§1- We first point out the symbols that will be used in the sequel. We will write Maxwell's equations in Gaussian units and denote the mean current density by $\bar{J}$, the electric and the magnetic fields by $\bar{E}$ and $\bar{B}$, the dielectric constant by ε, the magnetic permeability by μ, the speed of light in vacuum by c, the conductivity by σ. Furthermore we set:

$\rho(P,t)$=mean charge density inside the conductor at point P, time t

$\rho_c(P,t)$ = mean charge density at P,t of charge carriers

ρ_s = mean charge density of non-moving charges

$\bar{v}(P,t)$ = mean velocity at P, t of the charge carriers. (We observe explicitly that $\rho=\rho_s+\rho_c$, and that ρ_s does not depend on P,t).

We now intend to discuss briefly the unsatisfactory character of some results which follow from Ohm's law when it is introduced into Maxwell's equations inside conductors without appropriate criticism. For simplicity we will limit ourselves in the main part of this work to consider homogeneous isotropic conductors (some indications on possible generalizations will be given in §6). Then, the conductivity σ is a scalar and Ohm's law in differential form reads

$$\bar{J} = \sigma \bar{E} \tag{1.1}$$

It is well known that, if we assume that eq.(1.1) is correct, we get from Maxwell's equations

$$\rho(P,t) = \rho(P,0)\ e^{-t/\tau} \quad (\tau=\varepsilon/4\pi\sigma) \tag{1.2}$$

Now, let us consider an homogeneous and isotropic conductor where the current is due to the motion of negative charge carriers of a given type only.

Let S be a closed surface internal to the conductor, let D be the domain enclosed by it, and suppose that $\rho(P,0) = 0$ outside S; then, $\rho(P,t) = 0$, because of eq.(1.2), outside S. The total charge Q(t) inside S at time t is given by

$$Q(t) = \int_D \rho(P,t)\, dD = Q(0) e^{-t/\tau}$$

Let S' be another closed surface internal to the conductor and enclosing S, let D' be the domain enclosed by S' and let $\bar{n}(P,t)$ be the normal to S' directed outwards. The current I' through S' is given by

$$I' = \int_{S'} \bar{J} \cdot \bar{n}\, dS' = - \frac{d}{dt} \int_{D'} \rho(P,t) dD' = - \frac{d}{dt} \int_D \rho(P,t) dD =$$

$$= - \frac{dQ}{dt} = + \frac{1}{\tau} Q(0)\, e^{-t/\tau}$$

Now, observe that $\rho_c = - \rho_s$ at any point of S' and at any time, since $\rho(P,t) = 0$ if P belongs to S'. Hence we get

$$\bar{J} = \rho_c \bar{v} = - \rho_s \bar{v}$$

Substituting into the expression of I', we have

$$\int_{S'} \bar{v} \cdot \bar{n}\, dS' = - \frac{1}{\rho_s \tau} Q(0)\, e^{-t/\tau}$$

Let A' be the area of S' and set $v_n = \bar{v} \cdot \bar{n}$; then, we get

$$\frac{1}{A'} \int_{S'} \bar{v} \cdot \bar{n}\, dS' = \langle v_n \rangle = - \frac{1}{\rho_s \tau} e^{-t/\tau} \frac{Q(0)}{A'}$$

Since Q(O) and A' are arbitrary, for any $t_1 > 0$ we can choose Q(O) and A' such that

$$\frac{Q(O)}{A'} = - c\tau\rho_s e^{t_1/\tau}$$

so that

$$<v_n> = c\ e^{(t_1 - t)/\tau}$$

Hence, $<v_n> > c$ for any t such that $0<t<t_1$; this obviously contradicts the special relativity.

As a second example of the inadequacy of Ohm's law in its form (1.1), consider a conductor where a current $\bar{J}$ flows in the presence of a stationary magnetic field. Then, the potential φ of the electric field inside the conductor follows the Laplace's equation, while the boundary conditions that we must associate with this equation to determine φ do not change because of the magnetic field. Therefore, eq.(1.1) cannot explain, not even qualitatively, any effect, such as the Hall effect, caused by the presence of a magnetic field. For this reason most textbooks, when dealing with the Hall effect in the framework of the classical electromagnetism consider only particular cases and obtain equations that, while correct, are not consistent with the usual practice of writing $\bar{J} = \sigma\bar{E}$ into Maxwell's equations inside conductors.[4]

Finally, we remark that even simple conduction models for media in which the current is due to only one kind of charge carrier show that an equation of the form

$$\bar{J} = \pm\alpha\rho_c\bar{E} \qquad (1.3)$$

(where α is the mobility of the charge carriers and it is always positive, and the sign is plus if ρ_c is positive, minus if ρ_c is negative) is more realistic than eq.(1.1). Anyway, it is easy to show that not even eq. (1.3) is consistent with the special relativity.[5] Analogously, the magnetic field does not enter into eq.(1.3), so that we can again predict that no effect such as the Hall effect can be interpreted by it.

