The remaining part of the work is organized as follows: Section 2 describes consumers' and firms' behavior, while the innovative sector is more deeply analyzed in Section 3; Section 4 draws the characteristics of the equilibrium outcomes which are "moving" with particular changes in the distribution of workers; Section 5 presents some comments on the results, while Section 6 concludes.

2 Consumers' and firms' behavior

We consider a closed economy in which consumers love variety and their preferences are described by the following intertemporal utility function

$$U = \int_0^\infty e^{-\rho t} \log \left(\sum_{c=1}^{n(t)} D_c(t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} dt \tag{1}$$

where D_c is the consumption of variety c, ρ is the rate of time preference and $\sigma > 1$ is both the elasticity of substitution between any pair of varieties and the own-price elasticity of demand for any variety. The elasticity of intertemporal substitution in (1) is constant and equal to 1, while n is the total number of produced varieties in t.

Total consumers' expenditure E is defined as

$$E = \sum_{c=1}^{n} p_c D_c$$

where p_c is the price of variety c.

Consumers' demand x_c for any variety c is

$$x_c = \frac{p_c^{-\sigma}}{\sum_{c=1}^{n} p_c^{1-\sigma}} E \tag{2}$$

All varieties are produced by firms which need to buy a patent from the R&D sector to start their activity and which employ γ workers to produce a unit of their output. Given the assumptions of consumers' love for variety and the fact that there are no scope economies, all firms produce different varieties.

Moreover, firms are not all supposed to use the same production process, given that γ is not necessarily equal for all firms. More precisely, we assume that there are i different groups of firms, each of which is characterized by a particular value of γ, that is γ_m, which expresses the specific units of labor required to produce one unit of the output by the n_m firms of the group of type m, with $m = 1, 2, ..., i$.

We assume that the higher the value of m, the lower the value of γ_m is and, therefore, the higher the productivity of workers employed by firms of type m is. Each period is characterized by a particular value of i, which increases when a new group of goods is made available through R&D efforts in the innovative sector. We assume that any time the value of i increases, a new, more productive process is made available and new firms use the more productive production process. More details on how new values of γ are made available will be given in next section which describes the innovative sector. For the moment, we only anticipate that by producing new patents, researchers in the R&D sector exploit the knowledge accumulated by past innovative activities and that the development of a sufficiently large number of varieties allows them to introduce new patents characterized by higher productivity levels (that is, by lower γ values).

Hence, if there are only i groups of variety, each of numerousness n_m, we derive from (2) the demand x_m for any firms characterized by γ_m

$$x_m = \frac{p_m^{-\sigma}}{\sum_{j=1}^{i} \sum_{j=1}^{n_j} p_j^{1-\sigma}} E \tag{3}$$

with $m = 1, 2, ..., i$.

Given that all varieties of type m are symmetric, total expenditure in varieties of the same type is

$$n_m p_m x_m = \frac{n_m p_m^{1-\sigma}}{\sum_{j=1}^{i} n_j p_j^{1-\sigma}} E \tag{4}$$

with $m = 1, 2, ..., i$.

Considering the intertemporal component of consumers' allocation problem, following Grossman and Helpman (1991) we define the index of the manufactured output

$$D \equiv \left(\sum_{m=1}^{i} n_m x_m^{\alpha} \right)^{\frac{1}{\alpha}}$$

where $\alpha = \frac{\sigma-1}{\sigma}$, and the ideal price index of final goods

$$p_D \equiv \left(\sum_{m=1}^{i} n_m p_m^{1-\sigma} \right)^{\frac{1}{1-\sigma}}$$

Given that $D = E/p_D$, the intertemporal utility function (1) becomes

$$U = \int_0^{\infty} e^{-\rho t} \left(\log E - \log p_D \right) dt$$

As Grossman and Helpman (1991, pag. 48) show, the maximization of the previous expression "subject to an intertemporal budget constraint requires that spending evolve according to"

$$\frac{\dot{E}}{E} = r - \rho$$

Then we normalize prices in such a way that total expenditure E is equal to 1.[3] This implies that we have

$$r = \rho \tag{5}$$

Consumption varieties are produced by monopolistically competitive firms, which sustain a fixed cost in order to acquire a patent produced in the innovative sector and a variable cost of production. Since each firm maximizes profits, we know that price p_m is

$$p_m = \frac{1}{\alpha} \gamma_m w \tag{6}$$

where $\alpha = (\sigma - 1)/\sigma$ and w is the nominal wage.

We notice that the ratio between prices of any pair of varieties is proportional to the ratio of labor required to obtain one unit of each type of good with

$$\frac{p_m}{p_j} = \frac{\gamma_m}{\gamma_j} \tag{7}$$

[3] Cfr. Grossman and Helpman (1991)

where $m, j = 1, 2, ..., i$.

Operating profits realized by a single firm of type m are

$$\pi_m = \left(\frac{1-\alpha}{\alpha}\right) w\gamma_m x_m$$

In equilibrium, when supply, x_m, is equal to demand (3), substituting prices from (6), we obtain that profits π_m are

$$\pi_m = (1-\alpha)\frac{\gamma_m^{1-\sigma}}{\sum_{j=1}^{i} n_j \gamma_j^{1-\sigma}} < 1 \tag{8}$$

From the previous expression, we know that profits, π_m, decrease when the total number of firms increases, while they increase as productivity increases. In particular, for the more productive varieties, characterized by $m = i$, we know not only that profits decrease as n_i increases, with $\partial\pi_i/\partial n_i < 0$, but also that they increase as γ decreases, given that $\partial\pi_i/\partial\gamma_i < 0$.

Total labor demand by firms of type m, L_m, is given by

$$L_m = n_m \gamma_m x_m \tag{9}$$

Moreover, considering (4) together with (6), we derive that L_m is equal to

$$L_m = \frac{\alpha n_m p_m^{1-\sigma}}{w\sum_{j=1}^{i} n_j p_j^{1-\sigma}} \tag{10}$$

Given that the total number of units of workers in the economy is L, the labor market clearing condition requires that

$$L = L_R + \sum_{m=1}^{i} L_m \tag{11}$$

where L_R is the total amount of labor employed in the innovative sector and will be described in the following section.[4] Finally, we define L_C as the total amount of labor employed in the production of consumption goods which corresponds to

$$L_C = \sum_{m=1}^{i} L_m$$

[4] We simply assume that the switching technology cost for existing firms consists in a different, too high fixed cost which firms that are already in the market have to sustain to be able to use the process innovation generated within the R&D sector. This enables us to avoid considering the case of old firms switching technology.