1 – Introduction

In the past twenty years many European and OECD\(^1\) countries have accomplished tax structure reforms. In many cases one of the policy objectives was, and still continues to be it, to decrease the total tax burden in the economy, redistributing in some way the gains over tax units.

In this paper it presents some methods to redistribute the gains (or losses) of a tax reform and a procedure to identify the value judgements according to the tax reform outcome; they will be useful to analyse the first proposal of personal income tax (IRPEF) reform presented in Italy by the current Executive on 21 December 2001. In the past years also in Italy a reorganization of current income tax code was operating and, in contrast, this proposal chooses to modify it radically. IRPEF provides around ¼ of the total revenue for the government and is around 15 percent of the total GDP. The Italian Constitutional Law states that IRPEF had to be progressive, hence in some way it has to allow to redistribute a given amount of income from the richer component of the population to the poorer one.

The rest of the paper is organized as follows. First I present a review of the principal results of the literature concerning the rationales for a progressive income tax, especially by looking for the normative significance of outcomes of a tax code, as well some indexes able to summarize the redistributive effect and departure from disproportionality (that is, how much the tax burden is differently distributed than income between the population of income units).

In section 3 I offer an empirical survey. As we shall see, after the proposal was announced many practitioners have investigated its possible redistributive consequences. The principal outcome will show that ‘the redistribution continues to go from the richer group to the poorer, but with a reduction of intensity’. Section 4 concludes the paper by a deeper theoretical analysis of the design of the original tax reform, showing how theoretical instruments can be used to verify the degree of the progression of tax structure and resultant distributive effects.

In accordance with these effects, it seems that there could be some reasons for changes in the structure of the tax reform proclaimed more recently by the current Executive. Other contributions (Baldini and Bosi, 2004) study the effects of these changes, affirming the presence of a lower ‘reduction of intensity’. However, the principal conclusion does not modify so much: the Executive main interesting appears to be the relationship between the tax cut and the expenditure side of budget (how large does it would become the public spending

\(^{1}\) - Organization for Economic Cooperation and Development.
deficit? Is this acceptable?) and how the tax cut affect the economic growth, rather than which distribution of income.

2 – A Progressive Income Tax Schedule: Theoretical Results

The large majority of European and OECD countries have a progressive income tax. Given the respect, or not, of horizontal equity (HE) prior principle (a normative goal) “progression arises from principles of vertical equity” (Lambert, 2001, p. 175): vertical equity states taxpayers in unequal circumstances bear appropriately unequal taxes (a matter of judgment, that is an ethical norm referring to the treatments of unequals)\(^2\).

The literature suggests some rationales, for instance the Equal sacrifice principle (by using an increasing concave utility of income function equal for all individuals, this calls for a equal utility loss for all)\(^3\) as well as the Benefit principle (by relating the tax liability to the expenditure side of budget)\(^4\); the automatic stabilizing effect on the macro-economy; the envy reducer capacity.

After the seminal Atkinson (1970) and Shorrocks’ (1983) papers another way to justify progression was on hand to the researchers: I shall give an idea about it referring to the papers where it is opportune. Before to do that I provide some definitions about what it defines as a progressive income tax as well as some other useful instruments.

2.1 Some Definitions

An income tax schedule embodies a succession of upward and fixed marginal tax rates (the rate structure) on bands of taxable income with different specified threshold values.

If \( t(x) \) is the tax liability\(^5\) of the pre tax money income \( x \) and is a differentiable function, an increasing with income average tax rate \( t(x)/x \) is the condition for strict progression

\[
\frac{d \left( [t(x)/x] \right)}{dx} > 0 \quad \forall x \quad \text{iff} \quad t'(x) > \frac{t(x)}{x} \quad \forall x > 0
\]

In this case the tax burden is function only of money incomes while the typical real income tax code usually is also function of other features: we shall see how to take account of these non-income characteristics.

\(^2\) In a second best-world there is a potential trade-off between HE and VE objectives (see Musgrave 1990).

\(^3\) See Samuelson (1947).