§2- The inconsistencies discussed in the last section suggest that a more appropriate formulation of the conduction law inside conductors should be given. To this end we remind that eq.(1.1) becomes

$$\bar{J} = \sigma(\bar{E} + \bar{E}^{+})$$

when an "effective" field $\bar{E}^{+}$ adds to $\bar{E}$.
Then, if we consider a homogeneous isotropic conductor where the current is due to the motion of charge carriers of one type only, one immediately thinks that the general law of conduction has the form[2][3]

$$\bar{J} = \pm\, \alpha\, \rho_c(P,t)\ \bar{f}(P,t) \qquad (2.1)$$

where we call $\bar{f}$ the total force acting on the unitary charge inside the conductor[6] and the sign is chosen as in eq.(1.3). Note that eq. (2.1) takes into account eq.(1.3), and that α may depend on the frequence ω of the force (this implies Fourier transforms[7]). The force $\bar{f}(P,t)$ generally includes the magnetic field; if only electromagnetic fields act inside the conductor, eq.(2.1) takes the form

$$\bar{J} = \pm\, \alpha\, \rho_c \bar{E} \pm \frac{\alpha\rho_c}{c}\, \bar{v}\times\bar{B} = \pm\, \alpha\, \rho_c \bar{E} \pm \frac{\alpha}{c}\, \bar{J}\times\bar{B} \qquad (2.2)$$

Thus, we are led to think that eq.(11) must be replaced by eq. (2.2). Attaining a full generalization of this equation goes beyond the limits of the present article. We restrict ourselves here to observing that, considering an isotropic and homogeneous conductor where n kinds of charge carriers exist, eq.(2.2) holds separately for any kind of charge carrier. Denoting any quantity related to the carriers of type ℓ by the suffix ℓ, we get

$$\overline{J}_\ell = \pm \alpha_\ell \rho_{c\ell} \overline{E} \pm \frac{\alpha_\ell}{c} \overline{J}_\ell \times \overline{B} \qquad (2.3)$$

where the sign, as usual, is plus for positive carriers and minus for negative carriers. Hence the total current density will be given by

$$\overline{J} = \sum_\ell \overline{J}_\ell = (\sum_\ell \pm \alpha_\ell \rho_{c\ell}) \overline{E} + \frac{1}{c} (\sum_\ell \pm \alpha_\ell \overline{J}_\ell) \times \overline{B} \qquad (2.4)$$

Note that $\sum_\ell \pm \alpha_\ell \overline{J}_\ell$ is not generally proportional to $\overline{J}$; the eq. (2.4) is then more difficult to discuss than eq. (2.2).

§3- In § 2 we have introduced the conduction equations (2.2) and (2.4). We note that these equations have been written on a phenomenological basis, without any "justification" in the framework of a conduction model and without taking explicitly into account special relativity.
Thus, in particular, we cannot pretend that they are relativistically consistent, even if we expect that any relativistic correction will be neglegible for most practical purposes.

We intend now to show that, making use of an elementary conduction model, a form for the conduction law can be suggested which is relativistically correct (because of the way it has been obtained) and which coincides with eq.(2.2) if we assume that in the latter the coefficient α is independent of $\bar{E}$ and $\bar{B}$ only where small conduction velocities are concerned. To this end, we consider the simple conduction model used by some textbooks to give an elementary "explanation" of Ohm's law, taking into account the magnetic force acting on the charge carriers and using relativistic instead of classical mechanics. The following assumptions will be made[8]:

a) the conductor is made of neutral atoms and charge carriers, and the atoms interact with the carriers only at short range: if no field exists, the carriers move freely between two collisions.

b) the density of the carriers is very low, so that we can ignore their mutual interactions.

c) no statistical correlation exists between the velocity and momentum of a carrier before a collision and its velocity and momentum afterwards.

d) the electric and magnetic fields inside the conductor are so small that they do not alter the mean free time T between two collisions of a given carrier with an atom (if the temperature is fixed), and they vary so slowly in space and time that they are nearly constant during this time and along the mean free path of the charge carriers.

e) (Ergodic hypothesis). The mean statistical value of any dinamical quantity relative to a charge carrier, taken over a great number of carriers, is equal to the mean time value of this quantity during a sufficiently large number of collisions of a given charge carrier (under the assumption that $\bar{E}$ and $\bar{B}$ are nearly constant during this period).

