1 Introduction

The role played by innovation in determining economic growth is commonly recognized. In recent years an ever increasing amount of literature has been devoted to the subject and many authors, starting from the contributions by Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992), have written about its relevance. Many of these works underlined the importance of Arrow's (1962) claim that the development of new ideas spurs growth and they brought innovation to be one of the most relevant topics in contemporaneous economic literature giving start to the so called Endogenous Growth Theory or New Growth Theory.

The results produced by this stream of literature are many and, as is well known, very articulated.[1] Here, we recall the role played by the intentional research activity carried out in the innovative R&D sector described in the works by Grossman and Helpman (1991) and Aghion and Howitt (1992). While, on the one hand, Grossman and Helpman draw particular attention to the fact that research efforts may result in an extension of the number of available consumption varieties; on the other hand, Aghion and Howitt underline that these efforts may, among other things, improve the quality of the varieties already available.

The view that innovation is one of the engines of economic growth in the forms suggested by the above mentioned authors is widely recognized. However, we think that some new insights might be gained by means of a joint analysis on the effects of different types of innovation, which, following Schumpeter (1934), we identify as *product* and *process innovation.* By considering both types of innovation in a general equilibrium framework, we should be able to give a more articulated description of the effects of the innovative activity on the economy. Hence, in this work we introduce *process innovation* in the general equilibrium framework proposed by Grossman and Helpman (1991, ch. 3) in order to study the complementary role this kind of innovation plays with product innovation, which increases the number of available consumption (or intermediate)

[1] These results are a too big a subject to be summarized in the present paper, and we refer the interested reader to the exhaustive descriptions by Grossman and Helpman (1991) and Aghion and Howitt (1992).

varieties, and which is the type of innovation considered by Grossman and Helpman (1991), while process innovation increase productivity levels and, thus, decrease variable production costs of new firms entering the final good sector.

The complementarity between process and product innovation is studied by Athey and Schmutzler (1995) in a different framework. Their aim is to underline the interactions between the short-run innovative activity of a firm and its organizational structure, which defines its long-run characteristics such as research capabilities and flexibility, which determines "how costly it will be to make changes to product designs and production processes, once the opportunities for innovation have been identified." (Athey and Schmutzler, 1995, p. 558). Moreover, Eswaran and Gallini (1996, p. 722) "examine the interactions between firms' product and process innovation decisions, and the role patent policy can play in directing technological change toward a socially efficient mix of innovations." Moreover, Eswaran and Gallini (1996, p. 723) show that there is a trade-off "between an entrant's incentives to engage in product and process innovation. The more differentiated the products are, the less is the entrant's marginal profit from competing against the pioneer through lower production costs, and vice versa."

Differently from the works mentioned in the previous paragraph, our work does not focus on the organizational structure of innovating firms, rather it focuses on the dynamic effects of contextual process and product innovations in a general equilibrium framework and it aims to give one possible explanation, among different existing ones, to the fact that firms producing at any moment in time are *heterogeneous* in their productivity levels.

In a certain sense, the particular form of process innovation we represent is related, even if it is different, to the argument of the learning-by-doing processes suggested by Romer (1990), who considers knowledge accumulation as a side effect of conventional production activity not resulting from deliberate research activity in the innovative sector. We analyze the effects of the complementarity between product and process innovations when product innovations take place as a deliberate effort of researchers employed in the R&D sector, and contextual process innovations take place as a side effect of the innovative activity in the same sector, resulting in higher productivity of new consumption goods. This is possible because researchers, when carrying out their activity, accumulate knowledge which leads them, at a certain point, to develop new patents of consumption goods characterized by more productive production processes. However, we explain in the paper that workers (firms) engaged in the R&D sector have incentives in pursuing process innovation, because the purchasing power of their wages is increased only if process innovation takes place and only in terms of the more productive goods which have been made available.

Indeed, the improvements in production processes which we introduce in this work are assumed to take place as a by product in the R&D sector and they are costlessly adopted by firms starting the production of newly developed consumption (or intermediate) varieties, while already producing firms continue to produce using older technology which was available when the products were developed, and, as a consequence, we will be able to obtain long-run equilibria characterized by many different kinds of firms with different production processes.[2] These assumptions allow us to capture the fact that often new producing firms are more productive than older ones, given that they start to produce when knowledge accumulated in the past allows new production techniques to be more efficient.

