Introduction

This paper analyzes the price-setting behavior of multiproduct firms in a differentiated product market. The structure considered is one where large companies offer either a set of close substitutes or a set of distant substitutes. The key feature of the model is the possibility for multiproduct companies to choose their internal organizational structure. Each company, consisting of \( n \) divisions, may either set prices from the above in order to maximize the joint profits (as in the traditional approach), or alternatively, it may assign an independent product manager to run each division. In other words, product managers of the same company may behave either independently or cooperatively.

The modelling strategy of the paper is to allow for two different elasticities of substitution: while \( \delta \) represents the intra-group (intra-company) elasticity of substitution, \( \sigma \) is the inter-group (inter-company) elasticity of substitution. Then, the product line of companies may consist of either a set of close substitutes (market segmentation) or a set of distant substitutes (market interlacing). Within this set-up, from the one side each brand competes more intensively with closer substitutes and less intensively with distant substitutes; from the other side two kinds of interactions must be considered: that among products of the same multiproduct firm, and that among the latter and the brands produced by rivals.

As is well known, multiproduct firms are typically established in order to exploit economies of scope or economies of scale in production. Moreover, they may be the outcome of a merging process between firms aimed at removing the main market constraints, and at reducing competition between them on the demand side. It must be stressed that the literature mainly concentrated upon the supply-side foundations of multiproduct firms, while the interactions on the demand side have not received the same attention. Exceptions are due to Katz (1984), Brander-Eaton (1984), Raubitschek (1987) and Ju (2003). While Katz (1984) examines the effects of competition on the price-quality schedule, Brander and Eaton (1984) analyze multiproduct firms under market segmentation and market interlacing. As far as market structure is concerned, Raubitschek (1987) studies multiproduct firms under monopolistic competition; Ju (2003) models multiproduct firms under oligopoly.

However, Raubitschek’s assumption of monopolistic competition is not innocuous. It amounts to assuming that each Product Manager (PM) believes that all other PMs (including the PMs of the same large company) will react to an individual unit expansion in output by a total reduction of the same amount \(^1\). Therefore, given a large number of varieties in the market, firms exhibit the

\(^1\) In footnotes 3 and 4, Raubitschek assumes competitive conjectural derivatives in
standard Cournot behavior\(^2\), but since they neglect whatever effect on the industry quantity-index, the equilibrium has the properties of the traditional Dixit-Stiglitz (1977) model of monopolistic competition. While Raubitschek’s conclusions are obviously correct given her assumptions, it must be stressed that in the framework of the analysis of multiproduct firms these assumptions seem to be questionable for at least three reasons. First, why should the PMs of different companies react in the same way? Second, is it reasonable that multiproduct firms decisions are taken as negligible for the market? Finally, is it sensible to assume that the elasticity of substitution across products of the same firm does not differ from that across products of different firms, thus limiting the scope for product interaction on the demand side?

Ju (2003) does not suffer from these criticisms. He allows for an oligopolistic market structure and he takes into account both the industry price-index effect and the interactions on the demand side, distinguishing between the inter-firm and the intra-firm elasticity of substitution. Another important difference between the above contributions concerns the decisional center in the first stage of the game. While Ju assumes that the PMs of the same firm behave cooperatively, Raubitschek allows for independent PMs. In other words, the price decisions come from a centralized General Direction (GD) in Ju’s paper, and from independent PMs for each variety in Raubitschek’s model.

In the analysis of multiproduct firms’ organizational structure it is common to assume that the GD is responsible for two basic preliminary decisions. The first is the so-called proliferation decision: how many varieties to produce. The second concerns the product line selection: which kind of variety to produce. The key issue, however, is the definition of the decisional center responsible for the price/quantity decisions. Should they be delegated to PMs or should they be centralized in the GD? Which is the best corporate organization at this decision level?

The main purpose of this paper is to provide a microfounded answer to this question. While our model does not consider either the proliferation or the product-line selection decisions, it deals with multiproduct firms’ price decisions under oligopolistic competition, providing useful insights about the question of whether and when a system of PMs decentralized decisions is better than a mechanism with a centralized GD. The paper describes a differentiated product market where goods are produced at constant and identical marginal costs. The degree of competition is characterized through the use of conjectural variations. To keep the analysis simple, for each PM we shall

\[
\frac{\partial p_{hk}}{\partial x_{ij}} = -\frac{1}{n-1}, \forall k \neq j \text{ and } \forall i, j; \text{ where } x \text{ denote quantities and } n \text{ denotes the total number of variety in the market}. \text{ Note that, differently by us, she solves for quantities instead of prices equilibrium.}
\]

\(^{2}\) For \(n \to \infty\), \(\frac{\partial p_{hk}}{\partial x_{ij}} = -\frac{1}{n-1} \to 0\).
assume symmetric conjectures $\lambda$ on the reaction of the PMs of the same multiproduct firm, and symmetric conjectures $\mu$ on the reaction of the PMs of rival firms. Different values of the conjectural variations $\lambda$ are equivalent to different internal organizational structures.

The paper is organized as follows. Section 1 describes the demand side of the model starting from a *compound* CES utility function. Under the assumption that each brand is produced by a mono-product firm, the market equilibrium is derived in section 2 through the use of different conjectural variations. The results are reinterpreted in section 3 in terms of optimal price-setting behavior of multi-product firms, where the organizational structure of the corporate firm is endogenous. In particular, it is shown that under market interlacing, independent PMs may be more profitable than a centralized GD. Some conclusions are gathered in section 4.

1 Preferences

Consider an economy with identical households. The economy produces a numéraire homogeneous good and $M \geq 1$ groups of differentiated goods. Each group consists of $n_i \geq 1$ ($i = 1, \ldots, M$) varieties or brands (indexed by $j = 1, \ldots, n_i$, $\forall i$), so that the total number of varieties in the industry is $N = \sum_{i=1}^{M} n_i$.

