P R E F A C E

Proposal.

This paper is the first of a number devoted to an axiomatic approach to classic and relativistic mechanics.

We analyse the foundations of mechanics, trying to reach a new unifying view and to get a systematic exposition of the matter. In these years it is actual in the literature a foundational research, even if along a little different lines.

We study, in a unique context and with a unified language, topics often treated by different authors with different points of view. We try to surpass critically the historical stratification of the matter. In fact, often theories develop under the push of motivations and in a cultural context, which after some time change completely. Nevertheless, the substantial validity of the theories remains. So, while it is historically essential to understand the birth and the development of theories in their real context, from a technical point of view, such an approach can be confusing with respect to the essential synctactical structure of the theory. Moreover, a new synthesis that, even taking into account the historical logic, tries to achieve an independent formulation, can lead to a new phylosophical view.

In these papers we are explicitly concerned only with a theoretical axiomatic treatment.

Phylosophical background.

We want to outline the phylosophical background common to the present and to the subsequent papers, without any claim of rigor and completeness.

We think that a physical theory consists of several mutually connected languages, with different synctaxes, objects and degrees of formal rigor.

There are at least

a) a mathematical synctactical language, which is deductive, selfconsistent, formal, whose object is the theoretical model of the theory;

b) a physical experimental language, which is intuitive, descriptive, whose object is the description of phenomena;

c) an interpretative semantical language, whose object is the relation between the previous two languages.

The appropriate order of exposition of the matter can be different for the mathematical and the physical languages. So the validity of the theory, namely the agreement between the previous languages, must be tested globally and it is meaningless to verify a single axiom or theorem out of their context.

Of course this structure of the theory is not more than an outline.

We are firmly convinced that an omnicomprehensive supertheory cannot exist. We must necessarily deal with a lattice of physical theories, with different physical objects and degrees of validity. The comparison among them is very important and physically expressive. For example the validity of a physical theory is often tested in the context of a more general one.

It is fit to distinguish the language of a theory (even if branched into several sublanguages) from metalanguages which have the theory itself as object. For example, the relativity principles are not part of a phisical theory, as they do not describe physical phenomena, but they are metalinguistic conditions imposed to the theory.

We keep quite distinct the inductive and the deductive construction of a physical theory. In fact, the former has a value more historical than logic, whereas the latter is selfcontained and gives a deep physical insight.

In this way, for each physical question we have to make explicit the experimental and theoretical context, the required degree of approximation and the background accepted as known. As an esample, let us consider inertial frames. We can take into account the geometrical properties of space-time observed by them, the comparison between real and apparent forces observed by them (that involves a theory of interactions), the classical and relativistic approximation, their local and global existence, their experimental determination, etc. The abstract question "what is an inertial frame of reference?" regardless of the previous statements is meaningless.

In the present and in the subsequent papers we deal essentially with the mathematical synctactical language. We follow the actual structuralistic tendency of modern mathematics. Our physical approach is based on a deep analysis of the structure of the fundamental spaces constituting the general framework.
If the good fitting between theory and experiments is not too occasional and limited, but has a deep validity, the choice of basic spaces of the theory cannot be of little relevance and they must contain implicitly all the physical development. We believe that in a good theory all the facts that are mathematically relevant have a great physical interest and viceversa.

We believe that the spirit of Klein's program, of classification of geometrical theories based on their invariance properties, can be surpassed. In fact it was natural in the context of a mathematical language strictly based on coordinates. The situation is quite different now, because we have the intrinsical language of algebra, topology,...

manifolds, fiber bundles,... .
Nevertheless theoretical physics is up to now deeply based on invariance groups and their representation. We think it is time for a change. This proposal requires a large inversion in the traditional sequence and dependence of topics. In such a way, the deep role of mathematics in physical theories gets more relevant and it does not reduce to a computational aspect.

We expect that differential geometry will play a more and more important role in physics. This tendency is present in literature but it does not develop its whole euristic power.

People often say that a high formalization of the theory and a large inversion in its traditional exposition is hard to understand and damages intuition.Not in the long rum, it is our opinion.. In fact we believe that intuition is a process that makes automatic and uncoscious logic proceedings, so that syntax becomes semantic, by means of a long exercise. Then, what to day is abstract to morrow can be intuitive. The more a theory is based on few and well organized axioms, the more the intuitive process will be fast and complete. This believing is supported by many historical examples. The most typical regards elementary geometry. The classic euclidean logic is to day an intuitive description of geometrical daily and familiar physical phenomena. However we make intuitive the description of the same phenomena by means of linear algebra.

Specific criteria.

We try to get a unifying view of classical and special and general relativistic theories. Namely we use the same kind of language and exposition line. We have very similar general frameworks for the three

theories so that the structural differences appear clear and directly comparable. In all cases we have four dimensional "absolute" event spaces, four-dimensional "absolute"motions, velocity and accelerations, four-dimensional forces and so on. In all cases we consider general frames of references and we define the "observed"phenomena by means of the splitting into space and time induced by frames. This point of view is commonly considered as proper to general relativity.
Then for all three theories a general principle of relativity holds! Moreover the constancy of light velocity has not an explicit role and it is completely replaced by the metrical structure of the event space. In all three theories we have well fitted electromagnetic theories, along similar lines.

For classical and special relativistic theories we make a large use of affine spaces.Thatis justified by the physical properties of event spaces. The main peculiarity of affine spaces are free vectors, that is a natural displacement of applied vectors. So we could treat the theory only in terms of free vectors,employng free derivatives Df and D^2f of maps f between affine spaces. But we have also to consider non affine entities, as submanifolds, general frames and coordinates. Then we use a mathematical formalism, which allows a view of affine spaces in terms of free or applied vectors, introducing tangent spaces, affine connection, etc.., hence considering affine spaces as special manifolds.

Affine spaces are the basic element that determines our intrinsical language, permitting a clear and deep distinction among absolute phenomena, frames of reference and coordinate systems.
Galilei's and Lorentz's maps turn out to be of little importance.
In fact these are implicit in the general framework and do not play any basic role in the following exposition. This point of view is upsetting

of current treatment and can influence physical theories based on group representation.

We try to unify in a unique context topics generally exposed in ordinary mechanics, in analytical mechanics, in continuum mechanics, in foundations, etc. Of course we limit ourselves only to a general introductive statement.