

1 Introduction

The past twenty years have witnessed a significant increase in earning inequality; more precisely, “income polarisation” or “shrinking middle-class” phenomena have occurred – see, e.g., Levy and Murnane (1992), Atkinson *et al.* (1995), Jenkins (1995). This trend has been common to most OECD countries, as well as to many less developed countries.

The effects of income distribution on households’ behaviour and growth performance are widely discussed: see, e.g., the recent contributions by Benabou (1996) and Gottshalk and Smeeding (1997), as well as the comprehensive article by Aghion *et al.* (1999). By contrast, the body of research about the effects of this change upon the firms’ behaviour and market structure is much more restricted. This is a little bit surprising, given that income distribution affects market demand functions, and hence should in principle influence the optimal behaviour of firms.

In this paper we show that, if the firms’ fixed costs are below a critical threshold, income polarisation may lead to market concentration, that is, to a smaller number of firms able to survive in the long run. This result is obtained for any unimodal density function describing income distribution of the consumers, in the framework of discrete consumers’ choices. Competition among firms is modelled as Cournot oligopoly, which obviously includes perfect competition and monopoly as limiting cases.¹

In a theoretical perspective, the set-up we work with is fairly general, within the limits imposed by a discrete choice model:² it deals with any income distribution (the limitation that it is unimodal seems acceptable), and with any market form covered by the Cournot setting. In this respect, the paper encompasses recent studies dealing with related points, but limited to specific forms for the density function of income or to peculiar forms of imperfect competition (Benassi *et al.*, 1999).

In the perspective of the empirical relevance of our contribution, several studies show that Western economies have witnessed increasing market concentration over the last decades in many sectors, and especially in the large-consumption goods sector: see De Jong (1993), and Lyons and Matras (1996). Well established explanations rest on the increased competition due to trade liberalisation. Here we argue that, for some sectors, income polarisation may well be among the reasons why market concentration has

¹A recent study on the reaction of oligopolistic firms to shifts in market demand is Hamilton (1999); like the present paper, it focuses on free-entry equilibria.

²The discrete-choice structure of demand, which is clearly most appropriate in the case of durables, is quite common in the literature (e.g., Deaton and Muellbauer, 1983, ch.13; Anderson *et al.*, 1992).

been increasing: in this sense, there might be a link running from income distribution to market structure, consistent with the observed data.

Two final points should be noticed: first, we treat income polarisation as an exogenous shock; secondly, we do not take into account possible income increases. Exogeneity of the income distribution is consistent with our partial equilibrium approach, in the sense that we assume away any feedback effects from market concentration to aggregate income distribution. As far as the second point is concerned, in the real world income polarization has been associated with increases in average income; however, we abstract from the latter and focus on mean-preserving shocks to income distribution, in order to sort out the effects of purely distributive changes.

The outline of the paper is as follows: section 2 presents the basics of the model; section 3 performs comparative statics exercises, taking into account the effects of income polarisation; comments and conclusions are gathered in section 4.

2 The basic model

We consider, in turn, (*i*) the demand side, describing the income distribution, the optimal decision of consumers, and the resulting market demand function; (*ii*) the optimal decision of symmetric firms in an oligopoly setting *à la* Cournot.

2.1 Income distribution and demand

We model the demand side of the market as a *continuum* of consumers, each of whom is identified by the income y he is endowed with. The latter is continuously distributed as $F : [y_{\min}, y_{\max}] \rightarrow [0, 1]$ over some support such that $0 \leq y_{\min} < y_{\max}$. The *only* assumptions we impose on F (apart from differentiability) are that (a) the density $f(y, \theta) = \partial F(y, \theta) / \partial y$ is unimodal; (b) it is subject to mean preserving shocks – i.e., if we take a real parameter θ as a mean preserving spread, an increase in θ translates itself into the distribution $f(\cdot, \theta)$ being more dispersed around the given mean.³ If we denote the interior mode by $m \in (y_{\min}, y_{\max})$, we can write formally (a) and (b) as the following properties:

³Using a mean preserving spread amounts to ranking equal-mean distributions by second-order stochastic dominance. It is well known that such ranking is equivalent to Lorenz dominance: θ is thus an inequality index satisfying the Pigou-Dalton’s “principle of transfers” (Atkinson, 1970).

