The following table summarizes the equilibrium profits for specific values of μ and λ:

$\mu = -\frac{1+\lambda(n-1)}{n(M-1)}$, $-\frac{1}{n-1} < \lambda < 1$	$\mu = 0,\ \lambda = 0$	$\mu = 1,\ \lambda = 1$
$\pi^{Mc} = \frac{Y}{M} \frac{1}{\delta+(\sigma-\delta)(1+\lambda(n-1))}$	$\pi^{Bc} = \frac{Y}{(1-\sigma)+M[\sigma+\delta(n-1)]}$	$\pi^{PC} = \frac{Y}{Mn}$

where again superscripts denote respectively Monopolistic competition, Bertrand Competition and Perfect Coordinated behavior.

3 The multiproduct firms

The literature on multiproduct firms has mainly focused on the incentive to create brands portfolios as opposed to mono-product strategies. Indeed, the production of an entire product line may be a powerful tool to deter entry and to escape from a too much intense competition (Schmalensee, 1978).

However, the literature has paid a relatively little attention to the optimal price policies of large companies selling an entire product line; moreover, it has not provided a full motivation of two alternative organizational structures: there are companies which directly control prices from the above and companies which delegate the price decisions to independent PMs. Many papers on mergers have shown that it is profitable to allow for independent divisions when the capacity constraints play a fundamental role, such as in the cigarette market and in the automobile industry, while it is better to control each decision centrally under price competition - examples being the fast-food and mineral water industries. In the fast-food industry, all customers of the Mac-Donald and Burger King groups know that prices are defined centrally and that no autonomy is left to the single division (store). On the contrary, Williamson (1975) and Milgrom-Roberts (1992) have stressed the importance of giving independence to product divisions of the same company. There is significant evidence that Philip Morris tobacco, General Motors, Fiat, and Ford encourage competition across their own divisions, and that the same applies to Procter-&-Gamble and Mitsubishi (Nikkei Weekly 1994), to the firms of the cosmetics sector (Low 1994) and to those offering high-tech services (Forbes 1992).

Whether and when a system of PMs decentralized decisions is better than a mechanism with a centralized GD is not a trivial question. The analytical framework developed in this paper may provide an adequate tool to deal with this problem on the basis of a key distinction: the profitability of one or the other organizational structure may depend on the characteristics of the multiproduct firm's product line: market segmentation or market interlacing.

Which of them occurs depend on the relationship between the intra-company (δ) and inter-company elasticity of substitution (σ). For $\sigma < \delta$, each product line consists of a set of close substitutes (i.e. market segmentation), while for $\sigma > \delta$, each consists of a set of distant substitutes (i.e. market interlacing).

Let us consider again the model developed in sections 1 and 2 and let us now reinterpret the idea of a 'group' of products as the set of brands produced and sold by a multiproduct firm. The industry is then composed by M large multiproduct firms, whereby each company i sells n_i differentiated products (brands). Each company consists in n_i product divisions. Therefore, the *j-th* division of the *i-th* firms produces the *ij-th* variety ($\forall i = 1, ..., M$ and $\forall j = 1, ..., n_i$). As a result, the total profits of the *i-th* multiproduct firm is given by the sum of the profits of its n_i divisions:

$$\pi_i = \sum_{j=1}^{n_i} \pi_{ij} \tag{22}$$

where π_{ij} are the same of (17).

Each company sets the prices of all its products in order to maximize (22). A PM is assigned to run each division. The PMs of the same company may set prices independently or cooperatively and they have to consider the effect of each price change both on q_i (the price index of the company) and on q (the industry price index). The first order condition for the *i-th* multiproduct firm to maximize (22) is given by:

$$\frac{\partial \pi_i}{\partial p_{ij}} = 0 \ \forall j = 1..n_i$$

$$\left[\sum_{k=1}^{n_i} (p_{ik} - 1) \frac{\partial x_{ik}}{\partial p_{ij}}\right] + x_{ij} = 0$$

