The quantity sold on the foreign market is instead

$$q_{rs}(p_{rs}) = q_{rs}^{L}(p_{rs})\frac{L}{2} + q_{rs}^{H}(p_{rs})(1-\lambda_r)H \tag{10}$$

Similar expressions can be obtained for firms that produce in region s.

Operating profits of a representative firm which produces in r are obtained by adding operating profits which derive from sales in r, π_{rr}, to those derived from sales in s, π_{rs}, which are, respectively,

$$\pi_{rr} = p_{rr}q_{rr} \quad \text{and} \quad \pi_{rs} = (p_{rs} - t)\, q_{rs} \tag{11}$$

The production cost of each firm in region $z = r, s$ is generated by the fixed cost that firms have to sustain in order to employ f skilled workers and are given by

$$TC_r = fw_r \tag{12}$$

Therefore, pure profits π_r of the representative firm which produces in region r are

$$\pi_r = \pi_{rr} + \pi_{rs} - fw_r \tag{13}$$

Finally, the assumption of full employment of workers implies that

$$H_r = \lambda_r H = n_r f \quad \text{and} \quad H_s = (1-\lambda_r)H = n_s f \tag{14}$$

3 Preference differences and equilibrium outcomes

In this section we derive equilibrium prices and quantities and skilled workers' indirect utility functions used to evaluate the stability properties of the different potential outcomes. First of all, from the first order conditions for the maximization of profits, we obtain the following equilibrium price for varieties sold at home

$$p_{zz}^{*}(\lambda_z, \rho) = \frac{td_L\left(\frac{L}{2} + \rho\lambda_z H\right)(1-\lambda_z)\,M + 2a\left(\frac{L}{2} + \lambda_z H\right)}{2\left(2b_L + d_L M\right)\left(\frac{L}{2} + \rho\lambda_z H\right)} \tag{15}$$

where $z = r, s$. The asterisk always denotes equilibrium values.

Moreover, the price of exported varieties from region z to region v is

$$p^*_{zv}(\lambda_z, \rho) = p^*_{vv} + \frac{t}{2} \tag{16}$$

where $v, z = r, s$ and $v \neq z$. From the previous expression we note that, even though prices differ from the original linear core-periphery model by Ottaviano et. al. (2002), the relationship between prices of locally produced varieties, p_{vv}, and the imported varieties, p_{zv}, is still the one found in the linear model.

In order to have positive exports from region z to region v, exporting prices, p^*_{zv}, must be higher than transport costs, t, and this requires that

$$t < t^*_{zv} = \frac{2a\,(L + 2H)}{(2b_L + d_L M)\,(L + 2\rho H)} \tag{17}$$

where $v, z = r, s$ and $v \neq z$.

It can be easily verified from (15) and (16) that

$$\frac{\partial p^*_{zr}(\lambda_r, \rho)}{\partial \rho} < 0 \tag{18}$$

with $z = r, s$. The result in (18) reflects the fact that when skilled workers' preference for the manufactured good and the variety in its consumption increase, that is when ρ decreases, the price of each variety, either locally produced or imported, increases.

Moreover, we obtain that

$$\frac{\partial p^*_{zr}(\lambda_r, \rho)}{\partial \lambda_r} = \frac{1}{2(2b_L + d_L M)} \left[-d_L M t + \frac{4LaH(1-\rho)}{(L + 2\lambda_r H \rho)^2} \right] \tag{19}$$

with $z = r, s$. Thus, we may notice that, as in Ottaviano et al. (2002), equilibrium prices are dependent on the distribution of the workers' demand and firms between the two regions. However, while Ottaviano et al. (2002, p. 417) find that "the prices charged by both local and foreign firms fall when the mass of local firms increases (because price competition is fiercer)", we find that this is true only when $\rho \geq 1$, that is, when skilled workers have a weaker preference for the modern good and variety in the consumption of the same. Thus, prices charged by both local and foreign firms are not obliged to fall whenever the mass of local firms increases, because expression (19) shows that if the intensity of skilled workers' preference for the modern good and its variety is stronger (with $\rho < 1$), prices charged by firms, either local or foreign, may even increase when the mass of local firms increases. This result arises in our work from the fact that, together with the *competition effect* on prices generated by changes in the distribution of workers and firms, already described in Ottaviano et al. (2002), there is another contextual effect on prices due to preference heterogeneity which acts through the change in the relative weight of demand for the modern goods with respect to the traditional good. We call this effect the *preference effect* and its action will be more deeply discussed in next section.

