2 - FRAMES AND THE REPRESENTATION OF $T\mathbb{E}$.

In this section we are dealing with the first order derivatives of the frame and tangent spaces.

Frame velocity and jacobians.

1 The velocity of the frame is the vector field on $\mathbb{E}$ constituted by the velocities of the world-lines of the frame. Hence it is the first derivative of the notion with respect to time. On the orther hand, the jacobians are the first derivatives with respect to event. We consider only free entities, for simplicity of notions, leaving to the reader to write them in the complete form.

DEFINITION.

a) The (FREE) VELOCITY-FUNDAMENTAL FORM - of $\mathcal{P}$ is the map

$$D_1\tilde{P} : \mathbb{T} \times \mathbb{E} \to \bar{\mathbb{E}} .$$

The (FREE) VELOCITY-EULERIAN FORM - of $\mathcal{P}$ is the map

$$\bar{P} \equiv D_1\tilde{P} \circ j : \mathbb{E} \to \mathbb{E}$$

b) The (FREE) JACOBIAN-FUNDAMENTAL-EULERIAN FORM - of $\mathcal{P}$ is the map

$$D_2\tilde{P} : \mathbb{T} \times \mathbb{E} \to \bar{\mathbb{E}}^* \otimes \bar{\mathbb{E}} .$$

The (FREE) JACOBIAN-EULERIAN-EULERIAN FORM - of $\mathcal{P}$ is the map

$$\hat{P} \equiv D_2\tilde{P} \circ j : \mathbb{E} \to \bar{\mathbb{E}}^* \times \bar{\mathbb{E}} \quad .$$

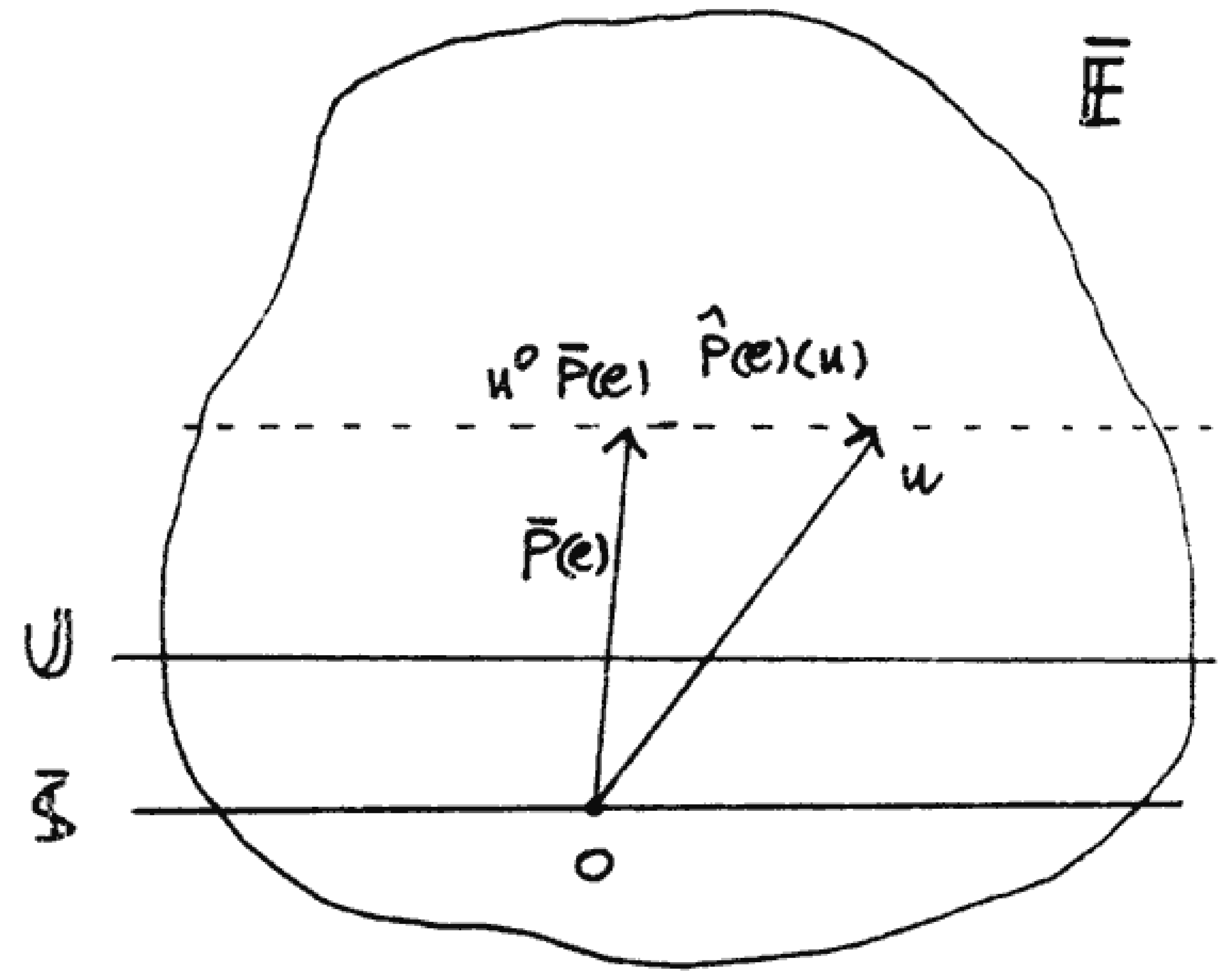
The (FREE) SPATIAL JACOBIAN-FUNDAMENTAL- EULERIAN FORM - of $\mathcal{P}$ is the map