$$\left\{ \begin{array}{l} \frac{\partial f(m, \theta)}{\partial y} = 0 \\ \frac{\partial f(y, \theta)}{\partial y} > 0 \text{ for } y < m \\ \frac{\partial f(y, \theta)}{\partial y} < 0 \text{ for } y > m \end{array} \right. \quad (\text{a})$$

$$\left\{ \begin{array}{l} \int_{y_{\min}}^y \frac{\partial F(x, \theta)}{\partial \theta} dx \geq 0, \quad y < y_{\max} \\ \int_{y_{\min}}^{y_{\max}} \frac{\partial F(x, \theta)}{\partial \theta} dx = 0 \end{array} \right. \quad (\text{b})$$

These properties yield a useful result, summarized in the following proposition:

Proposition 1 *If θ is a mean preserving spread of the distribution $F(y, \theta)$, then:*

- (i) *there exists a value $\hat{y} \in (y_{\min}, y_{\max})$ such that $\frac{\partial F(y, \theta)}{\partial \theta} \geq 0$ for all $y \in (y_{\min}, \hat{y})$, with strict inequality somewhere;*
- (ii) *there exists a value $\bar{y} < \hat{y}$ such that $\frac{\partial f(y, \theta)}{\partial \theta} \geq 0$ for all $y \in (y_{\min}, \bar{y})$, with strict inequality somewhere.*

Proof. For ease of notation, let $G(y, \theta) \equiv \frac{\partial F(y, \theta)}{\partial \theta}$. Then the following holds: (1) $G(y_{\min}, \theta) = G(y_{\max}, \theta) = 0$, which follows from θ not altering the distribution's range; (2) $\exists \hat{y}$ such that $G(y, \theta) \geq 0$ for all $y < \hat{y}$, with $\hat{y} > y_{\min}$ and strict inequality somewhere, since by the definition of mean preserving spread (property (b)) the integral cannot be negative around y_{\min} . This proves claim (i); (3) $G(y, \theta) < 0$ for some $y > \hat{y}$, by the same property, as the integral function is zero around y_{\max} . All of this implies that $G(y, \theta)$ crosses zero at least once in the interior of $[y_{\min}, y_{\max}]$, the first time at \hat{y} from above. Hence G exhibits a local maximum \bar{y} , between y_{\min} and \hat{y} . There follows that $\frac{\partial G(y, \theta)}{\partial y} \geq 0$ for all $y \in (y_{\min}, \bar{y})$ (with strict inequality somewhere). Then claim (ii) follows trivially, since by Young's theorem on cross derivatives $\frac{\partial G(y, \theta)}{\partial y} = \frac{\partial f(y, \theta)}{\partial \theta}$. ■

The basic idea is illustrated by the following figure, where the simple case of single crossing of the distribution is described (it should however be stressed that we do *not* impose single crossing): there is a value \bar{y} such that

for all $y \leq \bar{y}$ both the density and the distribution are raised by an increase of the mean preserving spread θ . Notice that \bar{y} might not be arbitrarily close to y_{\min} , as is apparent from Figure 1.⁴

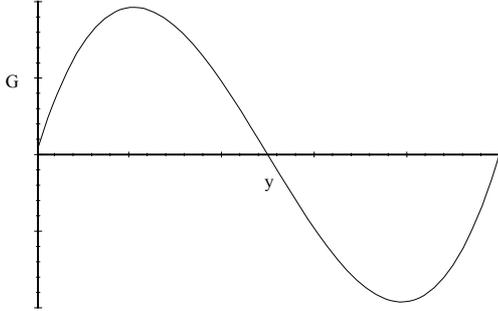


Figure 1: The function G

We assume that any given consumer buys one unit of the commodity whenever its price p is lower than his ‘income’ y – which of course amounts to interpreting y as the consumer’s reservation price.⁵ If one normalizes to unity the total population, market demand is then simply

$$Q(p, \theta) = 1 - F(p, \theta) \tag{1}$$

the (positive) elasticity of which can straightforwardly be derived as $\eta(p, \theta) = pf(p, \theta)/[1 - F(p, \theta)]$.

2.2 The Cournot equilibrium

The supply side of the model is described by a symmetric Cournot setup, with non-decreasing marginal and average variable costs, and non-negative fixed costs. Assume then that the market is served by n identical firms, and let $C(q_i)$, $i = 1, \dots, n$, denote variable costs and $C_M(q_i) = C(q_i)/q_i$ average variable costs: we have $C'(q_i) \geq C_M(q_i) \geq 0$, and $C''(q_i) \geq 0$ for $q_i \geq 0$. We also assume $C''(0) \in [y_{\min}, y_{\max})$.