Preferences are identical for all consumers. The representative household maximizes the utility function $U = U(x_0, V)$, where $x_0$ is the numéraire good and $U(\cdot)$ is homothetic in its arguments. Given this property, the utility maximization problem can be decomposed into two steps (Spence 1976). In particular, we assume that $V$ has a *compound* CES functional form:

$$V(x_i) = \left[ \sum_{i=1}^{M} x_i^\alpha \right]^{\frac{1}{\alpha}}$$  \hspace{1cm} (1)

$$x_i(x_{ij}) = \left( \sum_{j=1}^{n_i} x_{ij}^\beta \right)^{\frac{1}{\beta}}$$  \hspace{1cm} (2)

where $x_{ij}$ is the quantity consumed of the $j$-th product of the $i$-th group and $x_i$ represents the quantity index of the $i$-th group. Concavity of $V [x_i(x_{ij})]$ requires that $0 < \alpha < 1$ and $0 < \beta < 1$ \footnote{The love for variety could alternatively be modelled in a slightly different framework, by extending preferences over a continuous product space (Grossman and Helpman, 1989; Krugman, 1980).}. This utility function implies a
constant elasticity of substitution between any couple of varieties of different
groups of products (inter-group elasticity of substitution):

$$\sigma = \frac{1}{1 - \alpha} > 1$$  \hspace{1cm} (3)

and a constant elasticity of substitution between any couple of varieties of the
same group of products (intra-group elasticity of substitution):

$$\delta = \frac{1}{1 - \beta} > 1$$  \hspace{1cm} (4)

Let’s now denote with \( Y \) the consumer aggregate expenditure on the industry
products and with \( p_{ij} \) the price of the \( ij \)-th variety. The consumer’s problem
becomes:

$$\text{MAX}_{x_{ij}} V(x_{11}, \ldots, x_{ij}, \ldots, x_{MnM}) = \left( \sum_{i=1}^{M} \left( \sum_{j=1}^{n_i} x_{ij}^\beta \right)^\frac{1}{\beta} \right)^\frac{1}{\alpha}$$  \hspace{1cm} (5)

s.t. \hspace{1cm} \( Y = \sum_{i=1}^{M} \left( \sum_{j=1}^{n_i} p_{ij} x_{ij} \right) \)

First, the consumer maximizes \( x_i (x_{ij}) \) subject to the expenditure constraint
on the products of the group \( i \):

$$\text{MAX}_{x_{ij}} x_i = \left( \sum_{j=1}^{n_i} x_{ij}^\beta \right)^\frac{1}{\beta}$$  \hspace{1cm} (6)

s.t. \hspace{1cm} \( Y_i = \sum_{j=1}^{n_i} p_{ij} x_{ij} \)

where \( \sum_{i=1}^{M} Y_i = Y \) and \( Y_i \) represents the total expenditure on the \( i \)-th group.

In the second step, the household maximizes the utility \( V \) as a function of \( x_i \),
subject to the budget constraint on the overall of the \( M \) groups:

$$\text{MAX}_{x_i} V = \left( \sum_{i=1}^{M} x_i^\alpha \right)^\frac{1}{\alpha}$$  \hspace{1cm} (7)

s.t. \hspace{1cm} \( Y = \sum_{i=1}^{M} x_i q_i \)
where $q_i$, the price-index corresponding to the $i$-th group, is given by:

$$ q_i = \left( \sum_{j=1}^{n_i} p_{ij}^{1-\delta} \right)^{\frac{1}{1-\delta}} \\ (8) $$

and $q$, the industry price-index, is given by:

$$ q = \left( \sum_{i=1}^{M} q_i^{1-\sigma} \right)^{\frac{1}{1-\sigma}} \\ (9) $$

The solution to the first step gives us $x_{ij}(Y_i, p_{ij}) = \frac{Y_i}{p_{ij}} \left( \frac{p_{ij}}{q_i} \right)^{1-\delta}$, while the two-stage budgeting procedure requires that $Y = Y_i \left( \frac{q_i}{q} \right)^{\sigma-1}$.

Therefore, the demand schedule for the $j$-th brand in the $i$-th group ($\forall j \in [1, n_i]; \forall i \in [1, M]$) is:

$$ x_{ij}(p_{ij}) = \frac{Y}{p_{ij}} \left( \frac{p_{ij}}{q_i} \right)^{1-\delta} \left( \frac{q_i}{q} \right)^{1-\sigma} \\ (10) $$

Equation (10) will be used in the analysis of firms’ price-setting behavior. If we are interested in quantity competition, we may consider the corresponding inverse demand function:

$$ p_{ij}(x_{ij}) = \frac{Y}{x_{ij}} \left( \frac{x_{ij}}{Q_i} \right)^{\beta} \left( \frac{Q_i}{Q} \right)^{\alpha} \\ (11) $$

where $Q_i$ is the quantity-index of group $i$, given by:

$$ Q_i = \left( \sum_{j=1}^{n_i} x_{ij}^\beta \right)^{\frac{1}{\beta}} \\ (12) $$

and $Q$ is the industry quantity-index, given by:

$$ Q = \left[ \sum_{i=1}^{M} Q_i^\alpha \right]^{\frac{1}{\alpha}} \\ (13) $$

Notice the immediate interpretation of $\alpha$ and $\beta$ in terms of the structure of preferences. The parameters are indicators of the degree of substitutability between any couple of varieties: the lower $\alpha$, the lower the interdependence among varieties produced in different groups; the higher $\beta$, the higher the interdependence among brands produced within the same group. At the intra-group level, as $\delta \to 1$ the degree of product differentiation is maximum. As $\delta \to \infty$, there is no intra-group differentiation, the degree of substitutability
becomes infinite and varieties of the same group are homogeneous. In the inter-group perspective, as \( \alpha \to 0 \) the degree of substitution reaches the minimum level (i.e. \( \sigma \to 1 \)) and varieties of different groups become highly differentiated. As \( \alpha \to 1 \) there is no differentiation between varieties of different groups, the degree of substitutability becomes infinite (i.e. \( \sigma \to \infty \)) and any brand is perfectly substitutable with any others of the remaining \( M-1 \) groups.