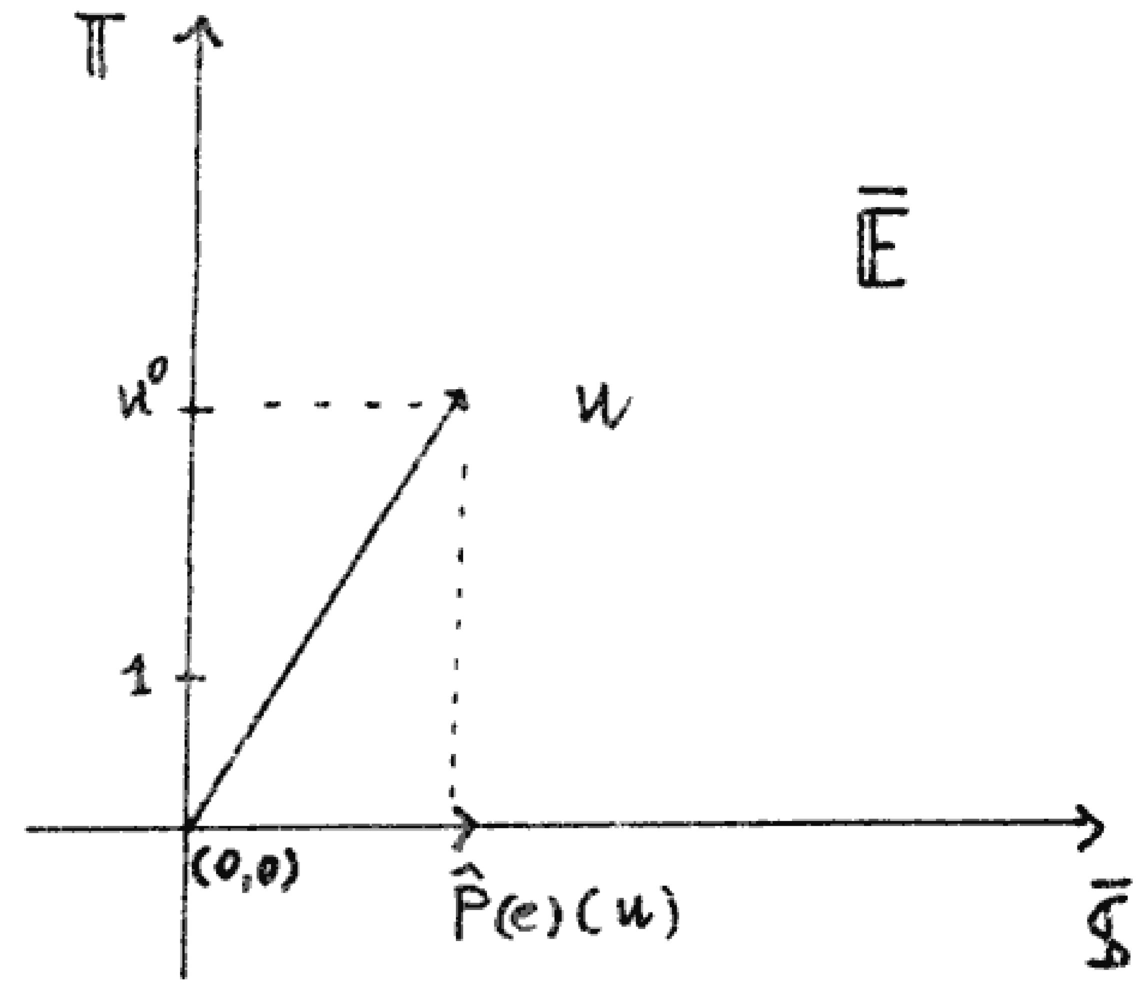
$$\check{P} \equiv \check{D}_2\tilde{P} : \mathbb{T} \times \mathbb{E} \to \bar{\mathbb{S}}^* \otimes \bar{\mathbb{E}}$$