Then, let us consider the carriers of a given type, having charge q and rest mass M_o, and let ΔN be their number in the volume ΔV.

Let $\bar{u}_i(t)$ and $\bar{p}_i(t)$ be the speed and the momentum of the i-th carrier of this kind at time t, suppose that its last collision with an atom took place at time t_i, and let $\bar{s}_i$ be its displacement in the interval (t_i, t). We have

$$\bar{p}_i(t) = \bar{p}_i(t_i) + \int_{t_i}^{t} (q\bar{E} + \frac{q}{c}\,\bar{u}_i(t') x \bar{B})\,dt' =$$

$$= \bar{p}_i(t_1) + q\bar{E}(t - t_i) - \frac{q}{c}\, B\, x \int_{t_i}^{t} \bar{u}_i(t')\,dt' =$$

$$= \bar{p}_i(t_i) + q\bar{E}(t - t_i) - \frac{q}{c}\,\bar{B}\, x\, \bar{s}_i$$

Summing over the ΔN carriers and dividing by ΔN we get

$$\frac{1}{\Delta N}\sum_i \bar{p}_i(t) = \frac{1}{\Delta N}\sum_i \bar{p}_i(t_i) + q\bar{E}\,\frac{1}{\Delta N}\sum_i (t - t_i) - \frac{q}{c}\,\bar{B}\, x\, \frac{1}{\Delta N}\sum_i \bar{s}_i$$

Now, $\frac{1}{\Delta N}\sum_i \bar{p}_i(t) = \bar{p}(t)$ is the mean momentum in ΔV of our carriers at time t(9); $\sum_i \bar{p}_i(t_i)$ is zero since the momenta $\bar{p}_i(t_i)$ are directed at random; $\frac{1}{\Delta N}\sum_i (t - t_i)$ and $\frac{1}{\Delta N}\sum_i \bar{s}_i$ are the mean time T and the mean displacement $\bar{s}$ between two collisions. Let us call $\bar{v}(t) = \frac{1}{\Delta N}\sum_i \bar{u}_i(t)$ the mean velocity in ΔV of our carrier at time t(9); then, $\bar{s} = \bar{v}T$, as can be easily demonstrated making use of assumption e), so that we can write (omitting t in $\bar{p}(t)$ and $\bar{v}(t)$)

$$\bar{p} = q\bar{E}T + \frac{q}{c}\; T\bar{v}\; x\; \bar{B}$$

Hence, setting $N = \Delta N/\Delta V$ and $M = \frac{|\bar{p}|}{|\bar{v}|}$ we get

$$Nq\bar{v} = \frac{q^2NT}{M}\bar{E} + \frac{q^2NT}{M}\frac{1}{c}\bar{v} \times \bar{B}$$

Now, Nq is the density ρ_c of the carriers and $Nq\bar{v} = \bar{J}$.
Therefore we obtain

$$\bar{J} = \frac{qT}{M}\rho_c\bar{E} + \frac{1}{c}\frac{qT}{M}\bar{J} \times \bar{B} \qquad (3.1)$$

Eq. (3.1) coincides with eq.(2.2) (hence with eq.(2.4) if more than one kind of charge carriers exist) if we set $|\frac{qT}{M}| = \alpha$. Anyway, it must be stressed that α depends now, through M, on the velocity of the charge carriers; hence, it does not depend on $\bar{E}$ and $\bar{B}$ only if these fields are so small that the velocity acquired by the charge carriers because of them is neglegible if compared with the speed of light.
Under these assumptions, and if, on top of this, the mean termal velocity is so low that for any charge carrier the relativistic mass cannot be distinguished from the rest mass, we can set $\alpha = |\frac{qT}{M}| \simeq |\frac{qT}{M_o}| = \alpha_o$ and write

$$\bar{J} = \pm\,\alpha_o\rho_c\bar{E} \pm \frac{\alpha_o}{c}\bar{J} \times \bar{B} \qquad (3.2)$$

This equation is actually the one which has a practical relevance, even if eq. (3.1) is theoretically interesting in solving the first paradox we discussed in §1.
Finally, we remark that the model discussed thus far is probably, even amongst the classical ones, much too simple for most physical conductors. We know that more complex non quantum models exist in literature (which, anyway, do not take into account relativistic mechanics and magnetic field); these

lead to substitute the mean time T with the "relaxation time" τ, and introduce the frequency when periodic fields are considered; but the linear dependence of $\bar{J}$ on $\bar{E}$ is preserved[3]. Thus, when we introduce the magnetic field, we expect that eq. (3.2) will be confirmed even by more sophisticated models.