⁴Consider e.g. a symmetric Beta distribution $f(y, \theta) = y^\theta(1 - y)^\theta/B(\theta)$, with $\theta > 0$, $y \in [0, 1]$ and $B(\theta) = \int_0^1 y^\theta(1 - y)^\theta dy$. Then a decrease in θ is a mean preserving spread in the distribution, and $\theta = 1$ (quadratic distribution) yields $\bar{y} \cong 0.253$ – that is, around the 25th percentile of the distribution.

⁵This is clearly the most direct way to link reservation prices to income. Since we are assuming no specific functional form for income distribution, our argument only requires that a unimodal income distribution generates a unimodal distribution of reservation prices, and that a wider spread in income distribution be mirrored into a wider spread of the distribution of reservation prices.

Given the market demand function (1), firm i maximizes its profits. In this setting it is easier to solve the Cournot model in prices along the lines suggested, e.g., by Kreps (1990, ch.10). This entails that the individual demand curve faced by firm i may be written as

$$q_i(p_i, p_{-i}; \theta) = 1 - F(p_i, \theta) - \sum_{j \neq i} q_j(p_j, p_{-j}; \theta)$$

where $p_{-i} = \{p_j\}_{j \neq i}$. The function to be maximized is

$$\pi_i(p_i, p_{-i}; K, \theta) = q_i(p_i, p_{-i}; \theta)p_i - C(q_i(p_i, p_{-i}; \theta)) - K \quad (2)$$

where $K \geq 0$ denotes fixed costs, and the Cournot conjecture entertained by firm i is $\partial q_j / \partial p_i = 0$. Firm i 's first order condition for profit maximization is

$$\frac{\partial \pi_i}{\partial p_i} = 1 - F(p_i, \theta) - [p_i - C'(\cdot)]f(p_i, \theta) - \sum_{j \neq i} q_j = 0$$

Invoking symmetry, $p_i = p_j = p$ for all $i, j = 1, \dots, n$, and hence

$$\sum_{j \neq i} q_j = \frac{n-1}{n}(1 - F(p, \theta)),$$

we obtain that in equilibrium

$$\frac{\partial \pi_i}{\partial p_i} \Big|_{p_i=p} = \frac{1}{n}(1 - F(p, \theta)) - [p - C'(\cdot)]f(p, \theta) = 0 \quad (3)$$

which of course amounts to the familiar Cournot condition $(1 - \frac{1}{\eta n})p = C'$. Equation (3) solves for the sought-for short-run equilibrium price $p^*(n, \theta)$.

Dropping the i subscript, the corresponding equilibrium profit of the generic firm is

$$\pi^e = \frac{1}{n}(1 - F(p^*, \theta))p^* - C\left(\frac{1}{n}(1 - F(p^*, \theta))\right) - K \quad (4)$$

Clearly, the equilibrium number of firms n^* is determined by the zero profit condition $\pi^e = 0$, which yields an implicit function $n^*(K, \theta)$.⁶ In the Appendix we show that, given our assumptions on costs and demand, one such equilibrium exists and exhibits the (standard) properties summarized in the following:

⁶We treat n as a continuous variable, following a well established practice (e.g., Mankiw and Whinston, 1986).

Proposition 2 Consider the normalized demand curve $Q(p, \theta) = 1 - F(p, \theta)$, where $F(p, \theta)$ is a unimodal income distribution and θ a mean preserving spread. Assume that (i) average variable costs $C(q)/q$ are nondecreasing in q ; (ii) marginal costs $C'(\cdot)$ are non-decreasing in q and such that $y_{\min} \leq C'(0) < y_{\max}$. Then (a) the symmetric equilibrium price $p^*(n, \theta)$ obtained from (3) is monotonically decreasing in n , that is $\frac{dp^*}{dn} < 0$; and (b) the long run Cournot equilibrium price $p^*(K, \theta) = p^*(n^*(K, \theta), \theta)$ decreases monotonically to its perfect competition level as K tends to zero, that is $\frac{dp^*}{dK} > 0$ and $\lim_{K \rightarrow 0} p^*(K, \theta) = \lim_{K \rightarrow 0} p^*(n^*(K, \theta), \theta) = C'(0)$.

Property (a) is known as ‘quasi-competitiveness’: it refers to industry output increasing as n increases. It should be noticed that property (b) (monotonic convergence to the competitive equilibrium) is not necessarily implied by the first (e.g., Ruffin, 1971).