\(^5\) I assume \( 0 \leq t'(x) < 1 \) (incentive preservation principle), then \( 0 \leq t(x) < x \) (where \( t'(x) \) is the first derivative).
Let $F(x)$ be the pre tax distribution of income and $f(x)$ the associated density function, then

(assuming no convergence problem at the top end of the distribution)

Total pre tax income $X = N \int_0^z x f(x) \, dx$

Total income tax revenue $T = N \int_0^z t(x) f(x) \, dx$

Total tax ratio $T/X = g = \frac{N \int_0^z t(x) f(x) \, dx}{N\mu} = \frac{\int_0^z t(x) f(x) \, dx}{\mu}$

where $\mu = \int_0^z x f(x) \, dx$ and $z$ could be described as ‘any income level in excess of the highest one actually occurred’ (Lambert, 2001, p. 20)

According to these integrals it can derive definitions for the Lorenz curve for pre tax income $L_X$ and the Concentration curves for post–tax income and tax liabilities, $L_{X-T}$ and $L_T$.

$p = F(y) \Rightarrow L_X(p) = \left[\frac{1}{\mu}\right] \int_0^y x f(x) \, dx$

$p = F(y) \Rightarrow L_{X-T}(p) = \left[\frac{1}{\mu(1-g)}\right] \int_0^y [x - t(x)] f(x) \, dx$

$p = F(y) \Rightarrow L_T(p) = \left[\frac{1}{\mu g}\right] \int_0^y t(x) f(x) \, dx$

By using these definitions it derives the relationship between $L_X, L_{X-T}$ and $L_T$

$L_X \equiv g L_{X-T} + (1-g) L_T$

The $L_X$ curve is a weighted average of tax and post tax income Concentration curves: as a result it can be shown that if and only if the tax burdens are distributed more unequally than before-tax incomes, the post tax incomes shares are more equal than the latter. Where $L_X$ and $L_T$ curves are superimposed also $L_{X-T}$ has to be it.

Furthermore Jakobsson (1976) and Fellman (1976) point out that this relationship is strictly linked with the progression of the tax schedule:

$$\frac{d}{dx} \left[\frac{t(x)}{x}\right] \geq 0 \quad \forall x \quad \text{iff} \quad L_{X-T} \geq L_X \geq L_T \quad \forall F(x)$$

Thus, with a tax code designed for any sub-population where the only differences among people are the incomes, a strictly progressive income tax is within group inequality reducing according to the dominance of post tax income concentration curve over the pre tax income Lorenz curve. According to $t(x)$ function only of money incomes I approach the normative issue.

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6 - A Lorenz Curve measures income shares plotting them against cumulative proportion of income units in the pre tax distribution. The units are arranged in ascending order of their income. The assumptions on the form of $t(x)$ function (5th footnote) allow to consider the $L_{X-T}$ concentration curve as a Lorenz curve (no reranking effect).
2.2 Progression, Progressivity and Welfare

According to Atkinson (1970) a dominance result by comparisons of Lorenz curves may fail: not any pair of income distributions can be clearly ranked (the Lorenz ordering is a partial ordering). By the Shorrocks (1983) theorem, sometimes it may go beyond the Lorenz curve comparison failing: by using generalized Lorenz curve concept it could be possible to rank unambiguously more (not any) distributions of income.

With these theorems it is possible to describe why progression should be a “good thing”, that is to verify if the welfare related with a progressive income tax is higher than the welfare associated with another way to raise the same revenue, in this case an equal yield flat tax applied to the same distribution of before tax incomes.

According to the fact that generalized Lorenz curves (by the Jakobsson-Fellman theorem) for post tax incomes, when progressive income tax and equal yield flat tax apply, are respectively

\[
p = F(y) \implies GL_{x-T}(p) = \mu(1-g) \int_0^y [x - t(x)] f(x) \, dx / \mu(1-g) = \mu(1-g) L_{x-T}(p)
\]

and

\[
p = F(y) \implies GL_{x(t-G)}(p) = \mu(1-g) \int_0^y x f(x) \, dx / \mu = \mu(1-g) L_{x}(p)
\]

it can be inferred that the distribution of post tax incomes associated with a progressive income tax rank dominates the distribution of post tax incomes related with the proportional tax. By Shorrocks theorem the dominance between GL curves implies (and it’s implied by) a higher level of Social Welfare reached by the dominant position:

\[
W_{X-T} \geq W_{X(t-G)} \quad \text{for all increasing and strictly concave } U(\cdot)
\]

This shows that choosing progressive income taxation rather than a proportional tax it obtains the smaller decrease of Social Welfare.

Hence the literature has developed measures of structural (or local) progression and effective progression, the former linked with the degree of progression along the income scale, the latter with the taxation effects on the after-tax income distribution.

Both give information about the consequences of different tax income schedules on their disproportionality degree and resultant redistributive effects and what is important for my

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7 - By \((x-x)\) = post-tax income, when equal-yield flat tax applies: the net incomes means in both cases are equal.