§4- In this section we intend to deduce some elementary consequences of eq. (2.2).

1) Joule effect inside conductors

It is clear that eq. (2.2) does not change the usual expression for the power dissipated by the current due to a given type of charge carrier. The total power W dissipated in the unity of volume inside a homogeneous isotropic conductor with many kinds of charge carriers has the form

$$W = \sum_{\ell} \frac{1}{\alpha_{\ell}\rho_{\ell}} \bar{J}_1^2 = \sum_{\ell} \frac{1}{\sigma_{\ell}} \bar{J}_1 \qquad (4.1)$$

where the conductivity $\sigma_{\ell} = \alpha_{\ell}\rho_{\ell}$ is not rigorously constant for the given medium since ρ_{ℓ} depends on the specific situation and may vary from point to point. On the contrary, the change in α_{ℓ} due to the relativistic mass of the charge carriers can usually be neglected.

II) Charge diffusion

It is not possible to deduce the behaviour of $\rho(P,t)$ starting from $\rho(P,0)$ without considering simultaneously the behaviour of $\bar{J}$, $\bar{E}$ and $\bar{B}$. Using the continuity equation, Maxwell's equations and eq. (2.2), we find non-linear equations even in the simple case of a conductor with only one kind of charge carriers; this is due to the term $\bar{J}\times\bar{B}$ that appears

in eq.(2.2)[2]. Furthermore, the dependence of α on the velocity of the charge carriers (and consequently on the fields) cannot be neglected now; indeed we are not "a priori" assured about $\bar{E}$ and $\bar{B}$ being small enough, and the presence of strong fields, like in the neigbourhood of a concentration of charge, causes a local increase of resistivity. In conclusion, no simple (and paradoxical) equation of the form (1.2) can be given now.
In particular, we shall show in § 5 that the limit of $\rho(P,t)$ as $t\to\infty$ is not necessarily zero.

III) Steady states, resistance and the Hall effect

Let us study a steady state inside a conductor with one type of charge carriers only. Generally, $\bar{E}$ and $\bar{B}$ will both be different from zero; in particular, $\bar{E}$ is a static field and its potential φ obeys Poisson's equation

$$\nabla^2\varphi = -4\pi\rho \qquad (4.2)$$

As we have already been, we cannot set $\rho = 0$, even as a limit when $t\to\infty$, since eq.(1.2) does not work now.
Let S_1 and S_2 be two surfaces (even internal) of the conductor that are maintained at potentials φ_1 and φ_2 respectively, and let S_1 be the "lateral surface" of the conductor (i.e. the set of the points of its surface which do not belong to S_1 or to S_2). Then, eq.(4.2) must be associated with the boundary conditions

$$\begin{cases} \varphi = \varphi_1 \text{ at } S_1 \\ \varphi = \varphi_2 \text{ at } S_2 \end{cases} \qquad (4.3)$$

and

$$\bar{J}\cdot\bar{n} = 0 \text{ at } S_1 \qquad (4.4)$$

Making use of eq.(2.2), we have from eq.(4.4)

$$\frac{d\varphi}{dn} = - \frac{1}{c\rho_c} (\bar{J} \times \bar{B}) \cdot \bar{n} \qquad (4.5)$$

Thus, we see that φ is now connected to the other unknown quantities of the problem (in particular, to the magnetic field) both by eq.(4.2) and by the boundary condition (4.5). Hence the usual discussions which lead to the concept of resistance as a characteristic of a given conductor (related to a given position of the terminals S_1 and S_2) do not hold any longer, even if this concept is still meaningful whenever the magnetic field can be ignored.

A general discussion of our problem is not simple, although we can get some results that do not require a complete solution of it. To do this, let us call $\bar{E}_o(P)$ a conservative field whose potential φ_o satisfies the eq. $\nabla^2\varphi_o = 0$ with the boundary conditions (4.3) and $\frac{d\varphi_o}{dn} = 0$.

