, a,b,c \in S : (a \times b)c = ac \times bc \,, \qquad c(a \times b) = ca \times c^{-1}b \,.$$

Proof. A few calculations show that the required operation is defined as follows:

$$g_1 g_2 \times h_1 h_2 = g_1 h_2 \quad \text{for every} \quad g_1, h_1 \in G_1,\; g_2, h_2 \in G_2 \,.$$

REMARK. Finally we observe that theorems analogous to theorems 1 and 2 can be proved if in place of (α) one has

(β) $$\forall\, a,b,c \in S : (a \times b)c = ac \times bc \qquad c(a \times b) = c^{-1} a \times cb$$

R E F E R E N C E S

[S] J. Szép : *On a finite algebra with two operations.* Act.Math.Academiae Scientiarum Hungaricae Tomus 26 (3-4),(1975),347-348.

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