Moreover, the difference between \( \sigma \) and \( \delta \) plays a fundamental role: when the inter-group elasticity of substitution is greater than the intra-group elasticity of substitution (i.e. \( \sigma > \delta \)), brands of different groups are closer substitutes rather than varieties of the same group; on the contrary, when \( \sigma < \delta \) each variety is more substitutable with a brand of the same group. This aspect is also captured by demand elasticities. In this model three price elasticities are defined: with respect to the own price, with respect to the price of brands produced in the same group and with respect to the price of goods produced in different groups. Let us denote with \( \eta_{x_{ik},p_{ij}} \) the cross price elasticity of the demand function of the \( ik \)-th variety with respect to the price of a brand of the same group, \( p_{ij} \); and with \( \eta_{x_{hk},p_{ij}} \) the cross price elasticity of the demand function of the \( hk \)-th variety with respect to the price of any other brand of a different group, \( p_{ij} \). The demand elasticity of the \( ij \)-th variety with respect to the own price, \( p_{ij} \), is:

\[
\eta_{x_{ij},p_{ij}} = -\frac{\partial x_{ij}}{\partial p_{ij}} \frac{p_{ij}}{x_{ij}} = \delta + (\sigma - \delta) \left( \frac{p_{ij}}{q_i} \right)^{1-\delta} + (1 - \sigma) \left( \frac{p_{ij}}{q_i} \right)^{1-\delta} \left( \frac{q_i}{q} \right)^{1-\sigma} \tag{14}
\]

The cross price elasticity of the demand of the \( ik \)-th variety with respect to the price of a brand produced within the same group, \( p_{ij} \), can be written as:

\[
\eta_{x_{ik},p_{ij}} = -\frac{\partial x_{ik}}{\partial p_{ij}} \frac{p_{ij}}{x_{ik}} = -(\sigma - \delta) \left( \frac{p_{ij}}{q_i} \right)^{1-\delta} - (1 - \sigma) \left( \frac{p_{ij}}{q_i} \right)^{1-\delta} \left( \frac{q_i}{q} \right)^{1-\sigma} \tag{15}
\]

The cross price elasticity of the demand of the \( hk \)-th variety with respect to the price of any other brand of a different group, \( p_{ij} \), is:

\[
\eta_{x_{hk},p_{ij}} = -\frac{\partial x_{hk}}{\partial p_{ij}} \frac{p_{ij}}{x_{hk}} = -(1 - \sigma) \left( \frac{p_{ij}}{q_i} \right)^{1-\delta} \left( \frac{q_i}{q} \right)^{1-\sigma} \tag{16}
\]

Notice that these elasticities correspond to the elasticities perceived by the price-maker for the relevant variety, when each of them believes that his decisions do not affect the rivals’ price decisions \(^4\) but takes into account the effect of his price on the relevant price indices. If \( \eta_{x_{ik},p_{ij}} < \eta_{x_{hk},p_{ij}} \) (i.e. \( \sigma > \delta \)), then \( x_{ij} \) and \( x_{ik} \) are distant substitutes, while \( x_{ij} \) and \( x_{hk} \) are closer substitutes.

\(^4\) That is in case each firm has a standard Bertrand conjectures. We shall generalize this assumption making use of non-zero conjectural variations in the next section.
If the opposite holds (for $\sigma < \delta$, i.e. $\eta_{ik,p_ik} > \eta_{hk,p_{hk}}$), $x_{ij}$ and $x_{ik}$ are close substitutes, while $x_{ij}$ and $x_{hk}$ are distant substitutes $^5$.

### 1.1 An example

In the above discussion, the varieties produced in the market are collected in groups, which may consist of either close substitutes or distant substitutes. In order to clarify this point an example may be of help.

Consider the Carbonated Beverages market. In this market it is possible to group the products according to two criteria. The first is based on intrinsic characteristics of the products themselves: e.g. Fruits drinks, Cola drinks and Fizzy drinks. In this case each group consists of close substitutes. The Fruits drinks are Fanta, Oransoda, and Lemonsoda etc.; in the second group we find Coke, Pepsi, and Virgin; while the last group is made by drinks such as Sprite, Schweppes, and Seven-Up.

According to a second criterion, however, it is possible to collect products on the basis of the different trade-marks under which the varieties are sold. In this case we have the group of the Coca-Cola Company which produces distant substitutes, such as Coke, Sprite, Fanta; the same occurs for the PepsiCo International Inc. which sells Pepsi-Cola, Mountain-Dew and Slice-Soda. On the other side, under their trade-marks both Cadbury-Schweppes plc. and Gruppo-Campari produce close substitutes. For example, in the product line of the former we can find Seven-Up, dnL, Schweppes, while the latter produces Lemonsoda, Oransoda, Pelmosoda and Tonicsoda.

In section 3, the product line of a multiproduct firm will coincide with a 'group' in the above definition. This allows to cover both the situations which Brander and Eaton (1984) define as market segmentation - the multiproduct firm produces close substitutes (e.g. the Cadbury-Schweppes plc. and Gruppo-Campari cases) - and those of market interlacing - each company produces distant substitutes (the Coke-Cola and Pepsi-Cola examples).

$^5$ The substitutability relationship may be expressed using direct or inverse demand function, and using either elasticities or derivatives. Moreover, the definitions do not necessary coincide. The concept of substitutability can be defined in terms of "q-substitutes" (with reference to the inverse demand function), or in terms of "p-substitutes" (with reference to the direct demand function) (Hicks (1956)). Since in the next sections we shall assume that prices are the firms’ strategic variable, the p-substitutes approach is more convenient.
2 Conjectural variations and market equilibrium

In this section market equilibrium is analyzed under the hypothesis that each variety is produced by a mono-product firm, which competes with both the other producers within its own group, and the producers belonging to other groups. In particular, we assume that there are \( n_i \) mono-product firms for each group \( i \) of products \((i = 1, \ldots, M)\), and that each of them produces a brand (indexed with \( j = 1, \ldots, n_i \)) of the \( i \)-th group.

Since there are \( n_i \) varieties per group, each firm simultaneously faces two different competitive environments. Horizontally, each firm competes with others producing imperfect substitutes of degree \( \sigma \) at the inter-group level. At the intra-group level, however, it competes with other firms producing imperfect substitutes with degree of substitutability \( \delta \). Therefore, there is an inter-group competition between firms of different groups, and an intra-group competition within the same group. We assume that prices are the firms’ strategic variable.

