Proposition 2 *Consider the normalized demand curve* $Q(p,\theta) = 1 - F(p,\theta)$, *where* $F(p,\theta)$ *is a unimodal income distribution and* θ *a mean preserving spread. Assume that (i) average variable costs* $C(q)/q$ *are nondecreasing in* q*; (ii) marginal costs* $C'(\cdot)$ *are non-decreasing in* q *and such that* $y_{\min} \leq C'(0) < y_{\max}$. *Then (a) the symmetric equilibrium price* $p^*(n,\theta)$ *obtained from (3) is monotonically decreasing in* n*, that is* $\frac{dp^*}{dn} < 0$*; and (b) the long run Cournot equilibrium price* $p^*(K,\theta) = p^*(n^*(K,\theta),\theta)$ *decreases monotonically to its perfect competition level as* K *tends to zero, that is* $\frac{dp^*}{dK} > 0$ *and* $\lim_{K\to 0} p^*(K,\theta) = \lim_{K\to 0} p^*(n^*(K,\theta),\theta) = C'(0)$.

Property (a) is known as 'quasi-competitiveness': it refers to industry output increasing as n increases. It should be noticed that property (b) (monotonic convergence to the competitive equilibrium) is not necessarily implied by the first (e.g., Ruffin, 1971).

3 Income distribution and the number of firms

The behaviour of the function $n^*(K,\theta)$ will tell us how the long-run equilibrium number of firms adjusts to changes in demand brought about by variations in θ, i.e. by mean-preserving increases in income dispersion. One can approach this problem by totally differentiating the zero profit condition. This gives

$$\frac{\partial \pi^e}{\partial p}\left(\frac{dp^*}{dn}dn + \frac{dp^*}{d\theta}d\theta\right) + \frac{\partial \pi^e}{\partial n}dn + \frac{\partial \pi^e}{\partial \theta}d\theta = 0 \tag{5}$$

Notice that the first term is not nil - that is, one cannot take advantage of the envelope theorem, since the firm does not maximize profit as defined by (4). There is an obvious externality involved, due to oligopolistic interaction: $\partial \pi^e/\partial p$ as derived from (4) is different from $\partial \pi_i/\partial p_i$ as defined in (3). The latter is clearly nil, while the former is not – indeed it is positive for $n > 1$, precisely because we know that $\partial \pi_i/\partial p_i = 0$: by comparing the two, it is easily checked that there is a factor $1/n$ of difference, such that the two collapse to the same (nil) value for $n = 1$ under monopoly, or under perfect competition as n tends to infinity - in both cases there is no externality.[7]

This being said, we are able to derive the paper's main result, to the effect that, if the fixed cost K is sufficiently low, shifting the mass of incomes towards the tails of the distribution *always* decreases the equilibrium number of firms surviving in the long-run.

[7] This is shown formally in the proof of Proposition 3, below.

Proposition 3 *Consider the normalized demand curve $Q(p,\theta) = 1 - F(p,\theta)$, where $F(p,\theta)$ is a unimodal income distribution and θ a mean preserving spread. Assume that (i) average variable costs $C(q)/q$ are nondecreasing in q; (ii) marginal costs are non-decreasing in q and such that $y_{\min} \leq C'(0) < \overline{y}$, where $\overline{y} < y_{\max}$ is defined by Proposition 1. Then the long-run zero profit condition of the symmetric Cournot equilibrium with n firms implies $\frac{dn}{d\theta} < 0$, if the fixed cost K is sufficiently low.*

Proof. We are interested in the sign of $\frac{dn}{d\theta}$, the expression for which is straightforward from (5)

$$\frac{dn}{d\theta} = -\frac{\dfrac{\partial \pi^e}{\partial p}\dfrac{dp^*}{d\theta} + \dfrac{\partial \pi^e}{\partial \theta}}{\dfrac{\partial \pi^e}{\partial p}\dfrac{dp^*}{dn} + \dfrac{\partial \pi^e}{\partial n}}$$

