REGULARITY RESULTS
FOR
ALMOST MINIMAL ORIENTED HYPERSURFACES IN $\mathbb{R}^n$

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Preface. - *This work is intended as an introduction to the regularity theory of oriented boundaries in* $\mathbb{R}^n$ *which are almost minimal for the area functional. It is based partly on an earlier manuscript which contained the proof of the main theorem presented below, and partly on lecture notes for a course by the author at the University of Lecce.*

The reader is presumed to have some knowledge of the basic facts concerning Caccioppoli sets: sections 2.1 to 2.4 of the book of Massari and Miranda (see [27] of the bibliography at the end of the volume) will serve the scope.

With the exception of a few "classical" inequalities, which proofs can also be found in [27], the exposition is essentially self-contained.

The first half of the work, Chapters 1 and 2, is introductory. We begin by recalling the (by now classical) Regularity Theorem of minimal boundaries, in the framework of Caccioppoli - De Giorgi Miranda's theory. Almost minimal boundaries are then defined, and a corresponding Regularity Theorem is stated. The remainder of Chapter 1 illustrates these concepts and results with several examples and applications.

A specialized version of the Theorem is derived in Chapter 2.

The proof utilizes important ideas of Campanato, originally introduced in the context of elliptic equations. The role of the "area

excess" as a regularity parameter is then emphasized, and De Giorgi's Lemma - the key result of the theory - is finally presented, in its original form.

An extended version of this Lemma is obtained in Chapter 3. While results of this type are usually proved "by contradiction" (see however the recent paper of Schoen and Simon [34]), we have been able to modify the original argument of De Giorgi, to get a simpler direct proof of the Lemma. Moreover this way the constants involved can be explicitly computed.

The proof of the Regularity Theorem is then completed in Chapter 4. Some notes and a bibliography conclude the work. Connections with related papers, notably with the important works of Almgren, Bombieri, and Schoen-Simon, are indicated, particulary at the beginning of Chapter 1 and at the and of Chapter 2.

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