Another new and significant result, strictly associated with the previous one, is that the increase of the mass of local firms in a region, for instance region r, is no longer always associated with an increase of the price of varieties sold in the other region, as it happens when $\rho = 1$. In fact, given that

$$\frac{\partial p_{zs}^*(\lambda_r, \rho)}{\partial \lambda_r} = \frac{1}{2(2b_L + d_L M)} \left[d_L M t - \frac{4LaH(1-\rho)}{(L + 2(1-\lambda_r)H\rho)^2} \right] \tag{20}$$

with $z = r, s$, it is easily verified that if skilled workers have a stronger preference for the modern good and variety in its consumption, that is if $\rho < 1$, then an increase of the mass of local firms in region r may also be associated with a decrease in prices of varieties sold in the other region s.

Moreover, we derive the equilibrium quantities which depend not only on the distribution of firms and workers between the two regions, but also on the value of ρ. Particularly, for any firm the equilibrium value of the quantity sold in the home region is

$$q_{zz}^*(\lambda_z, \rho) = \frac{(b_L + d_L M)\left[t d_L M (1-\lambda_z)(L + 2\rho\lambda_z H) + 2a(L + 2\lambda_z H)\right]}{4(2b_L + d_L M)} \tag{21}$$

where $z = r, s$. We also compute the equilibrium value of the quantity that any firm in v sells abroad, that is

$$q^*_{vz}(\lambda_z, \rho) = q^*_{zz}(\lambda_z, \rho) - \frac{t(b_L + d_L M)(L + 2\rho\,(1 - \lambda_z)\,H)}{4} \tag{22}$$

where $v, z = r, s$ and $v \neq z$.

It can be readily verified from (21) and (22) that

$$\frac{\partial q^*_{zz}(\lambda_z, \rho)}{\partial \rho} > 0 \quad \text{and} \quad \frac{\partial q^*_{vz}(\lambda_z, \rho)}{\partial \rho} < 0 \tag{23}$$

where $v, z = r, s$ and $v \neq z$. Therefore, a reduction in ρ, due to an increase in the preference for the manufactured good and the variety in its consumption for skilled workers, does always reduce equilibrium quantities of locally produced varieties, and increase those of imported varieties.

We notice from (23) and (8) we can derive that

$$q^*_{zz}(\lambda_z, \rho) > a \quad \text{and} \quad q^*_{vz}(\lambda_z, \rho) < a \tag{24}$$

with $v, z = r, s$ and $v \neq z$.

Skilled workers' indirect utility function in region r is given by the following expression

$$V_{Hr}(\lambda_r, \rho) = S_{Hr}(\lambda_r, \rho) + w^*_r(\lambda_r, \rho) + \overline{q}_0 \tag{25}$$

where the individual consumer surplus for skilled workers, $S_{Hr}(\lambda_r, \rho)$, is given by

$$\begin{aligned} S_{Hr}(\lambda_r, \rho) \quad = \quad & \frac{a^2 M}{2b_H} - a\ \left[n_r(\lambda_r, \rho) p^*_{rr}(\lambda_r, \rho) + n_s(\lambda_r, \rho) p^*_{sr}(\lambda_r, \rho)\right] + \\ & + \frac{b_H + d_H M}{2} \left[n_r(\lambda_r, \rho)(p^*_{rr}(\lambda_r, \rho))^2 + n_s(\lambda_r, \rho)(p^*_{sr}(\lambda_r, \rho))^2\right] + \\ & - \frac{d_H}{2} \left[n_r(\lambda_r, \rho) p^*_{rr}(\lambda_r, \rho) + n_s(\lambda_r, \rho) p^*_{sr}(\lambda_r, \rho)\right]^2 \end{aligned} \tag{26}$$

and the equilibrium skilled wage in region r, $w^*_r(\lambda_r, \rho)$, is derived from the free entry condition, which implies that profits in (13) are equal to zero in equilibrium.

We follow the myopic adjustment process adopted in Ottaviano et al. (2002), from which we know that a spatial equilibrium corresponds to the case in which each mobile worker located in a region cannot increase its utility level by moving to the other region. Therefore, we may write that a spatial equilibrium arises at an interior point, with $\lambda_r \in (0, 1)$, when

$$\Delta V_H(\lambda_r,\rho) \equiv V_{Hr}(\lambda_r,\rho) - V_{Hs}(\lambda_r,\rho) = 0 \tag{27}$$

or at the extreme point of full agglomeration in region s with $\lambda_r = 0$ (in region r with $\lambda_r = 1$) when $\Delta V_H(0,\rho) \leq 0$ ($\Delta V_H(1,\rho) \geq 0$).[8]

Finally, while it is easily verified that the agglomerated equilibria are always stable, the interior equilibria are stable when the slope of $\Delta V_H(\lambda_r,\rho)$ is negative.[9]