8 - Where \(W_f = \int_0^y U(x) f(x) \, dx\)

9 - See Jakobsson (1976) and Kakwani (1977a). Note that, advocating the fundamental Atkinson theorem (1970), Formby and Smith (1986, p. 562) comment, “If Lorenz curves intersect, a social welfare function can always be found which ranks income distribution differently than does the Gini coefficient or other summary measures of
purposes is the possibility, by using measures of effective progression, to obtain significant conclusions about the level of Social Welfare that different tax schedules incorporate.

Usually when the analysts wish to evaluate it they first look at two indexes, the Reynolds-Smolensky (ΠRS) and the Kakwani (ΠK) indexes: they are scalar index numbers, able to resume a tax code and income distribution pair. They are defined in terms of separation of the relevant Lorenz curves (the higher the separation between the curves, the higher the index),

\[ Π^K = 2 \int_0^1 [L_X(p) - L_T(p)] dp \quad ; \quad Π^{RS} = 2 \int_0^1 [L_{X-T}(p) - L_T(p)] dp \]

or, in a short way,

\[ Π^K = C_T - G_X \quad ; \quad Π^{RS} = G_X - C_{X-T} \]

where \( G_X \) is the Gini coefficient for before-tax income, \( C_T \) and \( C_{X-T} \) (here, \( C_{X-T} = G_X - T \)) the concentration coefficients respectively for tax liabilities and post tax income. The higher is \( Π^{RS} \), the ‘more’ dominant is the distribution of post tax incomes over the distribution defined by \( L_X \), where as before the latter has to be interpreted as the distribution of after-tax incomes resulting from an equal yield flat tax.

Clearly it has to be a link between the ΠRS and ΠK indexes strictly connected with the identity that represent the \( L_X \) curve as a weighted average between \( L_{X-T} \) and \( L_T \).

By Kakwani (1977b) it can be shown that (if no reranking through the taxation process is present)

\[ Π^{RS} = \left[ \frac{g}{1-g} \right] Π^K \]

What result from the Social Welfare point of view? Do we can conclude that a ‘more’ progressive tax is a better tax? For instance, given the assumptions made and choosing the social evaluation function (where \( \mu \) and \( G \) are the mean and the Gini coefficient (Sen 1973)),

\[ V(\mu, G) = \mu(1-G) \]

it can be shown that\(^{10}\)

\[ W_{X-T} - W_{X(1-g)} = \mu(1-g) \left[ (1 - G_{X-T}) - (1 - G_X) \right] \]

\[ W_{X-T} - W_{X(1-g)} = \mu(1-g) Π^{RS} \]

\[ W_{X-T} - W_{X(1-g)} = \mu g Π^K \]

In both cases the higher is the relevant index the greater is the Welfare Premium from progression.

\(^{10}\) - By the equality \( C_{X-T} = G_{X-T} \).
2.3 Income Tax Reforms

The results described in the last paragraph allow to analyse any personal tax income reform: by means of a sample of the distribution of before-tax incomes it can make an estimate to predict the consequences for distributional effects, the disproportionality of the new tax burden and, where feasible, the question of normative significance. Assuming a fixed distribution of pre tax incomes \( F(x) \), there are three alternative cases according to a change in a tax schedule \( t(x) \):

*a yield neutral/increasing/decreasing tax reform according as \( g \) (total tax ratio)*

*is equal/ increased/decreased after the reform*

Also the measures of structural progression\(^{11}\) \( \Pi^{\text{RS}} \) and \( \Pi^{\text{K}} \) modify in accordance with this change.

The consequences for \( g \) and indexes may be linked and operate together, turning out different overall results of the reform\(^{12}\). Consistent with these, the literature is able to provide to practitioners normative assessments about some general cases:

- a progression neutral tax cut/hike for all;
- a not\(^{13}\) progression neutral tax cut/hike for all;
- a single-crossing tax reform and a double-crossing tax reform.

About a progressivity-neutral reform, consider a 'linear' income tax cut/hike preserving some local progressivity measure such as \( \text{RP} \) and \( \text{LP} \). From the old schedule \( t'(x) \) it has to be, for the RP case

\[
\hat{t}^*(x) = t'(x) - a \left[ x - t'(x) \right]; \quad x - \hat{t}^*(x) = [x - t'(x)] + a \left[ x - t'(x) \right]
\]

and for the LP case

\[
\hat{t}^*(x) = t'(x) - b t'(x); \quad x - \hat{t}^*(x) = [x - t'(x)] + b t'(x)
\]

where \( \hat{t}^*(x) \) is the new schedule and \( a \) and \( b \) are positive/negative constant parameters according to a tax cut/hike.