3 Income distribution and the number of firms

The behaviour of the function $n^*(K, \theta)$ will tell us how the long-run equilibrium number of firms adjusts to changes in demand brought about by variations in θ , i.e. by mean-preserving increases in income dispersion. One can approach this problem by totally differentiating the zero profit condition. This gives

$$\frac{\partial \pi^e}{\partial p} \left(\frac{dp^*}{dn} dn + \frac{dp^*}{d\theta} d\theta \right) + \frac{\partial \pi^e}{\partial n} dn + \frac{\partial \pi^e}{\partial \theta} d\theta = 0 \quad (5)$$

Notice that the first term is not nil - that is, one cannot take advantage of the envelope theorem, since the firm does not maximize profit as defined by (4). There is an obvious externality involved, due to oligopolistic interaction: $\partial \pi^e / \partial p$ as derived from (4) is different from $\partial \pi_i / \partial p_i$ as defined in (3). The latter is clearly nil, while the former is not - indeed it is positive for $n > 1$, precisely because we know that $\partial \pi_i / \partial p_i = 0$: by comparing the two, it is easily checked that there is a factor $1/n$ of difference, such that the two collapse to the same (nil) value for $n = 1$ under monopoly, or under perfect competition as n tends to infinity - in both cases there is no externality.⁷

This being said, we are able to derive the paper’s main result, to the effect that, if the fixed cost K is sufficiently low, shifting the mass of incomes towards the tails of the distribution *always* decreases the equilibrium number of firms surviving in the long-run.

⁷This is shown formally in the proof of Proposition 3, below.

Proposition 3 Consider the normalized demand curve $Q(p, \theta) = 1 - F(p, \theta)$, where $F(p, \theta)$ is a unimodal income distribution and θ a mean preserving spread. Assume that (i) average variable costs $C(q)/q$ are nondecreasing in q ; (ii) marginal costs are non-decreasing in q and such that $y_{\min} \leq C'(0) < \bar{y}$, where $\bar{y} < y_{\max}$ is defined by Proposition 1. Then the long-run zero profit condition of the symmetric Cournot equilibrium with n firms implies $\frac{dn}{d\theta} < 0$, if the fixed cost K is sufficiently low.

Proof. We are interested in the sign of $\frac{dn}{d\theta}$, the expression for which is straightforward from (5)

$$\frac{dn}{d\theta} = -\frac{\frac{\partial \pi^e}{\partial p} \frac{dp^*}{d\theta} + \frac{\partial \pi^e}{\partial \theta}}{\frac{\partial \pi^e}{\partial p} \frac{dp^*}{dn} + \frac{\partial \pi^e}{\partial n}}$$

From Proposition 2(a) above, we know that $\frac{dp^*}{dn} < 0$. From the same Proposition, we also know that $\lim_{K \rightarrow 0} p^*(n^*(K, \theta), \theta) = C'(0)$, this limit being reached monotonically as K decreases: hence there exists some $\hat{K} > 0$ such that, for all $K < \hat{K}$, $p^* \in (C'(0), \bar{y})$, where \bar{y} is defined in Proposition 1. For all such K , this directly implies $\frac{dp^*}{d\theta} < 0$: indeed, $\frac{dp^*}{d\theta}$ is obtained by totally differentiating (3)

$$\frac{dp^*}{d\theta} = -\frac{-\frac{\partial F(p, \theta)}{\partial \theta}(1 + C''(\cdot)f(p, \theta)) - [p - C'(\cdot)]\frac{\partial f(p, \theta)}{\partial \theta}}{-\frac{f(p, \theta)}{n}(1 + n + C''(\cdot)f(p, \theta)) - [p - C'(\cdot)]\frac{\partial f(p, \theta)}{\partial p}}$$

which is negative whenever $p^* < \bar{y}$, as by Proposition 1 $\frac{\partial f(p^*, \theta)}{\partial \theta} > 0$ and $\frac{\partial F(p^*, \theta)}{\partial \theta} > 0$, while the denominator is negative by the second order conditions. Also, $\frac{\partial \pi^e}{\partial p} > 0$ for $n > 1$, as $\frac{\partial \pi^e}{\partial p} = \frac{1}{n}(1 - F(p^*, \theta)) - \frac{f(p^*, \theta)}{n}(p^* - C'(\cdot)) > \frac{\partial \pi_i}{\partial p_i} = \frac{1}{n}(1 - F(p^*, \theta)) - [p^* - C'(\cdot)]f(p^*, \theta) = 0$. Finally, $p^* \leq \bar{y}$ implies $\frac{\partial \pi^e}{\partial \theta} = -\frac{1}{n}(p^* - C'(\cdot))\frac{\partial F}{\partial \theta} \leq 0$, while pricing above marginal cost implies $\frac{\partial \pi^e}{\partial n} = -\frac{1}{n^2}(p^* - C'(\cdot))(1 - F(p^*, \theta)) < 0$. There follows that $\frac{dn}{d\theta} < 0$. ■

