$$(31)' \quad \begin{cases} g_1 = g_4 - E(I_p \mid e_{00}) \geq 0 \\ g_2 = g_4 - E(I_p \mid e_{11}) \geq 0 \\ g_3 = g_4 - E(I_p \mid e_{01}) \geq 0 \\ g_4 = E(I_p \mid e_{10}) \geq 0 \end{cases}$$

As for the costs C_{bp} of effort, we define[23]:

$$(32)\ C_{bp}(e_{bp}) = \begin{cases} 0 & \text{if } e_{00} \\ C_{bp} & \text{if } e_{11} \\ C_{bp} - C_p & \text{if } e_{10} \\ C_{bp} - C_b & \text{if } e_{01}. \end{cases}$$

Also here C_{bp} is constant, as C_b and C_p.

6. Comparison of the two contracts in the electoral period.

We have seen that in the election period, if the politician appoints two agents, his expected utility is:

$E(U\text{-}u \mid e_{10}) = H - H_0$.

On the other hand, the utility expected by the politician, when he entrusts the task to a single agent is

$$(1)' \quad E(U\text{-}u \mid e_{10}) = H - H_0'$$

To prove that in the electoral period it is to the politician's advantage to give the two tasks to a single agent - for an appropriate allocation of the size of incentive payments T_{ij}, compatible with the constraints (C1)(see appendix C.) $g_k \geq 0$ for k = 1,2,3,4 - one needs to show that it is possible to have

$H_0' \leq H_0$.

[23] For the problem of constrained optimization see appendix C.).

Let us suppose that the politician rewards the central banker, by paying him the same amount set for the banking authority under conditions of banking stability[24], provided he makes an effort to achieve both banking stability and price instability. He will be punished, however, in all other cases. So if we put:

$T_{10} = T_b = C_b\,(1- P_{3b}) / (P_{1b} - P_{3b})$ and

$T_{11} = T_{01} = T_{00} = t_b = -C_b\,P_{3b} / (P_{1b} - P_{3b})$,

we get

$2\,H_0 = (T_b^2 + t_b^2)\,P_{1b} - t_b^2$,

while

$2\,H_0' = (T_b^2 + t_b^2)\,P_{1b}\,(1 - P_{3p}) - t_b^2$

and the result is therefore $H_0' < H_0$, as was foreseen.

For this result to be valid, the allocation considered needs to be compatible with the constraints $g_k \geq 0$ given by (C1)(see in appendix C.). Let us check it.

With the incentive payments considered, the result is:

$g_1 \geq 0$ equivalent to $C_b\,(1- P_{3b}) + C_p \geq C_{bp}$

$g_2 \geq 0$ equivalent to $C_b\,[P_{1b}\,(1-P_{3p}) - P_{2b}\,(1 - P_{1p})] / (P_{1b} - P_{3b}) + C_p \geq 0$.

From conditions (2)' and (3)' it follows that the expression in square brackets is positive and, therefore, the previous inequality is always true.

$g_3 \geq 0$ equivalent to $C_b\,[P_{1b}\,(1 - P_{3p}) - P_{0b}\,(1 - P_{1p})] / (P_{1b} - P_{3b}) + C_p - C_b \geq 0$.

For the same reasons as before, the expression in square brackets is proved to be positive and, therefore, if $C_b \leq C_p$ the third constraint is also shown to be valid.

$g_4 \geq 0$ equivalent to $C_b\,[P_{1b}\,(1 - P_{3p}) - P_{3b}] \geq (C_{bp} - C_p)\,(P_{1b} - P_{3b})$

which can also be written

$$C_{bp} \leq C_p + C_b\,[1 - P_{1b}\,P_{3p} / (P_{1b} - P_{3b})\,]$$

and is likely to be $P_{1b}\,P_{3p} / (P_{1b} - P_{3b}) \leq 1$, if P_{3b} is small enough compared to P_{1b}.

[24] This hypothesis fits in with the idea that there are economies of scope ($c_{bp} < c_b + c_p$), which emerges from the model.

We can therefore conclude that the choice of incentive payments is compatible with the constraints if:

$C_{bp} \leq C_p + C_b$ i.e. if there are economies of scope;

$C_b \leq C_p$ if the task of the central bank is more demanding (this is clear if one thinks of the fact that the central bank is also responsible for banking stability as lender of last resort);

$P_{3b} < P_{1b}$ it is natural to expect this given the definition of these probabilities (in other words it is normal to expect that it is easier to achieve banking stability if the authority responsible makes an effort in this direction).

In these cases, in the election period it is to the politician's advantage to appoint a single agent for the two roles.

7. Contract with a single agent in the non-electoral period.

In the post-electoral period, government authorities will want to contain the negative effects, in terms of inflation, deriving from the non-socially beneficial decisions taken, during the election period, to maximize the probability of re-election.

For this reason in the non-electoral period the politician will prefer price stability and stability in the banking system. He will therefore offer the CB the following payments

T_{11} if $E_1 = Bs \cap Ps$

T_{10} " $E_2 = Bs \cap -Ps$

T_{01} " $E_3 = -Bs \cap Ps$

T_{00} " $E_4 = -Bs \cap -Ps$

with (presumably)

(33) $T_{11} \geq T_{10}$, $T_{01} \geq T_{00}$.

The politician's expected net utility in the non-electoral period will be:

$$E(U\text{-}u \mid e_{11}) = H' - K'$$