The \( j \)-th mono-product firm \((j \in [1, n_i])\), of the \( i \)-th group \((i \in [1, M])\), produces the \( ij \)-th variety according to a linear technology. Hence for the \( ij \)-th firm, the cost function is \( C(x_{ij}) = cx_{ij} \), where \( c \) is the constant marginal cost. Throughout the analysis, we normalize it to one (i.e. \( c = 1 \)). Each firm sets its own price in order to maximize profits:

\[
\pi_{ij} = x_{ij}(p_{ij})p_{ij} - x_{ij}(p_{ij})
\]  

(17)

The first order condition for profit maximization can be written in terms of the Lerner index of monopoly power:

\[
\frac{\partial \pi_{ij}}{\partial p_{ij}} = 0 \iff \frac{p_{ij} - 1}{p_{ij}} = -\frac{\frac{\partial x_{ij}}{\partial p_{ij}} p_{ij}}{\frac{\partial x_{ij}}{\partial x_{ij}} x_{ij}}
\]  

(18)

where \( -\frac{\partial x_{ij}}{\partial p_{ij}} x_{ij} = \eta^f_{x_{ij}, p_{ij}} \) is the demand price elasticity as perceived by the \( ij \)-th firm:

\[
\eta^f_{x_{ij}, p_{ij}} = \delta + (\sigma - \delta)\Delta_{ij} + (1 - \sigma)\Phi_{ij}
\]  

(19)

In the elasticity formula, \( \Delta_{ij} \) and \( \Phi_{ij} \) measure, respectively, the effects of the \( ij \)-th price variation on the own group price-index (group price-index-effect) and on the industry price-index (industry price-index-effect). The first is given by \( \Delta_{ij} = \frac{\partial q_j}{\partial p_{ij}} \frac{p_{ij}}{q_i} \), while the second is \( \Phi_{ij} = \frac{\partial q_j}{\partial p_{ij}} \frac{p_{ij}}{q_i} \).

\[\text{In particular, intra-group competition could be involved differences in quality, so that intra-group competition may turn to vertical product differentiation.}\]

\[\text{Notice the difference between demand elasticity in (14) and demand elasticity as perceived by firms.}\]
Clearly, the firm’s demand elasticity affects its market power, being related to the competitive environment perceived by firms. In particular, different market structures can be seen as the outcome of different assumptions about the impact of each firm’s price decision on the rivals’ behavior. Consider the effect of changing of \( p_{ij} \) both on \( q_i \) (i.e. \( \Delta_{ij} \)) and on \( q \) (i.e. \( \Phi_{ij} \)). If \( \Delta_{ij} = 0 \) and \( \Phi_{ij} = 0 \), then the firm’s price decisions have no effect respectively at the group and at the industry level; on the contrary, \( \Delta_{ij} = 1 \) and \( \Phi_{ij} = 1 \) denote full effects\(^8\). When the individual price decision influences the group price-index, an oligopolistic intra-group competition arises\(^9\). Differently, the intra-group competition is monopolistic when the firm’s price decisions are negligible and they do not influence the group price-index.

Moreover, the perceived effect on \( q \) synthesizes the nature of competition at the inter-group level. When the effect is not negligible, inter-group competition is oligopolistic; while inter-group monopolistic competition arises when such effect is neglected\(^{10}\).

2.1 The perceived market structure

In the standard monopolistic competition literature, the attention has often been focused on markets where the existence of a large number of operating firms implies that each individual decision is negligible in the previous sense. In particular the Dixit-Stiglitz (1977) model has been used to examine a wide range of issues.

However, the assumption of competitive behavior is independent of the number of agents in the market\(^{11}\): as long as the agents behave competitively, the competitive equilibrium can be solved for any number of firms. The idea that the competitive behavior and the negligibility assumption are related to the existence of a large number of sellers, depends on two main reasons. First, price-index-taking behavior seems more reasonable when the number of firms is large; second, the equilibrium in non-competitive market structures converges to the competitive one when the number of firms increases. Nevertheless, the definition of market structure is indeed independent of the number of firms. Rather, the specific environment faced by firms is closely related to the beliefs, the conjectures, about the rivals’ reactions (Bresnahan

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\(^8\) The admitted ranges are: \( 0 \leq \Delta_{ij} \leq 1 \) and \( 0 \leq \Phi_{ij} \leq 1 \).

\(^9\) Di Cintio (2005) studies a similar industry structure, where each group is composed by homogeneous products.

\(^{10}\) Reasonably, if firms neglect the effect on \( q_i \) they cannot take into account the effect on \( q \).

\(^{11}\) In frameworks of product differentiation, the term competitive is obviously referred to ”monopolistic competitive behavior”. 
(1981), Kamien-Schwartz (1983), Perry (1982)).

As a useful starting point, we first reinterpret the well-known market outcomes (commonly analyzed in the literature) in terms of conjectural variations. In this model two conjectural variations are considered. The first is the intra-group conjectural derivative: 

$$ \frac{\partial p_{ik}}{\partial p_{ij}} = \lambda_{ik} (\forall i \text{ and } \forall k \neq j), $$

which measures what the typical $ij$-th firm believes about the relationship between its own price variation and the price change of rivals of the same group.

The second is the inter-group conjecture 

$$ \frac{\partial p_{hk}}{\partial p_{ij}} = \mu_{hk} (\forall h \neq i \text{ and } \forall k), $$

which measures what the typical $ij$-th firm believes about the relationship between its own price change and the reaction of rivals of any other group.

For $\lambda_{ik} = 0$ and $\mu_{hk} = 0$, we are in the standard Bertrand case: each firm expects that if it changes its price, the rivals will not change theirs. In this model we also allow for non-zero conjectures. In particular, negative conjectures mean that each firm believes that the rivals will react to a price increase through a reduction of their prices; while positive conjectures mean that rivals will react in the same direction of the $ij$-th price change.

Taking into account the above definitions, we may express the effect (of a change in the $ij$-th price) on the group and industry price indices, in terms of the conjectural variations $\lambda$ and $\mu$:

$$ \Delta_{ij} = \frac{p_{ij}}{q_i} \left( p_{ij}^{-\delta} + \sum_{k \neq j} \sum_{k} n_{ik} \lambda_{ik} \right) $$

and

$$ \Phi_{ij} = \frac{p_{ij}}{q_i} \left[ \frac{q_i^{-\delta}}{q_i} \left( p_{ij}^{-\delta} \sum_{k \neq j} \sum_{k} n_{ik} \lambda_{ik} \right) + \left( \frac{\sum_{h \neq i} \sum_{h=1}^M q_h \mu_{hk}}{\sum_{h=1}^M} \right) \right] $$

By making use of conjectural variations, we allow for the dependence of market structure on the beliefs of agents, thus mitigating the predictive power of those theories which link the structure of the market to the number of active firms. Any market structure is therefore conceivable for any given number of firms, and different competitive environments may result.

