4 The full agglomeration and the symmetric equilibria

In this section, we analyze the full agglomeration and the symmetric equilibria when the regional levels of technology (a_r) are given.[7] In other words, equation (23) is not considered here, but will be introduced in the following section.

4.1 Agglomeration

For given regional levels of the technology (a_r), agglomeration of the modern sector in region v is a *sustainable equilibrium* when no firm finds it profitable to relocate or start its production in region r (where $v, r = s, n$ and $v \neq r$). In other words, full agglomeration of the modern sector in region v is a sustainable equilibrium if, and only if, with all firms located in region v, the sales of a (potential) firm relocating to region r (Q_{mir}) are less than the level required to break even (Q^*_{mir}):

$$Q_{mir} < Q^*_{mir}$$

Following Puga and Venables (1996) and Puga (1999), we compute in appendix A the conditions for the full agglomeration in region v to be a sustainable equilibrium when the regional levels of technology a_v and a_r are given.

Moreover, in appendix A it is shown that two cases may arise according to two different ranges of the parameters of the model. The first one arises when $0 < \mu_c \leqslant \mu_c^*$, and is characterized by the fact that the wages of unskilled workers are the same in the two regions ($w_{lv} = w_{lr} = 1$). The second case arises when $\mu_c > \mu_c^*$, and is characterized by a higher wage for unskilled workers in the region (v) in which the agglomeration of the modern sector takes place. Moreover, while in the first case the traditional sector may be active in both regions, in the second case region r is completely specialized in the production of the traditional good, and region v in the production

[7] There could be also a third type of equilibrium when $0 < \psi < 1$. In fact, there are two asymmetric equilibria characterized by technological level equal to $a_n = 1$ in the north and respectively $a_s = 1 - \sqrt{\psi}$ or $a_s = 1 + \sqrt{\psi}$ in the south. However, because these would never be stable equilibria for equation (23), we do not consider this type of equilibrium.

of the varieties of the modern good. The threshold value μ_c^* is given by:

$$\mu_c^* = \frac{(1-\mu)}{2(1-\mu)-\gamma}$$

4.1.1 Case I. $0 < \mu_c \leqslant \mu_c^*$

First, we consider the case in which the wages of unskilled workers in the core region v are equal to those of unskilled workers in the periphery r (that is, $w_{lv} = w_{lr} = 1$), and in which the traditional good may be produced in both regions. In this case, agglomeration in region v is a sustainable configuration when:

$$\frac{Q_{mir}}{Q_{mir}^*} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} \tau^{1-\sigma(1+\mu+\gamma\mu_c)} \left(\frac{\left(\tau^{2(\sigma-1)}-1\right)(1-\mu_c\gamma-\mu)}{2}+1\right) < 1 \tag{24}$$

Expression (24) cannot be "easily turned into a closed-form solution for the range of trade costs for which agglomeration is sustainable".[8] However, following Puga (1999), we notice that the value of Q_{mir}/Q_{mir}^* approaches $\left(\frac{a_v}{a_r}\right)^{1-\sigma}$ when τ tends to 1, and that its derivative is negative for τ close to 1. Moreover, when τ becomes infinitely large so does Q_{mir}/Q_{mir}^*, provided that $\sigma > 1/(1-\mu-\gamma\mu_c)$.

Let us define τ^* as the value of τ, below which agglomeration becomes sustainable because $Q_{mir}/Q_{mir}^* < 1$.

The graphic in Figure 3 plots Q_{mir}/Q_{mir}^* as a function of trade costs, for given values of other parameters such that $0 < \mu_c \leqslant \mu_c^*$.[9] When trade costs are higher than τ^*, the full agglomeration of the manufacturing sector in region v is not a sustainable configuration because a firm may start its production in region r without suffering losses, given that $Q_{mir}/Q_{mir}^* \geq 1$. On the contrary, full agglomeration in region v is a sustainable equilibrium if trade costs are smaller than τ^* because $Q_{mir}/Q_{mir}^* < 1$.

Insert Figure 3 about here

[8] See Puga (1999), p. 318.

[9] The graphic is obtained for the following parameter values: $\gamma = 0.1$, $\mu = 0.3$, $\mu_c = 0.4$, $\sigma = 5$, $a_v = a_r = 1$.

