introduced the assumption in (19) that increases in L may produce process innovations. These continuous subsequent process innovations due to increases in L may contribute to continuously lowering the value of b_i and keeping g from increasing.

In particular, this could not happen as long as subsequent patent innovations are related to varieties characterized by the same value of γ. In fact, partially differentiating (36) with respect to L_R, L and b_i, after few steps we obtain

$$\frac{dL_R}{L_R} = \frac{(1-\alpha)\, b_i}{(Lb_i\,(1-\alpha) - a\rho\alpha)} \left(L\frac{dL}{L} + \frac{\alpha\,(L+a\rho)}{((1-\alpha)\, b_i + \alpha)} \frac{db_i}{b_i} \right) \tag{38}$$

From (38) we know that L_R, and consequently g, would be constant only if

$$\frac{db_i}{b_i} = -\frac{L\,((1-\alpha)\, b_i + \alpha)}{\alpha\,(L+a\rho)} \frac{dL}{L} \equiv b^* < 0 \tag{39}$$

This is never the case when varieties of type i remain along the technological frontier given that b_i would continuously increase over time and, therefore, db_i/b_i can only be positive.

However, when L increases, continuous process innovations could continuously lower b_i. If these two effects on b_i balance each other, b_i will be constant implying that L_R, in turn, is constant with no change in the growth rate of the number of varieties. In appendix B we show that this would imply a constant growth of the real gross domestic product (GDP).

Therefore, we may conclude that when process innovations are associated to product innovations, we can obtain equilibrium paths characterized by a stable distribution of workers between the two sectors, which corresponds to a fixed growth rate, provided that b_i continuously decreases over time due to subsequent process innovations.

6 Conclusion

Scholars in the field of international economics and economic growth have devoted great attention to the subject of heterogeneity of firms in the last few years. Productivity differences across firms are, for instance, analyzed in a general equilibrium framework by Melitz (2003) which analyzes

the effects of international trade on intra-industry reallocation of firms and on aggregate industry productivity.

Our paper adopts a growth approach to explain how heterogeneous firms producing in a particular period of time are the result of subsequent waves of process innovations which allow more recent firms to produce using more productive techniques. The contemporaneous production of firms characterized by different productivity levels results in a variety of prices set by firms which reflect productivity differences. The latter are also responsible for patents' price differences, given that patent prices of more profitable varieties are higher. Moreover, demand and market shares of older less efficient firms decrease over time as long as new patents, which allow the production of new goods along the technological frontier, are produced in the innovative sector.

In this work we assume that old firms are unable to implement the more productive production processes due to high switching or implementing costs required to adopt the new production processes. However, demand for these firms is still positive given the Dixit and Stiglitz approach, which postulates love for variety in consumption. The assumption of goods which are imperfect substitutes together with that of productivity heterogeneity results in different equilibrium prices for different varieties. Moreover, our results suggest that policy intervention may have a role given that specific policies could be set in order to reduce switching costs when they are particularly high in order to implement a redistribution of the production activity toward the innovative sector with an associated higher rate of innovation and of growth of the overall economy.

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