The (FREE) SPATIAL JACOBIAN-LAGRANGIAN-LAGRANGIAN FORM, RELATIVE TO THE INITAL TIME $\tau \in \mathbb{T}$ AND TO THE FINAL TIME $\tau' \in \mathbb{T}$, of $\mathcal{P}$ - is the map

$$\check{P}_{(\tau'\tau)} \equiv D\tilde{P}_{(\tau'\tau)} : \mathbb{S}_\tau \to \bar{\mathbb{S}}^* \otimes \bar{\mathbb{S}} \ .$$

We will denote by


E
u° P(e)
P(e)(u)
u
P(e)
U
S
o


T
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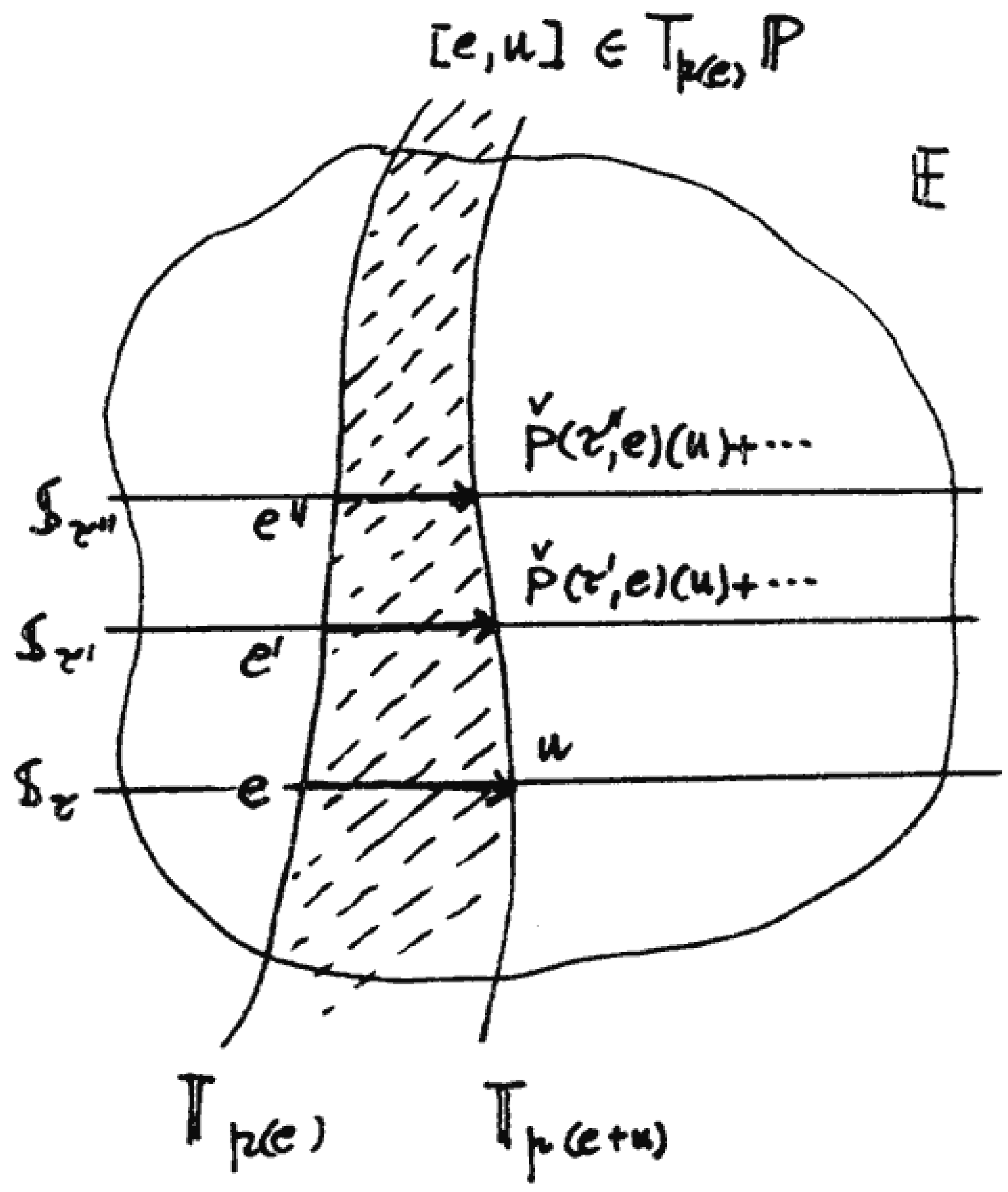
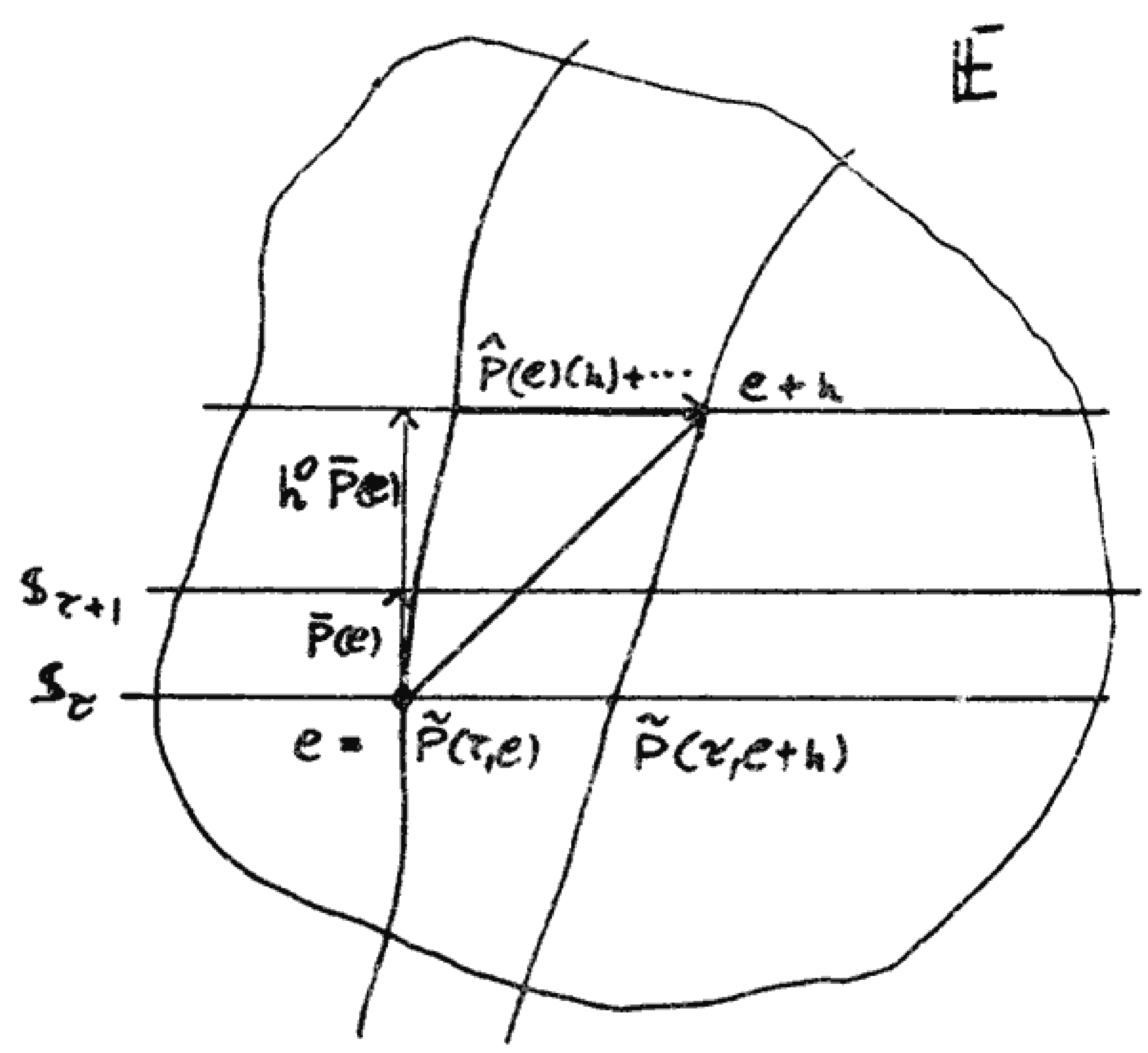

[e,u] ∈ T_p(e) P
E
P(τ'',e)(u)+···
S_τ''
e''
P(τ',e)(u)+···
S_τ'
e'
u
S_τ
e
T_p(e)
T_p(e+u)


E
P(e)(h)+···
e+h
h° P(e)
S_τ+1
P(e)
S_τ
e = P(τ,e)
P(τ,e+h)