COROLLARY : In PG(2,q) there exists a complete k-arc with $k=\frac{1}{2}(q+1-t)$ for every t satisfying 16.8 when either (a) q is odd, $q \geq 311$, t is even; or (b) q is even, $q \geq 256$, t is odd.

18. k-ARCS IN PG(2,q).

Let $\mathcal{K}$ be a complete k-arc in PG(2,q); that is, $\mathcal{K}$ has no three points collinear and is not contained in a (k+1)-arc. We define three constants m(2,q), n(2,q), m'(2,q).

$$m(2,q) = \max k = \begin{cases} q+2, & q \text{ even} \\ q+1, & q \text{ odd}, \end{cases}$$

$$n(2,q) = \min k.$$

If $m(2,q) \neq n(2,q)$,

$$m'(2,q) = \text{second largest } k;$$

if $m(2,q) = n(2,q)$, let $m'(2,q) = m(2,q)$. So, if a k-arc has $k > m'(2,q)$, then it is contained in an m(2,q)-arc. For q odd, every (q+1)-arc is a conic. For q even, the (q+2)-arcs have been classified for $q \leq 16$; see [4], [6].

The value of n(2,q) seems to be a difficult problem. By elementary considerations ([6] p.205),

$$n(2,q) \geq \sim \sqrt{2q}.$$

Constructions have been given for complete k-arcs with k having the following values (up to an added constant):

$\frac{1}{2}q$, see [6], §9.4;

$\frac{1}{3}q$, [1];

$\frac{1}{4}q$, [11]

$2q^{9/10}$, q large, [15];

cq , $c \leq \frac{1}{2}$, q large [16];

These examples all lie an rational curves, namely conics or singular cubics; to be precise the k-arcs of order $\frac{1}{2}q$ have one point off a conic. The examples of §17 are the only other ones known.

Conjecture: For each k such that

$$n(2,q) \leq k \leq m'(2,q),$$

these exists a complete k-arc in PG(2,q).

In fact, although the conjecture is true for $q \leq 13$, it is probably more realistic to ask for the smallest value of q for which the conjecture is false.

In Table 2, we give m, m' and n for $q \leq 13$.

q	2	3	4	5	7	8	9	11	13
m	4	4	6	6	8	10	10	12	14
m'	4	4	6	6	6	6	8	10	12
n	4	4	6	6	6	6	6	7	8

Upper bounds for m'(2,q) are as follows:

$m'(2,q) \leq q - \frac{1}{4}\sqrt{q} + \frac{25}{16}$, q odd, [17];

$m'(2,q) \leq q - \sqrt{q} + 1$, $q = 2^h$, [6], theorem 10.3.3.

$m'(2,q) = q - \sqrt{q} + 1$, $q = 2^{2r}$,[2].