INTRODUCTION

Summary.

The general framework of classical mechanics is the absolute event space $\mathbb{E}$. It is an affine four dimensional space, representing the set of absolute events. A three dimensional subspace $\bar{\mathbb{S}}$ of its vector space $\bar{\mathbb{E}}$ is fixed to represent the set of couples of symultaneous events. Then $\mathbb{E}$ results into the disjoint union of parallel three dimensional affine subspaces $\mathbb{S}_\tau$, generated by $\bar{\mathbb{S}}$, which represent the equivalence classes of symultaneous events. The set $\mathbb{T}$ of these equivalence classes is a one dimensional affine oriented space, which represents absolute time.

The quotient projection $t : \mathbb{E} \to \mathbb{T}$ is the time function. The triple $(\mathbb{E},t,\mathbb{T})$ is an affine trivial bundle (but not canonically a product), whose fibers are the(not canonically isomorphic among themselves) equivalence classes $\mathbb{S}_\tau$. The map $Dt : \bar{\mathbb{E}} \to \bar{\mathbb{T}}$ associates with each four vector u its absolute time component u°. We have not an absolute projection $\bar{\mathbb{E}} \to \bar{\mathbb{S}}$, or an absolute inclusion $\bar{\mathbb{T}} \hookrightarrow \bar{\mathbb{E}}$. Then we have not an absolute splitting $\bar{\mathbb{E}} = \bar{\mathbb{T}} \oplus \bar{\mathbb{S}}$ (whereas it is induced by a frame of reference). The inclusion $\bar{\mathbb{S}} \hookrightarrow \bar{\mathbb{E}}$ admits the vertical (along $\bar{\mathbb{S}}$,i.e. at a fixed time) derivatives Df of maps defined on $\mathbb{E}$. On the vector space $\bar{\mathbb{S}}$ we have absolute euclidean metric $\check{g}$, defined up to a conformal factor, which describes the classical geometry. For pratical reasons we choose a unit of measure on $\bar{\mathbb{S}}$ and on $\bar{\mathbb{T}}$, selecting the conformal factors. The unit of measure on $\bar{\mathbb{T}}$ determines the identification $\bar{\mathbb{T}} \cong \mathbb{R}$. Then we get the subspace $\mathbb{U} \hookrightarrow \bar{\mathbb{E}}$, constituted by the vectors normalized by $\underline{t} \equiv Dt$, which represents the space of velocities. We define a Poincare's map as a map $G : \mathbb{E} \to \mathbb{E}$, which preserves the structure of $\bar{\mathbb{S}}$.

Then we consider rigid frames, which are affine frames such that $\epsilon_{\mathcal{P}}$ is zero. The spatial derivative of their motion is unitary. Their motion is determined by the motion of one of their particles and by its spatial unitary derivative(by a time derivation we obtain, from this fact, the classical formula for the velocities of the rigid frame). As the motion preserves, along the time, the spatial metric, $\mathbb{P}$ results into an affine euclidean space,namely $g_{\mathcal{P}}$ results time independent and we can write $g_{\mathcal{P}} : T\mathbb{P} \to \mathbb{R}$. Then we consider translating frames, which are rigid frames such that $\Omega_{\mathcal{P}}$ is zero. The spatial derivative of their motion is zero. Their motion is determined by the motion of one of their particles. As the motion preserves, along the time, the spatial vectors, the vector space $\bar{\mathbb{P}}$ results to be equal to $\bar{\mathbb{S}}$. Finally we consider inertial frames, which are translating frames such that $\bar{\bar{P}}$ is zero. The total derivative of their motion is zero. Their motion is determined by the inertial motion of one of their particles. The projection $\hat{P}$ is constant, hence $\check{\Gamma}_P = \Gamma'_P$.