Then the field

$$\bar{E}_1 = \bar{E} - \bar{E}_o$$

is conservative and its potential $\varphi_1 = \varphi - \varphi_o$ satisfies eq. (4.2), is zero at S_1 and S_2 and satisfies eq.(4.5) at S_1. Thus, we see that $\bar{E}$ can be divided into the sum of two fields: the first, $\bar{E}_o$, is the "usual" field, that is, the one which would appear in the conductor if eq.(1.1) were correct, and $\bar{E}$ reduces to $\bar{E}_o$ only if $\bar{B} = 0$. The second, $\bar{E}_1$ is a conservative, generally non-solenoidal field, which appears when $\bar{B}$ differs from zero: we shall call it "transverse" in the sense that its line integral along any path connecting S_1 with S_2 is zero. Now, observe that $\bar{B}$ can be written as the sum of the field $\bar{B}_i$ generated by $\bar{J}$ itself plus the "external" field $\bar{B}_e$

$$\bar{B} = \bar{B}_i + \bar{B}_e$$

Generally, $\bar{B}_i$ cannot be zero, so that $\bar{E}_1$ is not rigorously zero even if $\bar{B}_e = 0$; that is,a transverse electric field always exists. Yet, $\bar{B}_i$ is often neglegible for any practical purpose: is this case, $\bar{E}_1$ appears only if $\bar{B}_e \neq 0$, giving rise to the Hall effect, and it can be found solving eq.(4.2) with the boundary conditions given above. However, this cannot be made without considering the whole problem of charge and fields inside the conductor.

§5- By considering particular simple problems we can achieve a better understanding of the implications of eq. (2.2).

For instance, we can study a steady state in absence of any external magnetic field for a cylindrical conductor of infinite length with one type of charge carrier only. In this case, the conditions (4.3) must be slightly changed because of the conductor being infinite: we shall suppose that two given cross sections S_1 and S_2 of the cylinder, d cm. apart, are equipotential surfaces, and that their potentials are φ_1 and φ_2 respectively, with $\varphi_1 > \varphi_2$. We choose the axis of the cylinder as the polar axis, oriented from S_1 to S_2, for a cylindrical coordinate system (r, θ, z), and denote the unitary vectors tangent to the coordinate lines by $\hat{r}$, $\hat{\theta}$, $\hat{z}$. Then the symmetry of the system implies that ρ, $\bar{J}$, $\bar{E}$, $\bar{H}$, depend only on r; moreover, being $\nabla \cdot \bar{J}=0$, $\bar{J} \cdot \hat{r}$ must be zero, while $\bar{J} \cdot \hat{\theta} = 0$ since, if it were not, no cross section of the conductor could be an equipotential surface.Therefore we can set

$$\bar{J} = J(r)\,\hat{z} \tag{5.1}$$

Eq.(5.1) implies that the condition (4.4) is automatically satisfied and that $\bar{B} = B(r)\,\hat{\theta}$, where

$$B_r(r) = \frac{4\pi\mu}{rc}\int_0^r J(r')r'\,dr'.$$

Moreover, $\bar{E}$ being conservative, its component along the polar axis cannot depend on r, while $\bar{E}\cdot\hat{\theta}$ must be zero;then, it is

$$\bar{E} = E_z\hat{z} + E_r(r)\hat{r}$$

where $E_z = (\varphi_1 - \varphi_2)/d$ and

$$E_r(r) = \frac{4\pi}{\varepsilon r}\int_0^r \rho\,(r')dr'.$$

Our problem is thus reduced to the evaluation of $\rho(r)$ and $J(r)$. Using eq.(2.2) we get the scalar equations

$$\begin{cases} J(r) = \pm\,\alpha\,\rho_c E_z \\ 0 = \pm\,\rho_c E_r(r) \mp \frac{1}{c}\,J(r)B(r) \end{cases} \tag{5.2}$$

where the upper sign holds when ρ_c is positive and the lower one holds when ρ_c is negative. Hence, we have

$$\frac{1}{\alpha E_z}\,E_r(r) = \pm\,\frac{1}{c}\,B(r)$$

that is

$$\frac{1}{\alpha E_z}\ \rho(r) = \pm\ \frac{\varepsilon\mu}{c^2}\ J(r)$$

and therefore

$$J(r) = \pm\ \frac{c^2}{\varepsilon\mu}\ \frac{1}{\alpha E_z}\ (\rho_s + \rho_c) \qquad (5.3)$$

