

- Dixit, A., and J. Stiglitz. (1977). “Monopolistic Competition and Optimum Product Diversity”, *American Economic Review* 67, 297–308.
- Eswaran, M., and N. Gallini. (1996). “Patent policy and the direction of technological change”, *RAND Journal of Economics* 27, No. 4, 722–746.
- Grossman, G., and E. Helpman. (1991). *Innovation and Growth in the Global Economy*, Cambridge, MA: MIT Press.
- Jones, C. I. (1999). “Growth: With or without Scale Effects?”, *American Economic Review* 89, No. 2, 139–144.
- Melitz, M. (2003). “The impact of trade on intra-industry reallocations and aggregate industry productivity”, *Econometrica* 71, No. 6, 1695–1725.
- Romer, P. (1990). “Endogenous Technological Change”, *Journal of Political Economy* 98, S71–S102.
- Schlicht, E. (1985). *Isolation and aggregation in economics*, Springer-Verlag, Berlin Heidelberg.
- Schlicht, E. (1997). “The Moving Equilibrium Theorem again”, *Economic Modelling* 14, 271–278.
- Schumpeter, J. (1934). *The Theory of Economic Development*, Harvard University Press, Cambridge Mass.

Appendix A

As in the text, we define h in such a way that $m = 1, 2, \dots, (h = i)$. Once there is an improvement along the learning curve described by (19), the series continues in the following way: $m = 1, 2, \dots, h, (h + 1 = i)$. In this appendix we show when process innovations which increase the value of h as defined above, end up with a smaller (higher) value of b_i . In other words, we show when b_h is higher (lower) than b_{h+1} .

We know from the definition (26) that

$$b_h \equiv \frac{n_h \gamma_h^{1-\sigma}}{\sum_{j=1}^h n_j \gamma_j^{1-\sigma}} \quad \text{and} \quad b_{h+1} \equiv \frac{n_{h+1} \gamma_{h+1}^{1-\sigma}}{\sum_{j=1}^{h+1} n_j \gamma_j^{1-\sigma}}$$

Hence, we derive that $b_h > b_{h+1}$ when

$$n_h \gamma_h^{1-\sigma} \sum_{j=1}^{h+1} n_j \gamma_j^{1-\sigma} > n_{h+1} \gamma_{h+1}^{1-\sigma} \sum_{j=1}^h n_j \gamma_j^{1-\sigma}$$

or, equivalently, when

$$n_h \gamma_h^{1-\sigma} \sum_{j=1}^h n_j \gamma_j^{1-\sigma} + n_h \gamma_h^{1-\sigma} n_{h+1} \gamma_{h+1}^{1-\sigma} - n_{h+1} \gamma_{h+1}^{1-\sigma} \sum_{j=1}^h n_j \gamma_j^{1-\sigma} > 0 \quad (40)$$

Expression (40) is true when

$$\frac{\sum_{j=1}^h n_j \gamma_j^{1-\sigma}}{h-1} > \frac{n_{h+1} \gamma_{h+1}^{1-\sigma}}{n_h \gamma_h^{1-\sigma}}$$

We substitute n_{h+1} from (21) and we obtain

$$l \equiv \frac{\sum_{j=1}^h n_j \gamma_j^{1-\sigma}}{h-1} > \frac{L_R \gamma_{h+1}^{1-\sigma}}{a \gamma_h^{1-\sigma}} = \frac{L_R}{a} \left(\frac{\gamma_h}{\gamma_{h+1}} \right)^{\sigma-1}$$

where the left term in the inequality, l , is always larger than 1. Therefore, given that $\gamma_{h+1} < \gamma_h$, we may at least state that $b_h > b_{h+1}$ is true, when $\frac{L_R}{a} \left(\frac{\gamma_h}{\gamma_{h+1}} \right)^{\sigma-1} < l$. That is when

$$1 < \left(\frac{\gamma_h}{\gamma_{h+1}} \right)^{\sigma-1} < \frac{a}{L_R} l \quad (41)$$

Expression (41) says that when the process innovation produces a reduction in γ which is not relatively high, then b_i decreases.

Appendix B

Following Grossman and Helpman (1991, p. 63) we define the index of the manufactured output

$$D \equiv \left(\sum_{m=1}^i n_m x_m^\alpha \right)^{\frac{1}{\alpha}}$$

where $\alpha = \frac{\sigma-1}{\sigma}$, while the ideal price index of final goods is p_D .

The gross domestic product (GDP), G , is defined as the sum of the value added in manufacturing and in the R&D sector

$$G \equiv p_D D + v_i \dot{n}_i$$