Following Pfähler (1984) if we have to choose between these reforms, the best way to share between income units the required changes in the income tax yield is to follow a RP neutral policy if we apply a tax cut and a LP neutral policy for the tax hike. In this way post tax income inequality is less and social welfare is higher, according to the Lorenz (or, more

\(^{11}\) - See Jakobsson (1976) and Kakwani (1977a).
\(^{12}\) - Realistically we can exclude the case of equality of \( g \) (pre and post reform) with the constancy of two indexes, although this is theoretically possible.
\(^{13}\) - From the empirical evidence, we shall see that this seems to be the case of the Italian tax reform proposal.
\(^{14}\) - Residual progression = \( \text{RP} = e^{\hat{t}(x)} = (d (x - \hat{t}(x)) / dx) (x /x - \hat{t}(x)) \); Liability progression = \( \text{LP} = e^{\hat{t}(x)} = \)
precisely, Concentration) dominance criterion and Atkinson theorem (it can be shown those two curves for post tax incomes, or taxes \( L_{X-T} \) or \( L_{T} \) - do not intersect and have the same mean for both).

Looking at a single-crossing reform, as before the revenue could be neutral, increasing or decreasing. About the distributional effects, there are two cases: the old and new tax schedule cross once, the new one crossing the old one from below or from above. The former leads to redistribution from rich to poor, for the latter the opposite is the case.

For instance, according to Dardanoni-Lambert (1988), it is possible to give some normative evaluation when the tax reform is inequality reducing:

1. for any before-tax distribution : \( g^2 \leq g^1 \) (by \( t'(x) \rightarrow t^2(x) \)) and the new schedule crosses the old one from below. The mean of post tax incomes is higher or equal, the redistribution operates to reduce inequality (accepting the Dalton’s Transfer Principle), hence the \( GL \) curve after the tax reform is higher relatively to the \( GL \) curve for the old schedule and social welfare level follows by Shorrocks’s theorem: it is higher for tax reform, for all increasing and strictly concave \( U(\cdot) \);

2. for any before-tax distribution : \( g^2 > g^1 \) (by \( t'(x) \rightarrow t^2(x) \)) and the new schedule crosses the old one from below.

The conditions are more stringent than [1] because \( GL_{X-T}(p) \) after the tax reform crosses the old \( GL_{X-T}(p) \) from above (inequality reducing but with a lower mean, \( \mu(1 - g^2) \leq \mu(1 - g^1) \), then with an inferior efficiency): to approve the tax reform normatively we need to decrease ‘enough’ the variance in the ‘new’ distribution of post tax incomes.

With reference to reform where a non-progression neutral tax cut/hike for all applies, I postpone the analysis to the last section of this essay whereas, concerning a double-crossing reform (of new and old tax schedule) justifiable by welfare approval, we refer to Lambert (2001, pp. 231-235).

### 2.4 The Typical Income Tax: Social Heterogeneity

Until now we are assessing by using some uncomplicated basis:

\( a) \) non-income factors are completely neglected by income tax schedules, they apply only on money incomes;

\[(d t(x) / dx) (x / t(x)) \]; Average rate progression = \( ARP = d( t(x)/x) / dx \).
b) as a result every individual utility, heart of the Welfare ‘measure’ (the average utility in society), is function only of money incomes;  

c) \[ \begin{align*} 0 \leq t(x) &< x, \\ 0 \leq t'(x) &< 1 \end{align*} \]  

d) the allocation of pre tax incomes is given: a partial equilibrium analysis where incentive effects are excluded by construction (ceteris paribus condition).

As a result of the assumption a), we employed a personal tax schedule \( t(x) \) where \( x \) is pre–tax money income: this could be considered a correct procedure only if all households are equal in size (and, possibly, composition)\(^{15} \) that is, if there is social homogeneity. Clearly this looks an improbable case and in the real world tax systems should take into account this matter.

The usual way used by practitioners is to apply an equivalence scale to deflate appropriately household money incomes, to have a common base of measure and rise above the heterogeneity problem. The equivalence scale uses a coefficient \( (z_n) \) for each type of family size and relative composition and it obtains a distribution of equivalent pre tax income (and a distribution of equivalent post tax income) dividing every households money income by these coefficients. They should reflect the differing needs of the relevant households: this is a matter of value judgment and it is not unusual to find differing opinions about the fairness of the coefficients of applied equivalence scales (moreover, how do they take full account of all non-income significant factors?).