$$\left[\sum_{k \neq j}^{n_i} \frac{(p_{ik} - 1)\, x_{ik}}{p_{ij}} \eta^f_{x_{ik},p_{ij}}\right] + \frac{(p_{ij} - 1)\, x_{ij}}{p_{ij}} \eta^f_{x_{ij},p_{ij}} + x_{ij} = 0 \tag{23}$$

where $\eta^f_{x_{ij},p_{ij}}$ and $\eta^f_{x_{ik},p_{ij}}$ denote respectively the demand own price elasticity and the cross (intra-company) price elasticity as perceived by the PMs. If the PMs of the same company share the same conjectures, (i.e. $\lambda_{ik} = \frac{\partial p_{ik}}{\partial p_{ij}} = \lambda_i$ $\forall j = 1..n_i$ - $\forall i = 1..M$), we have:

$$\eta^f_{x_{ij},p_{ij}} = -\frac{\partial x_{ij}}{\partial p_{ij}} \frac{p_{ij}}{x_{ij}} = \delta + (\sigma - \delta)\Delta_{ij} + (1 - \sigma)\Phi_i \tag{24}$$

$$\eta^f_{x_{ik},p_{ij}} = \frac{\partial x_{ik}}{\partial p_{ij}} \frac{p_{ij}}{x_{ik}} = -(\sigma - \delta)\Delta_{ij} - (1 - \sigma)\Phi_i \tag{25}$$

where $\Delta_{ij} = \frac{p_{ij}}{q_i^{1-\delta}} \left[p_{ij}^{-\delta} + \sum_{k \neq j} p_{ik}^{-\delta} \lambda_i \right]$ and $\Phi_i = \frac{p_{ij}}{q^{1-\sigma}} \left[q_i^{\delta-\sigma} \left(p_{ij}^{-\delta} + \sum_{k \neq j}^{n_i} p_{ik}^{-\delta} \lambda_i \right) \right]$ [18].
Using (10), the first order condition yields:

$$-\sum_{k=1}^{n_i} \frac{(p_{ik}-1)\, x_{ik}}{p_{ij}} \left((\sigma-\delta)\Delta_{ij} + (1-\sigma)\Delta_{ij} \left(\frac{q_i}{q} \right)^{1-\sigma} \right) =$$

$$= \left(\frac{(p_{ij}-1)}{p_{ij}} \delta - 1 \right) \frac{Y}{p_{ij}} \left(\frac{p_{ij}}{q_i} \right)^{1-\delta} \left(\frac{q_i}{q} \right)^{1-\sigma}$$

which can be rewritten as:

$$-\sum_{k=1}^{n_i} (p_{ik}-1)\, x_{ik} \left((\sigma-\delta) + (1-\sigma) \left(\frac{q_i}{q} \right)^{1-\sigma} \right) =$$

$$\left[\frac{(p_{ij}-1)}{p_{ij}} \delta - 1 \right] \frac{Y}{\Delta_{ij}} \left(\frac{p_{ij}}{q_i} \right)^{1-\delta} \left(\frac{q_i}{q} \right)^{1-\sigma} \tag{26}$$

the left-hand side of (26) turns out to be the same for all $j = 1..n_i$. Therefore, all prices set within the company are equal, $p_{ij} = p_i$, for all $j = 1..n_i$. Hence, $\left(\frac{p_{ij}}{q_i}\right)^{1-\delta} = \frac{1}{n_i}$ and $\Delta_{ij} = \Delta_i$: each variety is produced in the same amount, $x_{ij} = x_i$ for all $j = 1..n_i$. Now for a given n_i, $\Delta_i = \frac{1+\lambda_i(n_i-1)}{n_i}$ shows the degree of independence (or coordination) between the PMs of the same company i.