The proof of Proposition 3 is based on the idea that lower values of the fixed costs increase competition *via* a higher number of firms: the equilibrium price is accordingly driven down to the point where $p^* < \bar{y}$, so that Proposition 1 applies.

The economics behind this can be summed up as follows. Income polarisation towards the tails has a twofold effect on the demand faced by firms selling at a sufficiently low price. On the one hand, demand decreases due to some consumers becoming too poor to be able to buy, while the parallel

higher density of consumers in the upper tail of the distribution is immaterial, as it results only in a higher consumers' surplus. On the other hand, given the number of firms, demand becomes more elastic, due to a higher density of consumers whose reservation price is closer to the initial price. Accordingly, firms are subject to both a decrease in demand and a higher competitive pressure dictated by the new demand conditions. This results in lower profits which leads to a decrease in the number of firms able to survive, i.e., to higher market concentration.

4 Final comments

The endogenisation of market structure has always been a key topic in economic research. This paper contributes to this issue, suggesting a role for personal income distribution — a role which, to our knowledge, has not yet been investigated in detail. In particular, in this paper we have shown that the degree of income dispersion may affect the number of firms, *via* the market demand size and its elasticity.

This theoretical point can also shed light on some recent observed phenomena: specifically, polarisation in income distribution and increasing market concentration are two facts, that have characterised the last twenty years, both in the United States and in the EU countries. In a partial equilibrium perspective, these facts may be brought together along the lines suggested by our theoretical model – where the general framework is that of discrete-choice, unimodal income density and oligopoly behaviour *à la* Cournot on the firms' part. In this context, we envisage a causal link running from income polarization to market concentration.

Clearly, having consumers choosing discretely, and working in partial equilibrium proved to be quite helpful in two ways. The former assumption allowed us to establish a link between income and consumption, which does away with the issue of preference homotheticity; the latter allows to neglect possible feedback effects from market structure (and hence functional distribution) to personal income distribution. While both aspects are obviously relevant, our results are nevertheless robust with respect to two important features: they hold for any unimodal distribution, and can be applied to any market structure covered by the Cournot model.

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Appendix

Here we prove equilibrium existence, as well as the properties stated in Proposition 2. Existence and quasi-competitiveness are proved mainly for later convenience – indeed, our model satisfies the conditions studied by Amir and Lambson (2000), who generalize previous work by MacManus (1964) and Roberts and Sonnenschein (1976). Convergence to the competitive limit has been studied by Ruffin (1971) under conditions on demand more restrictive than ours.

We gather here our assumptions:

- (1) $C(0) = 0$, $C'(q) \geq 0$ and $C''(q) \geq 0$ for $q > 0$;
- (2) $C'_M(q) \geq 0$ for $q > 0$
- (3) $y_{\min} \leq C'(0) < y_{\max}$
- (4) $f(y) > 0$ for $y \in (y_{\min}, y_{\max})$

Existence

At a symmetric equilibrium, the two (first and second order) conditions are

$$\frac{1}{n}(1 - F(p^*, \theta)) - [p^* - C'(\frac{1}{n}(1 - F(p^*, \theta)))]f(p^*, \theta) = 0 \quad (\text{A.1})$$

$$-f(p^*, \theta)[2 + C''(\cdot)f(p^*, \theta)] - [p^* - C'(\cdot)]\frac{\partial f(p^*, \theta)}{\partial p} < 0 \quad (\text{A.2})$$

We show that (i) for any $n \geq 1$, there is a p^* such that (A.1) and (A.2) are satisfied; (ii) profits are non negative, depending (obviously) on K . Of course, (i) and (ii) together make up a short run Cournot equilibrium.