After this type of conversion, only now it can apply all the theoretical and social welfare results described above: from Ebert (1997, 1999) it can be shown that the best way, especially for the distributional analysis, is first to construct a standard individual, an artificial population of income units, substituting an household of size \( n \) “by a set of \( z_n \) equivalent adults”.

By using this fictional individual concept the literature showed (Lambert, 2001, par. 10.1) which has to be the right income tax procedure when the population of households is socially heterogeneous. The goal is to avoid non-horizontally inequitable households’ income tax. To ensure this, families with equal pre tax utility level must confirm this equality also after the taxation.

Unfortunately many problems may take place and is very likely that, by a typical income tax, unequal treatment of equals rise up: this occurs when the tax system does not use a \( t(x) \) function with \( 0 \leq t'(x) < 1 \) (homogenous case), or when \( t_n (x) = z_n \tau_n \left( x / z_n \right) \) and \( \tau_n \) is not the same schedule for all the families (heterogeneous case). In these cases:

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\(^{15} \) It seems obvious that a family of size \( n \neq 1 \) where only an individual gains should be treated in a different way relatively to a single person household: starting from equal pre-tax incomes situation, we have different level of pre tax purchasing power in these families and it is the living standard that is linked with the well-being.
• in all formulas defined on this section, it has to use equivalent income terms (the living standard);
• all the formulas derive results about households, not individuals;
• \( L_{X - T} \) is no more a Lorenz curve but a concentration curve; to define the position of the post tax Lorenz curve it has to take into account a correction factor reflecting the extent of reranking \( (R) \): when \( R \) is at work a difference between the pre tax and post tax rankings of (equivalent) income units is present and the post tax Lorenz curve is always dominated by the \( L_{X - T} \). The original measures of effective progression change relationship, from\(^{16}\)
  \[
  \Pi^{RS} = \frac{g}{1 - g} \Pi^K \quad \text{to} \quad \Pi^{RS} = \frac{g}{1 - g} \Pi^K - R
  \]
  It can refine the decomposition of the Reynolds-Smolensky index, capturing as well as the Kakwani index an effect (negative) of classical horizontal inequity\(^{17}\) not present if equals had been treated equally;
• it still holds the last assumption of this section.

Now, it is time to verify if these instruments are useful to assess, where feasible, the Italian income tax reform using the empirical evidence.

3 - Empirical Evidence: First Results

The first stage of the Italian income tax reform started in 2003 (then, with the income tax document of 2004). The Executive method is proceeding by various steps: in accordance with the parliamentary majority objectives, at the beginning the tax cut has been concentrated on the bottom part of income parade (Bosi and Baldini 2004). Clearly only at the end of the transition period, the characteristics and the redistributive effect of the tax reform will be definitive.

All papers focus on the government proposal, but during period between 2000 up to July 2002 distinct information about the design of tax reform became available for the analysts: for instance, the first paper was on hand in 2000 and redistributive results went out according to the electoral plan of the upcoming government.

During time doubts are decreased by an improved definition of the proposal and, finally, by the appearance of ‘official’ case studies simulating the tax reform effects on individual tax liability, average rates and post tax income.

It can divide papers into three categories, according to different available information

\(^{16}\) - See Kakwani (1984, 1986).
\(^{17}\) - See Aronson et al. (1994).
• from electoral plan (Baldini and Bosi 2000);
• from Law proposal on 2001, 3\textsuperscript{rd} article (Arachi and Zanardi 2002; Padoa Schioppa, 2002; Baldini and Bosi 2002);
• from the conference of Law proposal reporter, the Member of Parliament V.E. Falsitta (Baldini et al. 2002).

It seems obvious to state that the degree of correctness of evaluations increases with the growth of qualifications.

The absence of important parameters or general information constrained the researchers to present various case studies. We shall see that consistent with these, different decreases in the total tax burden - as well as a variety of redistributive effects - was calculated, the ultimate pattern of gainers and losers changing in accordance with different chosen parameters magnitudes.

Especially by the outcomes of Baldini et al. (2002) a first evaluation of the proposal may be defined and therefore it pays more attention on this paper, summarizing the principal results that preceding papers achieve. Before to do that I am going to illustrate which are fundamental points that should be clarified by an exhaustive design of an income tax schedule.