Therefore, in this work we adopt a growth approach in order to identify one potential mechanism whose action will result in productivity heterogeneity of firms. Hence, if firms are heterogeneous in their productivity levels, this should result in a variety of prices, and of demand and market shares which reflect firms' productivity differences. Moreover, these differences should, in turn, be reflected in patents' price differences, given that we would expect that patent prices of more profitable varieties are higher.

[2] This is a simplifying assumption, given that we could consider a more general case in which old firms could adopt the new more productive production processes provided that they sustain a certain switching or implementing cost. The presence of different *implementing costs* would imply that not all preexisting firms would be able to adopt the new more productive production processes once available. Thus, as in our simplified case, there would be equilibria characterized by many different kinds of firms with different production processes. Hence the nature of the results which we would obtain would be similar to those we obtain, with the sole difference that the distribution of firms among the available production processes would be more biased towards the new ones. However, we chose to adopt a simplified framework in which no technology change is possible in order to avoid the complications which would arise describing the switching processes, and given that it would not change the heterogeneous nature of our results.

In particular, we introduce process innovations in the setup proposed by Grossman and Helpman (1991) characterized by the assumption of consumers' love for variety by Dixit and Stiglitz (1977). This last assumption identifies consumers' love for variety as one of the causes of economic growth, because it pushes firms to innovate in order to satisfy consumers' demand for new varieties. Moreover, it allows us to introduce a further assumption related to how process innovation takes place, given that we assume that they are more likely to occur when the market is larger. In fact, in this case, researchers are induced to increase their efforts to find improvements in the available production technology given that, ceteris paribus, the number of more productive varieties on which relative prices would be reduced and purchasing power would be higher, would be larger. We could also justify this assumption with the argument that larger markets allow researchers to exploit increasing returns to scale.

Thus, the explicit purpose we try to assess in this work is to understand how the steady state outcomes by Grossman and Helpman (1991) are affected by process innovations that accompany product innovations, investigating the effects of this complementarity on the long run growth rate of the economy, productivity heterogeneity of firms, worker distribution among different sectors and on prices of all available varieties and patents.

Finally, our work will also try to address, or better mitigate, the scale effect problem which affected the original contribution by Grossman and Helpman (1991). In fact, Jones (1999, p. 139) writes that, when there is a scale effect, "the growth rate of the economy is proportional to the total amount of research undertaken in the economy. An increase in the size of the population, other things equal, raises the number of researchers and therefore leads to an increase in the growth rate of per capita income. Taken at face value, this prediction is problematic because it means that population growth should lead to accelerating per capita income growth". Thus, we would like to suggest that the introduction of a series of continuous process innovations, along the lines described in the following sections, could add another mechanism through which this scale effect may be tackled or better, as we stated, mitigated to those reviewed by Jones (1999).

The remaining part of the work is organized as follows: Section 2 describes consumers' and firms' behavior, while the innovative sector is more deeply analyzed in Section 3; Section 4 draws the characteristics of the equilibrium outcomes which are "moving" with particular changes in the distribution of workers; Section 5 presents some comments on the results, while Section 6 concludes.

2 Consumers' and firms' behavior

We consider a closed economy in which consumers love variety and their preferences are described by the following intertemporal utility function

$$U = \int_0^\infty e^{-\rho t} \log \left(\sum_{c=1}^{n(t)} D_c(t)^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}} dt \quad (1)$$

where D_c is the consumption of variety c, ρ is the rate of time preference and $\sigma > 1$ is both the elasticity of substitution between any pair of varieties and the own-price elasticity of demand for any variety. The elasticity of intertemporal substitution in (1) is constant and equal to 1, while n is the total number of produced varieties in t.

Total consumers' expenditure E is defined as

$$E = \sum_{c=1}^{n} p_c D_c$$

where p_c is the price of variety c.

Consumers' demand x_c for any variety c is

$$x_c = \frac{p_c^{-\sigma}}{\sum_{c=1}^{n} p_c^{1-\sigma}} E \quad (2)$$

All varieties are produced by firms which need to buy a patent from the R&D sector to start their activity and which employ γ workers to produce a unit of their output. Given the assumptions of consumers' love for variety and the fact that there are no scope economies, all firms produce different varieties.