$g = \frac{1}{2}(d-1)(d-2)$,

d_o= number of K-rational roots of $f(x,1)$,

$$A_m = \begin{cases} \frac{1}{24}m(m-1)\{4(d-m-1)(m+4)+(m-2)(m-5)\} & \text{for } m \leq d-3 \\ \frac{1}{24}(d-1)(d-2)(d-3)(d+4) & \text{for } m > d-3, \end{cases}$$

$$B_m = \begin{cases} dm - \frac{1}{2}m(m+3) & \text{for } m \leq d-3 \\ g & \text{for } m > d-3. \end{cases}$$

Note: When $m \leq p/d$, then $|mD|$ is Frobenius classical.

A Fermat curve is a special case of a Thue curve given by

$$\mathscr{F}_d : ax^d + by^d = z^d$$

with $a,b \in K\setminus\{0\}$.

THEOREM 14.3: For $\mathscr{F}_d$ with the same conditions as above,

$$N \leq (n-1)(g-1) + \frac{1}{n}\{md(q+n)-3d\ A_m-d_1\ B_m\}$$

with n,g,A_m,B_m as above, but d_1 is the number of points of $\mathscr{F}_d$ with $xyz = 0$.

15. THE MAXIMUM NUMBER OF POINTS ON AN ALGEBRAIC CURVE

In Table 1, we give the value of $N_q(g)$ or the best, known bound for $g \leq 5$ and $q \leq 49$ arising from results of Serre [12],[13] and the preceding sections. Also included in the table is the bound $S_g = q+1+g[2\sqrt{q}]$; see §2.

TABLE 1

The maximum number points on an algebraic curve

q	$[2\sqrt{q}]$	$N_q(1)$	$N_q(2)$	S_2	$N_q(3)$	S_3	$N_q(4)$	S_4	$N_q(5)$	S_5
2	2	5	6	7	7	9	8	11	9	13
3	3	7	8	10	10	13	12	16	$\leq$ 15	19
4	4	9	10	13	14	17	15	21	$\leq$ 18	25
5	4	10	12	14	16	18	18	22	$\leq$ 22	26
7	5	13	7	18	20	23	24-25	28	$\leq$ 29	33
8	5	14	18	19	24	24		29	$\leq$ 32	34
9	6	16	20	22	28	28	26-30	34	$\leq$ 36	40
11	6	18	24	24	28	30	32-34	36	$\leq$ 40	42
13	7	21	26	28	32	35	36-38	42	$\leq$ 45	49
16	8	25	33	33	38	41		49		57
17	8	26	32	34	40	42	$\leq$ 46	50	$\leq$ 54	58
19	8	28	36	36	44	44	$\leq$ 50	52	$\leq$ 58	60
23	9	33	42	42	$\leq$ 48	51	$\leq$ 58	60	$\leq$ 66	69
25	10	36	46	46	56	56	66	66		76
27	10	38	48	48		58		68		78
29	10	40	50	50		60		70	$\leq$ 78	80
31	11	43	52	54		65	$\leq$ 74	76	$\leq$ 82	87
32	11	44	53	55		66		77		88
37	12	50	60	62		74		86	$\leq$ 94	98
41	13	54	66	68		81		94	$\leq$ 102	107
43	13	57	68	70		83		96	$\leq$ 106	109
47	13	61	74	74		87		100		113
49	14	64	78	78	92	92		106		120