From equations (18) and (19), we can derive the general solution of the model for the equilibrium price, quantity and profits, under generic conjectures:

<table>
<thead>
<tr>
<th>$p_{ij}^*$</th>
<th>$x_{ij}^*$</th>
<th>$\pi_{ij}^*$</th>
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<tbody>
<tr>
<td>$\frac{n_{ij}}{q_i} \left( \frac{q_i}{q} \right)^{1-\delta} \left( \frac{q_i}{q} \right)^{1-\sigma} (\delta-1)+(\sigma-\delta)\Delta_{ij}+(1-\sigma)\Phi_{ij}$</td>
<td>$Y \left( \frac{n_{ij}}{q_i} \right)^{1-\delta} \left( \frac{q_i}{q} \right)^{1-\sigma} \left( \frac{q_i}{q} \right)^{1-\sigma} (\delta-1)+(\sigma-\delta)\Delta_{ij}+(1-\sigma)\Phi_{ij}$</td>
<td>$\left( \frac{n_{ij}}{q_i} \right)^{1-\delta} \left( \frac{q_i}{q} \right)^{1-\sigma} \left( \frac{q_i}{q} \right)^{1-\sigma} (\delta-1)+(\sigma-\delta)\Delta_{ij}+(1-\sigma)\Phi_{ij}$</td>
</tr>
</tbody>
</table>
If we confine our attention to the symmetric equilibrium with the same number of products in each group \((n_i = n)\), we have that all \(p_{ij} = p \ \forall i, j\), \(\Delta = \frac{1+\lambda(n-1)}{n}\) and \(\Phi = \frac{1+\lambda(n-1)+\mu n(M-1)}{Mn}\).

In the limit, if \(\lambda = 1\) \((\Delta = 1)\) and \(\mu = 1\) \((\Phi = 1)\)\(^{12}\) each firm expects that its own price change will be followed exactly by rivals; this means that in the market a perfect coordination of decisions is achieved (full intra- and inter-coordination).

When \(\lambda = 1\) \((\Delta = 1)\) and \(-\frac{1}{M-1} < \mu < 1\) \((0 < \Phi < 1)\) we have some kind of inter-group oligopolistic competition, while firms of the same group behave cooperatively in order to maximize the profits of the group (intra-group coordination).

For \(\lambda = -\frac{1}{n-1}\) \((\Delta = 0)\)\(^{13}\) each firm believes that its price variation will be offset by a price reduction of all its rivals in the same group, aimed at maintaining the group price index constant. But if a firm may not affect the group price-index, it is reasonable to assume that it cannot influence the industry price-index (i.e. if \(\Delta = 0\) then \(\Phi = 0\)), thus we have both intra-group and inter-group monopolistic competition.

For all \(\lambda\) such that \(-\frac{1}{n-1} < \lambda < 1\) \((0 < \Delta < 1)\), each firm enjoys some kind of market power within its own group; therefore if at the inter-group level \(\mu = -\frac{1+\lambda(n-1)}{n(M-1)}\) \(^{14}\) (so that \(\Phi = 0\)), we obtain intra-group oligopolistic competition and inter-group monopolistic competition.

Obviously, the standard Bertrand competition arises for \(\lambda = 0\) \((0 < (\Delta = \frac{1}{n}) < 1)\) and \(\mu = 0\) \((0 < (\Phi = \frac{1}{Mn}) < 1)\). Lastly, all other values of \(\lambda\) and \(\mu\), for which still \(0 < \Delta < 1\) and \(0 < \Phi < 1\)\(^{15}\), allow for 'unusual' intra- and inter-group oligopolistic competition.

Summing up, each firm may perceive the own price change as relevant or not with respect to the group price-index and it may preserve some degree of market power at the group perspective. However, when firms of different groups react in order to maintain the industry sales unchanged, an individual price change does not affect the industry price-index \(q\) (i.e. \(\Phi = 0\)) and, as a consequence, we have an \textit{inter-group monopolistic competition}\(^{16}\).

\(^{12}\) \(\lambda = 1\) and \(\mu = 1\) are the upper limits of both conjectures.

\(^{13}\) The lower limit for \(\lambda\).

\(^{14}\) The lower limit for \(\mu\).

\(^{15}\) That is \(-\frac{1}{n-1} < \lambda < 1\) and \(-\frac{1+\lambda(n-1)}{n(M-1)} < \mu < \frac{M}{M-1} - \frac{1+\lambda(n-1)}{n(M-1)}\).

\(^{16}\) In this case, elasticity in (19) yields: 
\[
\eta_{p_{ij}, p_{ij}}^f = \delta + (\sigma - \delta) \frac{p_{ij}}{q_i} \left[ p_{ij}^{-\delta} + \sum_{k \neq j} p_{ik}^{-\delta} \lambda_{ik} \right]
\]
In the following table we confine our attention to situations of inter-group monopolistic competition, allowing for different values of $\lambda$, i.e. for different intra-group competitive environments. Prices, quantities and profits are then evaluated at the symmetric equilibrium.

<table>
<thead>
<tr>
<th>$\lambda = -\frac{1}{n-1}$</th>
<th>$-\frac{1}{n-1} &lt; \lambda &lt; 1$</th>
<th>$\lambda = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\pi^{Mc} = \frac{Y}{Mn}\delta$</td>
<td>$\pi^{Oc} = \frac{Y}{M(1-\lambda)[\sigma+\delta(n-1)]+n\lambda\sigma}$</td>
<td>$\pi^{C} = \frac{Y}{Mn\sigma}$</td>
</tr>
<tr>
<td>$p^{Mc} = \frac{\delta}{\delta-1}$</td>
<td>$p^{Oc} = \frac{(1-\lambda)[\sigma+\delta(n-1)]+n\lambda\sigma}{(1-\lambda)[\sigma+\delta(n-1)]+n\lambda\sigma-1}$</td>
<td>$p^{C} = \frac{\sigma}{\sigma-1}$</td>
</tr>
<tr>
<td>$x^{Mc} = \frac{Y}{Mn}\delta-1$</td>
<td>$x^{Oc} = \frac{Y}{Mn}(1-\lambda)[\sigma+\delta(n-1)]+n(\lambda\sigma-1)+n\lambda\sigma}$</td>
<td>$x^{C} = \frac{Y}{Mn}\sigma-1$</td>
</tr>
</tbody>
</table>

where superscript denotes respectively Monopolistic competition, Oligopolistic competition and Coordinated behavior at the intra-group level. As expected, under all possible conjectures, profits are decreasing in the number of varieties and in the elasticities of substitutions ($\delta$, $\sigma$, or both).