Eq. (5.3), (where ρ_s has the meaning defined in § 1), together with the first of eqs. (5.2), gives J(r) and ρ_c; our problem is thus solved, since ρ_s is a characteristic parameter of the given medium. To obtain the final equations in a more transparent form, observe that, being $\bar{J} = \rho_c \bar{v}$, the quantity $\pm\ \alpha E_z$ represents the mean velocity $\bar{v}$ of the charge carriers in the direction $\hat{z}$. Then, setting $\sqrt{\varepsilon\mu} = n$, we can write

$$v = \pm\ \alpha(\varphi_1 - \varphi_2)/d$$

and

$$J(r) = (\rho - \rho_s)v$$

$$\rho = +\ \frac{n^2}{c^2}\ vJ(r)$$

that is

$$\rho = -\ \frac{n^2 v^2}{c^2}\ (1 - \frac{n^2 v^2}{c^2})^{-1}\ \rho_s$$

$$J = -v(1- \frac{n^2 v^2}{c^2})^{-1}\ \rho_s$$

The same calculation with Ohm's law in its usual form would have given $\rho=0$, $J = -\rho_s v$; the corrections that we have obtained are neglegible in most cases. However, our example exhibits two interesting theoretical features. First, we see that ρ can be different from zero in a steady state (10).

Secondly, let S be a cross section of the conductor; then the total current I will be given by

$$I = JS = \mp \alpha \frac{S}{d} \rho_s \left(1 - \frac{n^2 v^2}{c^2}\right)^{-1} (\varphi_1 - \varphi_2) \qquad (5.4)$$

Eq. (5.4) shows that I is not proportional to $(\varphi_1 - \varphi_2)$ because v depends on $(\varphi_1 - \varphi_2)$ and, moreover, α depends on v, so that the concept of resistance has a meaning only in the non-relativistic limit, that is, when $v \ll c$; in this case, eq. (5.4) reduces to its usual expression.

As a second example, we could study an infinite conductor having a rectangular cross section when an external magnetic field is applied which is normal to one side of the conductor. This is the case that is more frequently studied in the text-books; if we suppose that the magnetic field generated by the current flowing in the conductor itself is neglegible, set $\alpha\rho = \sigma$, and suppose that σ is nearly constant, eq.(2.2) would reproduce well known results.

§6- We will conclude this article by dealing briefly with a further generalization of eq.(2.2). Consider a homo-geneous, non isotropic conductor with only one type of charge carrier; let us denote the components of any vector in the x,y,z, directions by the indices 1,2,3, and use the summation convention over repeated indices. Then we get, substituting the mobility tensor (α_{ik}) for the scalar mobility α

$$J_i = \pm \rho_c \alpha_{ik} E_k \pm \frac{1}{c} \alpha_{ik} (\bar{J} x \bar{B})_k \qquad (i,k = 1,2,3) \qquad (6.1)$$

The homogeneity condition can be relaxed if α_{ik} is supposed to depend on the point; it any case, (α_{ik}) is a symmetric tensor[11].

Eq.(6.1) can also be written in the form

$$E_i = \pm \frac{1}{\rho_c} \left[(\alpha^{-1})_{ik} J_k \mp \frac{1}{c} (\bar{J} x \bar{B})_i \right] \qquad (6.2)$$

that is

$$E_i = (p_{ik} + q_{ik}) J_k \qquad (6.3)$$

where p_{ik} is a symmetric tensor and q_{ik} an antisymmetric tensor, and it is

$$\left\{ \begin{array}{l} p_{ik} = \pm \dfrac{1}{\rho_c} (\alpha^{-1})_{ik} \\ (q_{ik}) = - \dfrac{1}{c\rho_c} \begin{pmatrix} 0 & B_z & -B_y \\ -B_z & 0 & B_x \\ B_y & B_x & 0 \end{pmatrix} \end{array} \right. \qquad (6.4)$$

Eqs. (6.2) and (6.3) agree with Landau-Lifchitz's equations; moreover, eq.(6.4) gives, at least when α does not depend on the speed of the charge carriers, i.e. in the non-relativistic limit, an explicit form for p_{ik} and q_{ik} that does not appear in Landau's book.

Finally, this generalization can be trivially extended to conductors with many kinds of charge carriers, following the same methods used in § 2 to extend eq.(2.2).