For $\Delta_i = 1$, the PMs' decisions depend on the GD's instructions (centralized decisions); while the lower is Δ_i within the interval $0 < \Delta_i < 1$, the lower is the coordination among divisions; at the extreme, for $\Delta_i = 0$ we have independent PMs. Moreover, since we have assumed Bertrand competition between multiproduct firms, the industry price index effect is given by $\Phi_i = \Delta_i \left(\frac{q_i}{q}\right)^{1-\sigma}$. Recalling (26), for each variety we get:

$$\frac{p_i - 1}{p_i} = \frac{1}{\delta + (\sigma-\delta)\Delta_i + (1-\sigma)\Delta_i \left(\frac{q_i}{q}\right)^{1-\sigma}} \tag{27}$$

Because all firms are identical (except for product differentiation) we confine our attention to the symmetric equilibrium with the same number of products per firm ($n_i = n \ \forall i = 1..M$); if all companies show the same internal organizational structure ($\Delta_i = \Delta \ \forall i = 1..M$), equation (27) yields:

$$p_i^* = p^* = \frac{M\delta + M(\sigma-\delta)\Delta + (1-\sigma)\Delta}{M(\delta-1) + M(\sigma-\delta)\Delta + (1-\sigma)\Delta} \tag{28}$$

[18] In order to obtain meaningful analytical results we have confined our attention to Bertrand conjectures between PMs of different multiproduct firms (i.e. $\mu_{hk} = \frac{\partial p_{hk}}{\partial p_{ij}} = 0 \ \forall k, j$ and $\forall h \neq i$)

Hence for each variety, all firms produce:

$$x_i^* = x^* = \frac{Y}{Mn}\frac{M(\delta-1)+M(\sigma-\delta)\Delta+(1-\sigma)\Delta}{M\delta+M(\sigma-\delta)\Delta+(1-\sigma)\Delta} \tag{29}$$

As Katz (1984) noted, by lowering the price of one of its brands, a multiproduct firm can increase its sales. However, these additional sales come from three sources. On the one hand, the lower price increases the total amount purchased by consumers. On the other hand, consumers will substitute this cheaper brand for varieties produced by rivals (inter-company substitution effect). Finally, sales increase at the expenses of other varieties of the same firm (intra-company substitution effect). Therefore, a multiproduct firm must consider the effects of a price change on its whole product line, taking into account how the others PMs react to such price variation. This is perfectly consistent with the findings by Yang and Heijdra (1993), who noted that the demand elasticity (of demand curves derived from CES utility functional forms) appears like a weighted average of unity and both the intra- and the inter-sector elasticities of substitution [19] . In this perspective, the demand elasticity given in (24) can be rewritten as:

$$\eta^f = \delta + \Delta\sigma - \delta\Delta - \Delta\frac{\sigma-1}{M} \tag{30}$$

where the first element, δ, is the intra-company elasticity of substitution which corresponds to the Dixit-Stiglitz (1977) approximation of demand elasticity under the standard monopolistic competition (i.e. $\Delta = 0$). For $\Delta \neq 0$, demand elasticity incorporates the inter-company substitution effect (i.e. $\Delta\sigma$), the intra-company substitution effect (i.e. $-\Delta\delta$) and the industry price index effect (i.e. $-\Delta\frac{\sigma-1}{M}$).

It is important to notice that multiproduct firms may benefit from the possibility to make use of strategies, which are available to multi-divisional firms only. Each company can define the optimal corporate structure. It may choose to control and to coordinate all decisions centrally through a GD; or it may allow each PM to be independent. An increase in coordination (higher Δ) affects the Lerner index of monopoly power along three lines: first, it strengthen the (unprofitable) impact of the inter-company substitution effect ($\Delta\sigma$); second,

[19] There is also an income-feedback effect which affects the equilibrium. As a matter of fact, the consumer's income, I, is the sum of the value of the endowment (labor) and distributed profits: $I = w + \sum \pi_i$. Therefore, one could take into account not only the direct effect of a price change, but also an additional indirect effect. The latter, is the so-called *Ford-effect*: the effect upon demand of a change in prices through the income. Nevertheless assuming free entry and normalizing the wage to one, profits are zero and the income is constant and equal to the endowment ($I = 1$). However, D'Apremont et al. (1996) rejected this approximation.

it increases the (profitable) impact of the intra-company substitution effect ($\Delta\delta$); third, it reinforces the (profitable) impact of the industry price index effect ($\Delta\frac{\sigma-1}{M}$). The two latter effects lead to a gradual decrease in demand elasticity, and hence to a gradual increase of the market power. However, the profitable effects are reduced by an higher impact of the inter-company substitution effect, and though one would expect that firms gain from whatever form of coordination rather than competition, in multiproduct firms analysis the question of the profitability of coordination between PMs of the same company is neither granted or trivial.