From Proposition 2(a) above, we know that $\frac{dp^*}{dn} < 0$. From the same Proposition, we also know that $\lim_{K \to 0} p^*(n^*(K,\theta),\theta) = C'(0)$, this limit being reached monotonically as K decreases: hence there exists some $\widehat{K} > 0$ such that, for all $K < \widehat{K}$, $p^* \in (C'(0), \overline{y})$, where $\overline{y}$ is defined in Proposition 1. For all such K, this directly implies $\frac{dp^*}{d\theta} < 0$: indeed, $\frac{dp^*}{d\theta}$ is obtained by totally differentiating (3)

$$\frac{dp^*}{d\theta} = -\frac{-\frac{\partial F(p,\theta)}{\partial \theta}(1 + C''(\cdot) f(p,\theta)) - [p - C'(\cdot)]\frac{\partial f(p,\theta)}{\partial \theta}}{-\frac{f(p,\theta)}{n}(1 + n + C''(\cdot) f(p,\theta)) - [p - C'(\cdot)]\frac{\partial f(p,\theta)}{\partial p}}$$

which is negative whenever $p^* < \overline{y}$, as by Proposition 1 $\frac{\partial f(p^*,\theta)}{\partial \theta} > 0$ and $\frac{\partial F(p^*,\theta)}{\partial \theta} > 0$, while the denominator is negative by the second order conditions. Also, $\frac{\partial \pi^e}{\partial p} > 0$ for $n > 1$, as $\frac{\partial \pi^e}{\partial p} = \frac{1}{n}(1 - F(p^*,\theta) - \frac{f(p^*,\theta)}{n}(p^* - C'(\cdot)) > \frac{\partial \pi_i}{\partial p_i} = \frac{1}{n}(1 - F(p^*,\theta)) - [p^* - C'(\cdot)] f(p^*,\theta) = 0$. Finally, $p^* \leq \overline{y}$ implies $\frac{\partial \pi^e}{\partial \theta} = -\frac{1}{n}(p^* - C'(\cdot))\frac{\partial F}{\partial \theta} \leq 0$, while pricing above marginal cost implies $\frac{\partial \pi^e}{\partial n} = -\frac{1}{n^2}(p^* - C'(\cdot))(1 - F(p^*,\theta)) < 0$. There follows that $\frac{dn}{d\theta} < 0$. ■

The proof of Proposition 3 is based on the idea that lower values of the fixed costs increase competition *via* a higher number of firms: the equilibrium price is accordingly driven down to the point where $p^* < \overline{y}$, so that Proposition 1 applies.

The economics behind this can be summed up as follows. Income polarisation towards the tails has a twofold effect on the demand faced by firms selling at a sufficiently low price. On the one hand, demand decreases due to some consumers becoming too poor to be able to buy, while the parallel higher density of consumers in the upper tail of the distribution is immaterial, as it results only in a higher consumers' surplus. On the other hand, given the number of firms, demand becomes more elastic, due to a higher density of consumers whose reservation price is closer to the initial price. Accordingly, firms are subject to both a decrease in demand and a higher competitive pressure dictated by the new demand conditions. This results in lower profits which leads to a decrease in the number of firms able to survive, i.e., to higher market concentration.

4 Final comments

The endogenisation of market structure has always been a key topic in economic research. This paper contributes to this issue, suggesting a role for personal income distribution — a role which, to our knowledge, has not yet been investigated in detail. In particular, in this paper we have shown that the degree of income dispersion may affect the number of firms, *via* the market demand size and its elasticity.

This theoretical point can also shed light on some recent observed phenomena: specifically, polarisation in income distribution and increasing market concentration are two facts, that have characterised the last twenty years, both in the United States and in the EU countries. In a partial equilibrium perspective, these facts may be brought together along the lines suggested by our theoretical model – where the general framework is that of discrete-choice, unimodal income density and oligopoly behaviour *à la* Cournot on the firms' part. In this context, we envisage a causal link running from income polarization to market concentration.

Clearly, having consumers choosing discretely, and working in partial equilibrium proved to be quite helpful in two ways. The former assumption allowed us to establish a link between income and consumption, which does away with the issue of preference homotheticity; the latter allows to neglect possible feedback effects from market structure (and hence functional distribution) to personal income distribution. While both aspects are obviously relevant, our results are nevertheless robust with respect to two important features: they hold for any unimodal distribution, and can be applied to any market structure covered by the Cournot model.