Now we consider a fixed frame $\mathcal{P}$ and a fixed motion M, we define the quantities of M observed by $\mathcal{P}$ and we make a comparison between absolute and observed quantities. The observed motion is the map $M_{\mathcal{P}}:\mathbb{T} \to \mathbb{P}$, which associates with each time $\tau \in \mathbb{T}$ the position $p(M(\tau))$ touched by M at that time. The observed motion $M_{\mathcal{P}}$ characterizes the absolute motion M, since $M(\tau) = \tilde{P}(\tau, M_{\mathcal{P}},(\tau))$. Then we get the velocity of the observed motion $dM_{\mathcal{P}} : \mathbb{T} \to T\mathbb{P}$, which is the derivative of $M_{\mathcal{P}}$ performed by $\mathcal{P}$, by means of its differentiable structure. We get also the acceleration of the observed motion $\check{\nabla}_{\mathcal{P}} \check{d}M_{\mathcal{P}}:\mathbb{T} \to T\mathbb{P}$, which is the covariant derivative of the velocity of the observed motion, per

formed by $\mathcal{P}$, by means of its time depending affine connection $\check{\Gamma}_{\mathcal{P}}$.
The observed velocity of M is the projection on $T\mathbb{P}$ of the velocity dM. The observed velocity and the velocity of the observed motion are equal.
The observed acceleration is the proiection of the sum of the acceleration of the observed motion and of a generalized Coriolis term, plus a dragging term. Namely we can write $D^2M = D^2_{\mathcal{P}}M_{\mathcal{P}}+C_{\mathcal{P}}(D_{\mathcal{P}}M_{\mathcal{P}})+D_{\mathcal{P}}\circ(M_{\mathcal{P}})$.

Finally we consider two frames $\mathcal{P}_1$ and $\mathcal{P}_2$ and a motion M and we make a comparison among quantities observed by $\mathcal{P}_1$ and by $\mathcal{P}_2$. First we consider the quantities of $\mathcal{P}_1$ observed by $\mathcal{P}_2$. Then we find the addition velocities theorem and the generalized Coriolis Theorem. Specializing the kind of frame of references, we get the usual theorems.

Comparison with special and general relativity.

We want to show some surprising and important analogies with the special and general relativity, not involving the light velocity.

In both cases we have a four dimensional event space $\mathbb{E}$, which is affine in the classical and special relativistic case and which has not an absolute splitting into space and time. In the classical case we have a privileged three dimensional subspace $\bar{\mathbb{S}} \hookrightarrow \bar{\mathbb{E}}$, which determines the absolute simultaneity and the absolute time as the quotient space $\mathbb{T}\equiv\mathbb{E}_{/\bar{\mathbb{S}}}$.
These facts have not an absolute relativistic counterpart. On the other hand, in the relativistic case we have a Lorentz metric on the whole $T\mathbb{E}$ (it is constant and fixed in special relativity and it is matter depending in general relativity), while in the classical case we have

only an euclidean metric on $T\mathbb{E}$. The relativistical Lorentz metric determines, by orthogonality, a frame depending and pointwise (local if the frame is integrable) spatial section which replaces the classical $\bar{\mathbb{S}}$. The classical time orientation is given on $\mathbb{T}$, while the relativistic one is given on the light cone.

In all the three cases we can describe a motion (or its world-line) by a four dimensional map $M : \mathbb{T} \to \mathbb{E}$ (or by a one dimensional submanifold $\mathbb{M} \hookrightarrow \mathbb{E}$), which is absolute, i.e. not depending on any frame of reference. In the relativistic case the condition that $\mathbb{M}$ is time-like replaces the classical condition $t \circ M = id_{\mathbb{T}}$. In the relativistic case $\mathbb{T}$ is not the absolute time, but the proper time of the motion, namely it is $\mathbb{M}$ itself endowed with the affine euclidean structure induced by the metric of $\mathbb{E}$. In the classical case we get $<\underline{t},DM> = 1$ and $<\underline{t},D^2M>=0$.

These conditions are replaced in the relativistic case by $DM^2=-1$ and $DM \circ D^2M = 0$.

In all three cases we can consider the most general kind of frame of reference,while people often consider only rigid frames in classical mechanics and inertial frames in special relativistic. The definition of frame is essentially the same in all three cases: the only differences come from the implicit differences in the definition of the world lines of the continuum particles. Analogous considerations hold for the representation of $\mathbb{P}$, $T\mathbb{P}$ and $T^2\mathbb{P}$, for the time depending metric and connection, the Coriolis and dragging maps and the classification of special frames.

Since we deal with general frames of reference, we get for classical and special relativistic observed kinematics criteria currently used in general relativity. In fact under our statment of the absolute Coriolis Theorem we can recognize usual general relativistic formulas, commonly quoted in other form.