However, if we allow for inter-group oligopolistic competition, i.e. if we allow for strategic interaction both at the intra-group and at the inter-group level, we have to consider the price-index effect on both price indices $q_i$ and $q$. In this case, for any $-\frac{1+(n-1)\lambda}{n(M-1)} < \mu \leq 1$, firms’ market power arises both within groups and in the entire market.\(^{17}\)

In this case of \textit{inter-group oligopolistic competition} (i.e. for $0 < \Phi \leq 1$), if $0 < \Delta \leq 1$ we have the following expressions for the symmetric equilibrium prices, quantities and profits in terms of $\lambda$ and $\mu$:

| $p$ | $\frac{M((1-\lambda)[\sigma+\delta(n-1)]+n\lambda\sigma)+n\lambda\sigma(1-\sigma)+\mu(M-1)-1}{M((1-\lambda)[\sigma+\delta(n-1)]+n\lambda\sigma(1-\sigma)+\mu(M-1)+1)}$ |
| $x$ | $\frac{M((1-\lambda)[\sigma+\delta(n-1)]+n(\lambda\sigma-1)+n\lambda\sigma(1-\sigma)+\mu(M-1)-1)}{M((1-\lambda)[\sigma+\delta(n-1)]+n(\lambda\sigma-1)+n\lambda\sigma(1-\sigma)+\mu(M-1)+1)}$ |
| $\pi$ | $\frac{Y}{M((1-\lambda)[\sigma+\delta(n-1)]+n\lambda\sigma(1-\sigma)+\mu(M-1)+1)}$ |

Under Bertrand behavior ($\mu = 0$, $\lambda = 0$) we have $\Delta = \frac{1}{n}$ and $\Phi = \frac{1}{Mn}$, while full coordination arises for $\lambda = 1$ and $\mu = 1$. In this latter case firms choices have a full effect on the market ($\Delta = 1$ and $\Phi = 1$) and the perfect coordination between all firms allows them to extract the total consumers’ surplus.

\(^{17}\)In this case, elasticity in (19) yields: $\eta_{x_{ij}, p_{ij}} = \delta+$

\[
+ (\sigma - \delta) \frac{p_{ij}}{q_i} \left[ p_{ij}^{-\delta} + \sum_{k \neq j} p_{ik}^{-\delta} \lambda_{ik} \right] +
\]

\[
+ (1 - \sigma) \frac{p_{ij}}{q_i} \left[ q_i^{-\delta - \sum_{k \neq j} p_{ik}^{-\delta} \lambda_{ik}} \right] +\left( \sum_{h \neq i} q_h^{-\delta - \sum_{k \neq i} p_{hk}^{-\delta} \mu_{hk}} \right)
\]

13
The following table summarizes the equilibrium profits for specific values of $\mu$ and $\lambda$:

<table>
<thead>
<tr>
<th>$\mu$</th>
<th>$\lambda$</th>
<th>$\pi^{Mc}$</th>
<th>$\pi^{Bc}$</th>
<th>$\pi^{PC}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$-\frac{1+\lambda(n-1)}{n(M-1)}$, $-\frac{1}{n-1} &lt; \lambda &lt; 1$</td>
<td>$0$, $\lambda = 0$</td>
<td>$\frac{Y}{M \delta + (\sigma - \delta)(1+\lambda(n-1))}$</td>
<td>$\frac{Y}{(1-\sigma)+M[\sigma+\delta(n-1)]}$</td>
<td>$\frac{Y}{Mn}$</td>
</tr>
</tbody>
</table>

where again superscripts denote respectively Monopolistic competition, Bertrand Competition and Perfect Coordinated behavior.

### 3 The multiproduct firms

The literature on multiproduct firms has mainly focused on the incentive to create brands portfolios as opposed to mono-product strategies. Indeed, the production of an entire product line may be a powerful tool to deter entry and to escape from a too much intense competition (Schmalensee, 1978).

However, the literature has paid a relatively little attention to the optimal price policies of large companies selling an entire product line; moreover, it has not provided a full motivation of two alternative organizational structures: there are companies which directly control prices from the above and companies which delegate the price decisions to independent PMs. Many papers on mergers have shown that it is profitable to allow for independent divisions when the capacity constraints play a fundamental role, such as in the cigarette market and in the automobile industry, while it is better to control each decision centrally under price competition - examples being the fast-food and mineral water industries. In the fast-food industry, all customers of the Mac-Donald and Burger King groups know that prices are defined centrally and that no autonomy is left to the single division (store). On the contrary, Williamson (1975) and Milgrom-Roberts (1992) have stressed the importance of giving independence to product divisions of the same company. There is significant evidence that Philip Morris tobacco, General Motors, Fiat, and Ford encourage competition across their own divisions, and that the same applies to Procter-&-Gamble and Mitsubishi (Nikkei Weekly 1994), to the firms of the cosmetics sector (Low 1994) and to those offering high-tech services (Forbes 1992).

Whether and when a system of PMs decentralized decisions is better than a mechanism with a centralized GD is not a trivial question. The analytical framework developed in this paper may provide an adequate tool to deal with this problem on the basis of a key distinction: the profitability of one or the other organizational structure may depend on the characteristics of the multiproduct firm’s product line: market segmentation or market interlacing.
Which of them occurs depend on the relationship between the intra-company ($\delta$) and inter-company elasticity of substitution ($\sigma$). For $\sigma < \delta$, each product line consists of a set of close substitutes (i.e. market segmentation), while for $\sigma > \delta$, each consists of a set of distant substitutes (i.e. market interlacing).

Let us consider again the model developed in sections 1 and 2 and let us now reinterpret the idea of a 'group' of products as the set of brands produced and sold by a multiproduct firm. The industry is then composed by $M$ large multiproduct firms, whereby each company $i$ sells $n_i$ differentiated products (brands). Each company consists in $n_i$ product divisions. Therefore, the $j$-th division of the $i$-th firms produces the $ij$-th variety ($\forall i = 1, ..., M$ and $\forall j = 1, ..., n_i$). As a result, the total profits of the $i$-th multiproduct firm is given by the sum of the profits of its $n_i$ divisions:

$$\pi_i = \sum_{j=1}^{n_i} \pi_{ij}$$

(22)

where $\pi_{ij}$ are the same of (17).