(i) To ease notation, (A.1) can be written as $h(p^*, n, \theta) = 0$, where $h(p, n, \theta) \equiv \frac{1}{n}(1 - F(p, \theta)) - [p - C'(\frac{1}{n}(1 - F(p, \theta)))]f(p, \theta)$. For any given finite $n > 0$ and θ , $h(y_{\min}, n, \theta) = \frac{1}{n} - [y_{\min} - C'(\frac{1}{n})]f(y_{\min}, \theta) > 0$: trivially, if $f(y_{\min}, \theta) = 0$; but also if $f(y_{\min}, \theta) > 0$, since $C''(\cdot) \geq 0$ implies $C'(1/n) \geq C'(0) \geq y_{\min}$ by Assumption 3. On other hand, there is a $\hat{p} \leq y_{\max}$ such that $h(\hat{p}, n, \theta) < 0$: $\hat{p} = y_{\max}$ if $f(y_{\max}) > 0$, as $-[y_{\max} - C'(0)]f(y_{\max}, \theta) < 0$; while if $f(y_{\max}) = 0$ we note that $\frac{\partial h(y_{\max}, n, \theta)}{\partial p} = -[y_{\max} - C'(0)]\frac{\partial f(y_{\max}, \theta)}{\partial p} > 0$ by Assumption 4 (implying $\frac{\partial f(y_{\max}, \theta)}{\partial p} < 0$), so that $h(\hat{p}, n, \theta) < 0$ for any \hat{p} close enough to y_{\max} . By continuity, there exists a $p^* \in (y_{\min}, y_{\max})$ such that $h(p^*, n, \theta) = 0$. To prove (A.2), note that h crosses the 0-axis at least once from above; let p^* be one such crossing point, where clearly $\frac{\partial h(p^*, n, \theta)}{\partial p} < 0$: we have $0 < -\frac{\partial h(p^*, n, \theta)}{\partial p} = \frac{\partial f(p^*, \theta)}{\partial p}[p^* - C'(\cdot)] + [1 + \frac{1}{n}C''(\cdot)f(p^*, \theta)]f(p^*, \theta) + \frac{f(p^*, \theta)}{n} \leq \frac{\partial f(p^*, \theta)}{\partial p}[p^* - C'(\cdot)] + f(p^*, \theta)[2 + C''(\cdot)f(p^*, \theta)]$. The last inequality holds for any $n \geq 1$.

(ii) There has to be a value $\bar{K} > 0$, such that for all $K \leq \bar{K}$ equilibrium profits are non negative. This is directly implied by Assumption 2, as (for any finite n) $p^* > C_M(\frac{1}{n}(1 - F(p^*, \theta)))$: hence, for $K = 0$ profits are strictly positive at such n , and there follows that such \bar{K} exists.

Quasi-competitiveness

Since p^* satisfies $h(p^*, n, \theta) = 0$, clearly $dp^*/dn = -\frac{\partial h}{\partial n}/\frac{\partial h}{\partial p}$. We know from the above that at p^* , $\frac{\partial h}{\partial p} < 0$. On the other hand, for given p^* , $\frac{\partial h}{\partial n} = -\frac{1}{n^2}[(1 - F(p^*, \theta)) + C''(\cdot)f(p^*, \theta)] < 0$. Hence p^* decreases as n increases.

Monotonic convergence

In order to prove that $\lim_{K \rightarrow 0} p^*(n(K, \theta), \theta) = C'(0)$, we first note that $\lim_{K \rightarrow 0} n(K, \theta) = \infty$: by Assumption 2, for any finite n profit is positive at $K = 0$, since $p^* > C_M$. Now observe that $\lim_{n \rightarrow \infty} p^*(n, \theta) = C'(0)$. To this end, suppose to the contrary that

$$\lim_{n \rightarrow \infty} p^*(n, \theta) = \hat{p} \neq C'(0)$$

Then we would have

$$\lim_{n \rightarrow \infty} h(p^*, n, \theta) = 0 - [\hat{p} - C'(0)]f(\hat{p}, \theta) \neq 0$$

which cannot be, since it violates (A.1).

As to monotonicity, it is easily checked that the long run equilibrium number of firms $n^*(K, \theta)$ verifies $\partial n^*/\partial K < 0$. Indeed, by the zero profit condition $\hat{\pi}(p^*(n), \theta) - K = 0$ (where $\hat{\pi}(p^*(n), \theta)$ is gross profits), and using quasi-competitiveness, one can see that $d\hat{\pi}/dn < 0$ and hence $\partial n^*/\partial K < 0$. Monotonicity of p^* then follows.