For a given Δ, by (22) the company total profit are:

$$\pi^* = \frac{Y}{M\delta + M(\sigma-\delta)\Delta + (1-\sigma)\Delta} \tag{31}$$

We can therefore study the profitability of coordination, evaluating the variation of profit (31) with respect to Δ:

$$\frac{\partial\pi^*}{\partial\Delta} = Y\frac{\delta - \sigma + \frac{(\sigma-1)}{M}}{[M\delta + M(\sigma-\delta)\Delta + (1-\sigma)\Delta]^2} \tag{32}$$

The sign of (32) mainly depends on the relationship between the two elasticities of substitution. Borrowing the Brander-Eaton (1984) criterion, we have already defined the possible industry structures on the basis of the relationship between the cross price elasticities of demand: market segmentation ($\delta > \sigma$) arises when multiproduct firms produce closer substitutes, while market interlacing ($\delta < \sigma$) refers to multiproduct firms producing distant substitutes.

Simple inspection of (32) shows that an increase in coordination between PMs of the same company is undoubtedly profitable in presence of segmentation, while the sign of (32) may be negative under market interlacing[20].

Under market segmentation, coordination is always profitable because the (profitable) increase of the intra-company substitution effect (δ) always dominates the (unprofitable) increase of the inter-company substitution effect (σ). On the contrary, under market interlacing, the latter effect dominates the former and allowing for independent PMs may indeed be profitable. Coordination is still to be preferred if the increase in the net (unprofitable) effect ($(\delta-\sigma)$) is lower than the (profitable) increase in the industry price index effect ($\frac{\sigma-1}{M}$). If the opposite holds, companies who centralize price decisions get lower profits.

Therefore the choice of independent PMs (as in Raubitschek, 1987) is surely the best choice under market interlacing when the standard monopolistic competition arises. In this case the price index effect is obviously negligible and the

[20] In both cases the sign of (32) is independent of n.

net impact of the increase in coordination is negative: it reduces market power, prices and profits. In this case, by allowing for independent PMs ($\Delta = 0$), a multiproduct firm offsets the net unprofitable effect. Equation (32), however, shows that a decentralized organizational structure may be optimal also in case of oligopoly [21] , depending on size of the industry price index effect.

4 Conclusion

This paper analyzes the price-setting behavior of multiproduct firms in a differentiated product market. The structure considered is one where large companies offer either a set of close substitutes (market segmentation) or a set of distant substitutes (market interlacing).

The modelling strategy of the paper is to allow for two different elasticities of substitution: while δ represents the intra-company elasticity of substitution, σ is the inter-company elasticity of substitution. The key feature of the model is the possibility for multiproduct companies to choose their optimal internal organizational structure, according to the relative size of these two parameters.

Each company, consisting of n divisions, may either set prices centrally (as in the traditional approach), or alternatively, it may assign an independent product manager to run each division. In other words, product managers of the same company may behave either independently or cooperatively.

While the model does not consider either the proliferation or the product-line selection decisions, it deals with multiproduct firms' price decisions under oligopolistic competition making use of conjectural variations. Its main purpose has been to provide a microfounded answer about the question of whether and when a system of product managers decentralized decisions is better than a mechanism with a centralized general direction.

The paper has shown that coordination is always profitable under market segmentation; while under market interlacing, the strategy of relying on independent product managers is profitable when the standard monopolistic competition arises; it may also be profitable with oligopolistic (Bertrand) competition under some (not very restrictive) assumptions.

[21] With a not negligible price index effect.