Each company sets the prices of all its products in order to maximize (22). A PM is assigned to run each division. The PMs of the same company may set prices independently or cooperatively and they have to consider the effect of each price change both on $q_i$ (the price index of the company) and on $q$ (the industry price index). The first order condition for the $i$-th multiproduct firm to maximize (22) is given by:

$$\frac{\partial \pi_i}{\partial p_{ij}} = 0 \quad \forall j = 1..n_i$$

$$\left[ \sum_{k=1}^{n_i} (p_{ik} - 1) \frac{\partial x_{ik}}{\partial p_{ij}} \right] + x_{ij} = 0$$

$$\left[ \sum_{k \neq j}^{n_i} \frac{(p_{ik} - 1) x_{ik}}{p_{ij}} \eta_{x_{ik},p_{ij}}^f \right] + \frac{(p_{ij} - 1) x_{ij}}{p_{ij}} \eta_{x_{ij},p_{ij}}^f + x_{ij} = 0$$

(23)

where $\eta_{x_{ij},p_{ij}}^f$ and $\eta_{x_{ik},p_{ij}}^f$ denote respectively the demand own price elasticity and the cross (intra-company) price elasticity as perceived by the PMs. If the PMs of the same company share the same conjectures, (i.e. $\lambda_{ik} = \frac{\partial p_{ik}}{\partial p_{ij}} = \lambda_i \quad \forall j = 1..n_i - \forall i = 1..M$), we have:

$$\eta_{x_{ij},p_{ij}}^f = -\frac{\partial x_{ij}}{\partial p_{ij}} \frac{p_{ij}}{x_{ij}} = \delta + (\sigma - \delta) \Delta_{ij} + (1 - \sigma) \Phi_i$$

(24)

$$\eta_{x_{ik},p_{ij}}^f = \frac{\partial x_{ik}}{\partial p_{ij}} \frac{p_{ij}}{x_{ik}} = -\sigma \Delta_{ij} + (1 - \sigma) \Phi_i$$

(25)
where $\Delta_{ij} = \frac{p_{ij}}{q_i} \left[ p_{ij}^{-\delta} + \sum_{k \neq j} p_{ik}^{-\delta} \lambda_i \right]$ and $\Phi_i = \frac{p_{ij}}{q_i} \left[ q_i^{-\sigma} \left( p_{ij}^{-\delta} + \sum_{k \neq j} p_{ik}^{-\delta} \lambda_i \right) \right]^{18}$. Using (10), the first order condition yields:

$$- \sum_{k=1}^{n_i} \left( p_{ik} - 1 \right) x_{ik} \left( (\sigma - \delta) \Delta_{ij} + (1 - \sigma) \Delta_{ij} \left( \frac{q_i}{q} \right)^{1-\sigma} \right) = 0$$

$$= \left( \frac{p_{ij} - 1}{p_{ij}} \right)^{1-\delta} - 1 \left( \frac{p_{ij}}{p_{ij}} \right)^{1-\delta} \left( \frac{q_i}{q} \right)^{1-\sigma}$$

which can be rewritten as:

$$- \sum_{k=1}^{n_i} \left( p_{ik} - 1 \right) x_{ik} \left( (\sigma - \delta) + (1 - \sigma) \left( \frac{q_i}{q} \right)^{1-\sigma} \right) = 0$$

$$= \left[ \frac{p_{ij} - 1}{p_{ij}} \right]^{1-\delta} - 1 \left( \frac{p_{ij}}{p_{ij}} \right)^{1-\delta} \left( \frac{q_i}{q} \right)^{1-\sigma}$$

(26)

the left-hand side of (26) turns out to be the same for all $j = 1..n_i$. Therefore, all prices set within the company are equal, $p_{ij} = p_i$, for all $j = 1..n_i$. Hence, $(p_{ij}/q_i)^{1-\delta} = 1/n_i$ and $\Delta_{ij} = \Delta_i$: each variety is produced in the same amount, $x_{ij} = x_i$ for all $j = 1..n_i$. Now for a given $n_i$, $\Delta_i = \frac{1+\lambda_i(n_i-1)}{n_i}$ shows the degree of independence (or coordination) between the PMs of the same company $i$.

For $\Delta_i = 1$, the PMs’ decisions depend on the GD’s instructions (centralized decisions); while the lower is $\Delta_i$ within the interval $0 < \Delta_i < 1$, the lower is the coordination among divisions; at the extreme, for $\Delta_i = 0$ we have independent PMs. Moreover, since we have assumed Bertrand competition between multiproduct firms, the industry price index effect is given by $\Phi_i = \Delta_i \left( \frac{q}{q} \right)^{1-\sigma}$. Recalling (26), for each variety we get:

$$\frac{p_i - 1}{p_i} = \frac{1}{\delta + (\sigma - \delta) \Delta_i + (1 - \sigma) \Delta_i \left( \frac{q_i}{q} \right)^{1-\sigma}}$$

(27)

Because all firms are identical (except for product differentiation) we confine our attention to the symmetric equilibrium with the same number of products per firm ($n_i = n \forall i = 1..M$); if all companies show the same internal organizational structure ($\Delta_i = \Delta \forall i = 1..M$), equation (27) yields:

$$p_i^* = \frac{M - M \Delta + M (\sigma - \delta) \Delta + (1 - \sigma) \Delta}{M (\delta - 1) + M (\sigma - \delta) \Delta + (1 - \sigma) \Delta}$$

(28)

18 In order to obtain meaningful analytical results we have confined our attention to Bertrand conjectures between PMs of different multiproduct firms (i.e. $\mu_{hk} = \frac{\partial p_{hk}}{\partial p_{ij}} = 0 \forall k, j$ and $\forall h \neq i$)
Hence for each variety, all firms produce:

\[ x_i^* = x^* = \frac{Y}{M n} \left[ \frac{M(\delta - 1) + M(\sigma - \delta)\Delta + (1 - \sigma)\Delta}{M\delta + M(\sigma - \delta)\Delta + (1 - \sigma)\Delta} \right] \] (29)