The indirect utility differential is

$$\Delta V_H(\lambda_r,\rho) = \frac{(2\lambda_r - 1)\,M}{(2b_L + d_L M)^2}\left[(b_L + d_L M)\,(a_0 t^2 + b_0 t) + c_0\right] \tag{28}$$

where the three coefficients a_0, b_0 and c_0, respectively, are

$$\begin{aligned}
a_0(\rho) &= -\tfrac{(L+H\rho)d_L^2 M^2 + 2(3H\rho+L)b_L d_L M + 6b_L^2 H\rho}{4H} < 0 \\
b_0(\lambda_r,\rho) &= \frac{a\left\{\begin{array}{c} 2(L+2\lambda_r H\rho)[L+2(1-\lambda_r)H\rho]d_L M+ \\ +[(4-\rho)L^2+2(4-\rho)\rho HL + 12(1-\lambda_r)\lambda_r H^2\rho^2]b_L \end{array}\right\}}{[L+2(1-\lambda_r)H\rho](L+2\lambda_r H\rho)} \\
c_0(\lambda_r,\rho) &= -\frac{2a^2 HL(1-\rho)\left\{\begin{array}{c} [L+2(1-\lambda_r)H\rho](L+2\lambda_r H\rho)d_L M+ \\ +[(2-\rho)L^2+(3-\rho)\rho HL + 4(1-\lambda_r)\lambda_r H^2\rho^2]b_L \end{array}\right\}}{[L+2(1-\lambda_r)H\rho]^2(L+2\lambda_r H\rho)^2}
\end{aligned} \tag{29}$$

We observe that we obtain the results in the linear core periphery model by Ottaviano et al. (2002) when $\rho = 1$. In this particular case, $c_0 = 0$. We also note that when $\rho < 1$, it is always true that $b_0 > 0$ and $c_0 < 0$.

In table 1 we compare the case in the linear core periphery model ($\rho = 1$) to our extension ($\rho > 0$) and we draw the attention to the fact that in the latter case a_0 depends only on ρ, while b_0 and c_0 depend both on ρ and λ_r, while in the former case no coefficients depend on the distribution of skilled workers.

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[8] See Ottaviano et. al. (2002)

[9] The reader interested in the specification of the migration process may find its accurate description in Ottaviano et al (2002, p. 419).

	a_0	b_0	c_0
$\rho > 0$	$a_0(\rho)$	$b_0(\lambda_r, \rho)$ with $b_0(0,\rho) = b_0(1,\rho)$	$c_0(\lambda_r, \rho)$ with $c_0(0,\rho) = c_0(1,\rho)$
$\rho = 1$	$a_0 = -\frac{(L+H)d_L^2 M^2 + 2(3H+L)b_L d_L M + 6b_L^2 H}{4H}$	$b_0 = a(3b_L + 2d_L M)$	0

Table 1.

Fig. 2 plots the indirect utility differential $\Delta V_H(\lambda_r, \rho)$ when $\rho < 1$ and shows not only that agglomeration may result unstable for parameter values for which it was stable with $\rho = 1$, but also that asymmetric stable equilibria outcomes may arise when the symmetric equilibrium is unstable. In fact, when $\rho < 1$ there is another dispersion force at work which acts together with all traditional forces in determining the equilibria of the model. In particular, this force arises because in the region with the highest (lowest) density of workers, prices tend to increase (decrease) due to the stronger (weaker) demand for the differentiated good compared with that for the traditional good, and it accompanies the agglomeration competition effect on prices which tend to decrease (increase) in the same region because of the fiercer (weaker) competition originated by the greater (smaller) number of firms.

Insert figure 2 about here

Finally, we note that when $\rho < 1$, the indirect utility differential in (28) at $\lambda_r = 1$ depends on the the values of $a_0(\rho) < 0$, $b_0(1, \rho) > 0$ and $c_0(1, \rho) < 0$.[10] Clearly, the expression in square brackets in (28) depends on the level of economic integration. More precisely, it is a concave parabola in t, with its maximum for $t_1 = -b_0(1,\rho)/(2a_0(\rho)) > 0$. Hence, given that the sign of $\Delta V_H(1, \rho)$ depends on that of the parabola, we can state that full agglomeration is never a potential equilibrium for high and low trade costs, while it may be an equilibrium for intermediate

[10] In particular, $b_0(1,\rho) = a\left[(4-\rho)\,b_L + 2Md_L\right] > 0$ and $c_0(1,\rho) = -2Ha^2\frac{(1-\rho)[(2-\rho)Lb_L+(3-\rho)\rho Hb_L+(L+2H\rho)Md_L]}{(L+2H\rho)^2} < 0$.

trade costs.[11] In any case, we note that in the previous phrase we used the word potential to qualify the equilibrium, because we remember that trade costs must be compatible to positive prices and quantities, which require expression (17) to be satisfied.