When the level of integration between the two regions is high, that is, when τ tends to 1, we observe that

$$\lim_{\tau \to 1} \frac{Q_{mir}}{Q^*_{mir}} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} \tag{25}$$

Therefore, when the level of the technology of the core region v is higher than that of the periphery r, that is, when $a_v > a_r$, and the two regions are highly integrated ($\tau \to 1$), full agglomeration of the modern sector in region v is more likely to occur because (25) is:

$$\lim_{\tau \to 1} \frac{Q_{mir}}{Q^*_{mir}} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} < 1 \qquad \forall \sigma > 1$$

4.1.2 Case II. $\mu_c > \mu_c^*$

When $\mu_c > \mu_c^*$, the share of consumers' expenditures on manufacturing goods is high enough to yield full agglomeration of the manufacturing sector in region v as well as full specialization of the two regions. More precisely, in this case manufacturing goods are produced only in the core region v, and the agricultural good only in the periphery r, the wages of unskilled workers being now lower in the periphery than in the core. In fact, in appendix A we show that, while in region r unskilled workers earn 1, in region v their wage is given by:

$$w_{lv} = \frac{(1-\mu-\gamma)\mu_c}{(1-\mu)(1-\mu_c)}$$

where $\mu, \mu_c \neq 1$, $\mu_c \neq (1-\mu)/\gamma$.[10]

The previous expression outlines the fact that the wage w_{lv} of unskilled workers in region v increases when the share $(1-\mu-\gamma)$ of total cost of production of firms devoted to unskilled workers increases, when the share μ of total cost devoted to manufacturing intermediate goods decreases, and when the share μ_c of consumers' expenditures on manufacturing goods increases.

Agglomeration in region v is a sustainable configuration if:

$$\frac{Q_{mir}}{Q^*_{mir}} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} \tau^{1-\sigma(1+\mu+\gamma\mu_c)} \left(\frac{(1-\mu)(1-\mu_c)}{(1-\mu-\gamma)\mu_c}\right)^{-\sigma(1-\gamma-\mu)} \left[\left(\tau^{2(\sigma-1)}-1\right)(1-\mu)(1-\mu_c)+1\right] < 1 \tag{26}$$

[10] In Appendix A we show that when $\mu_c > \mu_c^*$, $w_{lv} = \dfrac{(1-\mu-\gamma)\mu_c}{(1-\mu)(1-\mu_c)} > 1$.

Again, it is worth noting that the value of Q_{mir}/Q^*_{mir} becomes infinitely large when τ tends to ∞, provided that $\sigma > 1/(1-\mu-\gamma\mu_c)$.

However, expression (26) implies that not only high trade costs, but also sufficiently low trade costs may lead some firms to locate their production in the periphery r where the wages of unskilled workers are lower. Indeed, Q_{mir}/Q^*_{mir} may become higher than (or equal to) 1 for low trade costs. In this case, the agglomeration of the modern sector in region v may be unsustainable not only for high, but also for low trade costs.

Let τ^{**} be the value of trade costs at which agglomeration becomes unsustainable for $\tau \leqslant \tau^{**}$ (with $\tau^* < \tau^{**}$). Figure 4 plots Q_{mir}/Q^*_{mir} when parameters are such that $\mu_c > \mu^*_c$ and that τ^{**} exists.[11]

Insert Figure 4 about here

A necessary condition for τ^{**} to exist is that $Q_{mir}/Q^*_{mir} \geq 1$ when $\tau = 1$, that is:

$$\frac{Q_{mir}}{Q^*_{mir}} = \left(\frac{a_v}{a_r}\right)^{1-\sigma} \left(\frac{(1-\mu)(1-\mu_c)}{(1-\mu-\gamma)\mu_c}\right)^{-\sigma(1-\gamma-\mu)} \geq 1 \tag{27}$$

Expression (27) is true when the wages of unskilled workers are such that:

$$w_{lv}^{\sigma(1-\gamma-\mu)} = \left(\frac{(1-\mu-\gamma)\mu_c}{(1-\mu)(1-\mu_c)}\right)^{\sigma(1-\gamma-\mu)} \geq \left(\frac{a_v}{a_r}\right)^{\sigma-1} \tag{28}$$