As Katz (1984) noted, by lowering the price of one of its brands, a multi-product firm can increase its sales. However, these additional sales come from three sources. On the one hand, the lower price increases the total amount purchased by consumers. On the other hand, consumers will substitute this cheaper brand for varieties produced by rivals (inter-company substitution effect). Finally, sales increase at the expenses of other varieties of the same firm (intra-company substitution effect). Therefore, a multiproduct firm must consider the effects of a price change on its whole product line, taking into account how the others PMs react to such price variation. This is perfectly consistent with the findings by Yang and Heijdra (1993), who noted that the demand elasticity (of demand curves derived from CES utility functional forms) appears like a weighted average of unity and both the intra- and the inter-sector elasticities of substitution. In this perspective, the demand elasticity given in (24) can be rewritten as:

\[ \eta^f = \delta + \Delta\sigma - \delta\Delta - \Delta\frac{\sigma - 1}{M} \] (30)

where the first element, \( \delta \), is the intra-company elasticity of substitution which corresponds to the Dixit-Stiglitz (1977) approximation of demand elasticity under the standard monopolistic competition (i.e. \( \Delta = 0 \)). For \( \Delta \neq 0 \), demand elasticity incorporates the inter-company substitution effect (i.e. \( \Delta\sigma \)), the intra-company substitution effect (i.e. \( -\Delta\delta \)) and the industry price index effect (i.e. \( -\Delta\frac{\sigma - 1}{M} \)).

It is important to notice that multiproduct firms may benefit from the possibility to make use of strategies, which are available to multi-divisional firms only. Each company can define the optimal corporate structure. It may choose to control and to coordinate all decisions centrally through a GD; or it may allow each PM to be independent. An increase in coordination (higher \( \Delta \)) affects the Lerner index of monopoly power along three lines: first, it strengthen the (unprofitable) impact of the inter-company substitution effect (\( \Delta\sigma \)); second, it is also an income-feedback effect which affects the equilibrium. As a matter of fact, the consumer’s income, \( I \), is the sum of the value of the endowment (labor) and distributed profits: \( I = w + \sum \pi_i \). Therefore, one could take into account not only the direct effect of a price change, but also an additional indirect effect. The latter, is the so-called **Ford-effect**: the effect upon demand of a change in prices through the income. Nevertheless assuming free entry and normalizing the wage to one, profits are zero and the income is constant and equal to the endowment (\( I = 1 \)). However, D’Apremont et al. (1996) rejected this approximation.
it increases the (profitable) impact of the intra-company substitution effect \( (\Delta \delta) \); third, it reinforces the (profitable) impact of the industry price index effect \( (\Delta \frac{\sigma-1}{M}) \). The two latter effects lead to a gradual decrease in demand elasticity, and hence to a gradual increase of the market power. However, the profitable effects are reduced by an higher impact of the inter-company substitution effect, and though one would expect that firms gain from whatever form of coordination rather than competition, in multiproduct firms analysis the question of the profitability of coordination between PMs of the same company is neither granted or trivial.

For a given \( \Delta \), by (22) the company total profit are:

\[
\pi^* = \frac{Y}{M\delta + M(\sigma - \delta)\Delta + (1 - \sigma)\Delta}
\]  

(31)

We can therefore study the profitability of coordination, evaluating the variation of profit (31) with respect to \( \Delta \):

\[
\frac{\partial \pi^*}{\partial \Delta} = Y \frac{\delta - \sigma + \frac{\sigma - 1}{M}}{[M\delta + M(\sigma - \delta)\Delta + (1 - \sigma)\Delta]^2}
\]  

(32)

The sign of (32) mainly depends on the relationship between the two elasticities of substitution. Borrowing the Brander-Eaton (1984) criterion, we have already defined the possible industry structures on the basis of the relationship between the cross price elasticities of demand: market segmentation \( (\delta > \sigma) \) arises when multiproduct firms produce closer substitutes, while market interlacing \( (\delta < \sigma) \) refers to multiproduct firms producing distant substitutes.

Simple inspection of (32) shows that an increase in coordination between PMs of the same company is undoubtedly profitable in presence of segmentation, while the sign of (32) may be negative under market interlacing\(^{20}\).

Under market segmentation, coordination is always profitable because the (profitable) increase of the intra-company substitution effect \( (\delta) \) always dominates the (unprofitable) increase of the inter-company substitution effect \( (\sigma) \). On the contrary, under market interlacing, the latter effect dominates the former and allowing for independent PMs may indeed be profitable. Coordination is still to be preferred if the increase in the net (unprofitable) effect \(( (\delta - \sigma)) \) is lower than the (profitable) increase in the industry price index effect \(( \frac{\sigma-1}{M}) \). If the opposite holds, companies who centralize price decisions get lower profits.

Therefore the choice of independent PMs (as in Raubitschek, 1987) is surely the best choice under market interlacing when the standard monopolistic competition arises. In this case the price index effect is obviously negligible and the

\(^{20}\) In both cases the sign of (32) is independent of \( n \).
net impact of the increase in coordination is negative: it reduces market power, prices and profits. In this case, by allowing for independent PMs ($\Delta = 0$), a multiproduct firm offsets the net unprofitable effect. Equation (32), however, shows that a decentralized organizational structure may be optimal also in case of oligopoly\textsuperscript{21}, depending on size of the industry price index effect.

### 4 Conclusion

This paper analyzes the price-setting behavior of multiproduct firms in a differentiated product market. The structure considered is one where large companies offer either a set of close substitutes (market segmentation) or a set of distant substitutes (market interlacing).

The modelling strategy of the paper is to allow for two different elasticities of substitution: while $\delta$ represents the intra-company elasticity of substitution, $\sigma$ is the inter-company elasticity of substitution. The key feature of the model is the possibility for multiproduct companies to choose their optimal internal organizational structure, according to the relative size of these two parameters.

Each company, consisting of $n$ divisions, may either set prices centrally (as in the traditional approach), or alternatively, it may assign an independent product manager to run each division. In other words, product managers of the same company may behave either independently or cooperatively.

While the model does not consider either the proliferation or the product-line selection decisions, it deals with multiproduct firms’ price decisions under oligopolistic competition making use of conjectural variations. Its main purpose has been to provide a microfounded answer about the question of whether and when a system of product managers decentralized decisions is better than a mechanism with a centralized general direction.

The paper has shown that coordination is always profitable under market segmentation; while under market interlacing, the strategy of relying on independent product managers is profitable when the standard monopolistic competition arises; it may also be profitable with oligopolistic (Bertrand) competition under some (not very restrictive) assumptions.

\textsuperscript{21} With a not negligible price index effect.
References