On the other hand, when $\lambda_r = 1/2$ the indirect utility differential in (28) is decreasing in λ_r, and therefore we have an equilibrium at $\lambda_r = 1/2$ only when

$$\left.\frac{\partial\left(\Delta V_H(\lambda_r,\rho)\right)}{\partial\lambda_r}\right|_{\lambda_r=1/2} = \frac{2M\left[(b_L + d_L M)\left(a_0(\rho)t^2 + b_0(1/2,\rho)t\right) + c_0(1/2,\rho)\right]}{(2b_L + d_L M)^2} < 0$$

Clearly, the previous inequality is true when the expression in square brackets is negative. We observe that, when $\rho < 1$, this expression is depicted by a concave parabola in t with $a_0(\rho) < 0$, $b_0(0.5,\rho) > 0$ and $c_0(0.5,\rho) < 0$. Thus, the symmetric equilibrium is stable only for high and low trade costs, provided that (17) is satisfied, while it is unstable for intermediate trade cost values.[12]

4 The competition effect and the preference effect in detail

In order to more deeply discuss the findings in the previous section, we recall that Ottaviano et al. (2002) find that there are different effects which give rise to the agglomeration and dispersion forces, whose interplay defines the properties of the equilibrium outcomes. These forces are the dispersion force originated by the demand of immobile unskilled workers, and the agglomeration force originated from the fact that a greater number of firms in a region implies that fewer varieties are imported, and that equilibrium prices of all varieties sold in this region are smaller (*competition effect on prices*).

In this work, we show that these effects are partially modified and enriched by the additional force which is generated when $\rho \neq 1$. In particular, the centrifugal force generated by immobile

[11] In particular, with $\rho < 1$, when $\lambda_r = 1$ and $t = t^*$, we know that $\Delta V_H(1,\rho) > 0$ if $b_0(1,\rho)^2 > \frac{4a_0(\rho)c_0(1,\rho)}{(b_L+d_L M)}$.

[12] In particular, $b_0(0.5,\rho) = 2ad_L M + \frac{a[(4-\rho)L(L+2\rho H)+3H^2\rho^2]b_L}{(L+H\rho)^2}$ and $c_0(0.5,\rho) = -2a^2H(1-\rho)\left\{(L+H\rho)d_L M + \left[\left((2-\rho)L+(3-\rho)\rho H\right)L + H^2\rho^2\right]b_L\right\} L\frac{1}{(L+H\rho)^4}$

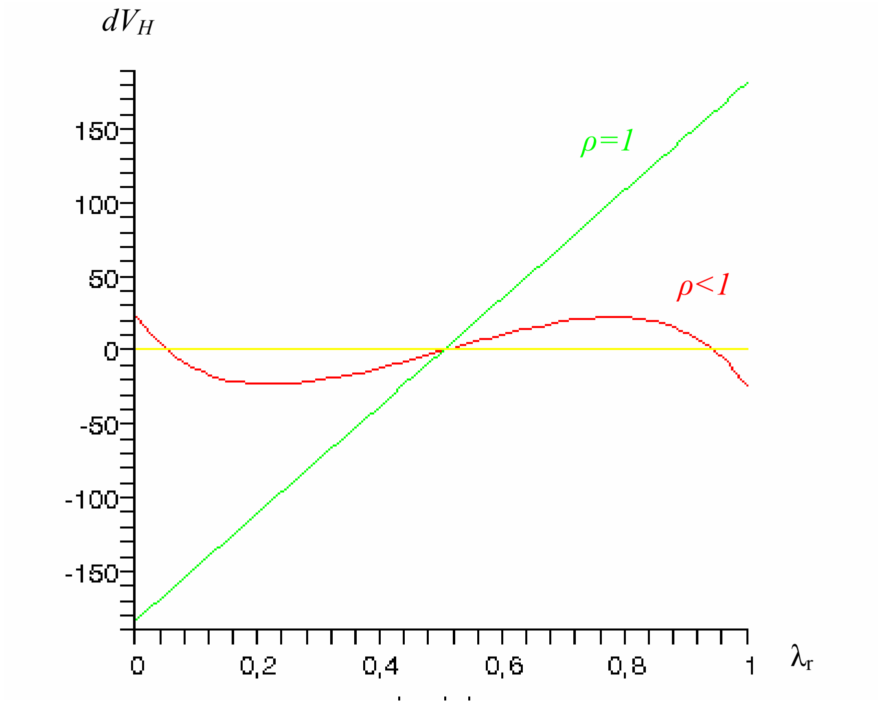

Fig. 2