When τ^{**}.exists, agglomeration of the manufacturing sector in region v is unsustainable for $\tau \leqslant \tau^{**}$. In other words, the high nominal wage of unskilled workers in region v is not supported by a sufficiently high level of technology available for firms in this region. We may compare this result with that of Puga (1999), who considers only one category of workers who can or cannot be interregionally mobile. While in Puga (1999) labor mobility implies a monotonic relationship between the sustainability of agglomeration and the levels of trade costs, in our work, the introduction of two types of workers, characterized by different mobility assumptions, allows us to show

[11] The graphic is obtained for the following parameter values: $\gamma = 0.1$, $\mu = 0.4$, $\mu_c = 0.6$, $\sigma = 5$, $a_v = a_r = 1$.

that *the existence of an immobile factor may give rise to a non-monotonic relationship.*[12] In fact, we may come across the $\cap$-shaped relationship found by Venables (1996) when $\mu_c > \mu_c^*$. When this is so, the existence of an immobile factor leads to the dispersion of the economic activity for high level of integration, because firms find it profitable to produce in the periphery, where the wages of unskilled workers are lower. However, from expression (28) we may conclude that this happens only if the technological advantage of the core region v is not too large, and if the wages of unskilled workers in the core are too high in relation to the technological gap $(a_v - a_r)$.

Finally, following Puga (1999), numerical simulations for (24) and (26) suggest the following results:

$$\frac{\partial \tau^*}{\partial \sigma} < 0 \qquad \frac{\partial \tau^*}{\partial \mu} > 0 \qquad \frac{\partial \tau^*}{\partial \gamma} > 0 \qquad \frac{\partial \tau^*}{\partial a_v} > 0$$

and

$$\frac{\partial \tau^*}{\partial \mu_c} \gtrless 0 \qquad if \qquad \mu_c \lesseqgtr \mu_c^*$$

4.2 Stable Symmetric Equilibrium

The free entry and exit condition implies that the number of manufacturing firms in region r (n_r) increases (decreases) when profits in the region are positive (negative).[13] Therefore, the evolution of the mass of firms in region r is given by:

$$\dot{n}_r = \delta \pi_{ir} \tag{29}$$

where δ is a positive constant.

Let us rewrite equation (29) as follows:

$$\dot{n} = \delta \pi_i \tag{30}$$

where $\pi_i = \begin{bmatrix} \pi_{in} \\ \pi_{is} \end{bmatrix}$ and $n = \begin{bmatrix} n_n \\ n_s \end{bmatrix}$.

[12] This is possible only if τ^{**} exists.

[13] See Puga (1999).

For given values of regional technological levels (a_r), the economy can be considered at a short run or at a long run equilibrium. When the variables are at their short run equilibrium values, the number of firms in each region is not necessarily at its long run equilibrium value, because profits of firms may be positive or negative. Therefore, we may divide the variables of the models between what we call "slow" and "fast" variables. That is, the number of firms in a region (n_n and n_s) is a *slow variable*, while the other variables of the model are referred to as *fast variables*.[14] This distinction underlines that, to carry out the stability analysis, we suppose that fast variables have already reached their short run equilibrium values (which depend on slow variables values) and move along them, while slow variables move towards their long run equilibrium values.[15]

This distinction allows us to rewrite expression (30) in the neighborhood of a long run equilibrium as follows:

$$\dot{n} = \delta u\,(n) \equiv z(n) \tag{31}$$

In fact, in appendix B we show that this is possible if profits in a neighborhood of a long run equilibrium can be expressed as a function of the number of firms n:

$$\pi_i = u(n)$$

Differentiating $\dot{n} = z\,(n)$ and taking Taylor's expansion of the first order evaluated at the equilibrium values for n (denoted by $*$) yields:

$$\partial\dot{n} = \dot{n} = z\,(n^*) + \frac{\partial z}{\partial n}\,(n^*)\,(n - n^*)$$

Given that $z\,(n^*) = 0$, this expression becomes:

$$\dot{n} = \frac{\partial z}{\partial n}\,(n^*)\,(n - n^*)$$

where the matrix $\frac{\partial z}{\partial n}\,(n^*)$ is the Jacobian matrix for equation (30) for given values of a_n and a_s.

[14] With the exception of $a_n = 1$ and a_s which, in this section, are considered given.

[15] See Boggio (1986, 1999). It should be noted that while we assume that fast variables are asymptotically stable. A more rigourous approach should prove it rather than assume it.

Let the Jacobian matrix evaluated at the equilibrium be:

$$J_1^* = \frac{\partial z}{\partial n}(n^*) = \delta \frac{\partial u}{\partial n}(n^*) = \delta M$$

where $M = \frac{\partial u}{\partial n}(n^*)$.

In appendix C we show how to compute matrix J_1 and we give the symmetric equilibrium solutions. It must be noted that this equilibrium is possible only if the levels of technology are the same in the two regions ($a_n = a_s = 1$).

Matrix J_1^* is symmetric when evaluated at the symmetric equilibrium and its eigenvalues are equal to the two eigenvalues λ_1 and λ_2 of matrix M multiplied by δ.[16] Moreover, we observe that the eigenvalues of matrices J_1^* and M at the symmetric equilibrium are real numbers because the two matrices are symmetric.

Let us define the level of the free-ness of trade t, that is, the level of integration between the two regions, as:[17]

$$t \equiv \tau^{1-\sigma}$$

where $0 \leqslant t \leqslant 1$. For a given level of the elasticity of substitution σ, the free-ness of trade increases (decreases) when trade costs decrease (increase).

Given the complexity of the eigenvalues, we are not able to find a closed-form solution for the range of trade costs for which the symmetric equilibrium is stable. However, numerical simulations are helpful to illustrate *how different level of trade costs and parameters may affect the stability of the symmetric equilibrium.*

When $a_n = a_s = 1$, simulations show that the symmetric equilibrium is stable for low levels of the free-ness of trade t, that is, for low levels of integration between the two regions. Figures 5a-b and Figures 6a-b plot, respectively, the two eigenvalues λ_1 and λ_2 from different angles when t

[16] Since δ is a scalar different from zero and $\delta \in C$, and $J_1 = \delta\, M$, if the eigenvalues of M are λ_1 and λ_2, the eigenvalues of J_1 are $\delta\lambda_1$ and $\delta\lambda_2$. See Lütkepohl (1996).

[17] See Baldwin and Forslid (2000).

and μ_c, change, for given values of the other parameters.[18] These figures show the complexity of eigenvalues evaluation. Nevertheless, we observe that both eigenvalues are negative, and, therefore, the symmetric equilibrium is stable, when the level of free-ness of trade t is low (that is trade costs τ are high) and the consumption share of expenditures on manufactured good μ_c is low. In fact, in this case centripetal forces are weaker than the centrifugal ones because pecuniary externalities may not be intensively exploited by firms and consumers, given the small share of consumers' expenditures on manufacturing goods and the high levels of trade costs. However, if the free-ness of trade t increases (trade costs decrease), for given low levels of μ_c, centripetal forces become strong enough to make the symmetric equilibrium unstable, and the eigenvalue λ_1 becomes positive. Moreover, a further increase in the free-ness of trade may yield a symmetric equilibrium that is again stable for the range of parameters for which the centrifugal forces are stronger than the centripetal ones. Figures 5a-b and Figures 6a-b allow us to point out that the symmetric equilibrium also becomes unstable if, for given trade costs values (τ), the share of the manufacturing good in expenditures (μ_c) increases. However, if this share becomes too high, and is associated with relatively high level of integration (t), manufacturing firms spread out and are uniformly distributed between the two regions, because the symmetric equilibrium becomes stable again.

Insert Figure 5a about here

Insert Figure 5b about here

Insert Figure 6a about here

Insert Figure 6b about here

[18] The two eigenvalues are computed for the following parameter values: $\sigma = 3$, $\gamma = 0.1$, $\mu = 0.3$, $\bar{L}_n = \bar{L}_s = 1$, $a_n = a_s = 1$, $\lambda_r = \lambda_v = 1$ and $\bar{H} = 1$.
Following the literature on the home bias, we adopt $\lambda_r = \lambda_v = 1$.