### 3.1 Income Tax Reform Design

First, it should identify completely each variable into the functional form

\[ s(y) = s(x - a_i - d_i(x)) - c_i \]

where \( s(\cdot) \) = rate structure; \( y \) = taxable income; \( x \) = pre tax income; \( a_i = \) lump sum allowance; \( d_i(x) = \) income related deductions; \( c_i = \) tax credits.

According to the 3\textsuperscript{rd} article of the fiscal system broad reform, it focuses on:

1. which are the income sources that are taxable and how much they are liable to tax;
2. the choice between notions of effective or ‘usual’ income;
3. the designation of the unit type: individual or household?;
4. the rate structure;
5. the relief typology \((d_i(x))\), the type and the amount of exemption \((a_i)\);
6. the role of tax credits.

Given these choices “an income unit’s tax liability depends very much on its non income characteristics as well as its income” (Lambert 2001, p.180). At the same time by using a
sample of pre tax incomes it is feasible to estimate the total income tax revenue as well as the redistributive effect of a tax reform.

1 – To define which is the taxable income we must characterize what the term \( x \) represents. Usually

\[
x = x_E + x_T + x_{SE} + x_F
\]

where \( x_E \) = employment income; \( x_T \) = retirements and net transfers; \( x_{SE} \) = self-employment income; \( x_F \) = income by financial assets (interest rate * Assets stock).

First, the tax reform does not change radically the old approach: the latter term, \( x_F \), continues to be excluded from the personal income tax, covered by a substitutive tax code.

On the other hand, the treatment of non-profit firms is included in the personal income tax, and different marginal rates should be applied on some income sources.

Different treatment of these income sources leads to HI among pre tax equals.

2 – Concerning the self-employment income and the individual firm performance the reform establishes the importance of ‘sector studies’, hence giving relevance to a notion of usual income rather than effective income.

In contrast the simulations presented by Falsitta do not show these criteria as well as no distinction between treatments of, for instance, \( x_E \) and \( x_{SE} \).

With this uncertainty about the choice and without improved information, the papers continue to use the old system approach: an effective income concept.

If not, relatively to the tax unit, differences among the effective and usual income, would lead also in this case to HI: given that the taxation should lead to an equal treatment of equals, in many cases the effective income may be different from the taxable income amount that sector studies define usual for that activity, or job, etc.

3 – As in the majority of the European and OECD countries\(^{18}\) the actual income tax is a personal one. The choice of the tax unit in the tax reform is not defined exactly, but looking at other official technical articles it can read that household should be designed as the central subject of the public finance activity. Also in such a case the implicit consequences of Falsitta’s case studies may allow us to apply the old procedures: the household relevance is maintained by the different lump sum allowances given to individuals living in different family sizes and not by an explicit definition of it as the passive subject.

In both cases the tax liability matter is relevant only if the tax income is progressive. If it were proportional (flat average rate for all the income level) for equal household incomes the

gain of splitting possibility is irrelevant: even if the household income is divided among the members, the total liability does not decrease. This matters only if, for different types of household income distribution among members of the family, the tax schedule is progressive.

On the other hand also with a personal income tax it is feasible to consider different composition of the household in which the individual subject lives.

According to the family size it may have several exemptions as well as reliefs\(^{19}\), usually correlated with family characteristics that the law considers correct to protect.

4 – Probably the new rate structure is the only point that seems well defined by the tax reform.

From 5 increasing marginal rates along the income scale and relative thresholds, the tax reform reduces them to only two increasing, with the sole explicit threshold at 100000 Euro (more or less 62000 £).

The two rates are 23% and 33%. There is only the 0.5% of the actual distribution of pre tax incomes approximately above the threshold. For the 99.5% of the population then it is possible to regard the Flat rate tax as the theoretical model, even with the occurrence of abatements and deductions.

5 – Given that it has been identified what is \( x \), now it can define the behaviour of \( a_i \) and \( d_i(x) \) to obtain a taxable income definition.

The abatement level must be equal to the level that identifies the poverty line. From Falsitta’s relation implicitly it can derive that for a single adult the poverty line should correspond to 6300 Euro (around 3900 £).

Note that in a personal income tax it has to consider the existence of economies of scale into the family own life (and the different needs). Equivalence scale become the necessary instrument as means of adjusting measured income into comparable quantities to avoid dangerous interference with respect to the prior principle of Horizontal Equity, the typical causes of differentiation between households being the number of adults and the number and ages of their dependants.

For the other kind of tax-free incomes and income related deductions the tax reform focuses on: family, house, health care, education, research, retirement; non-profit, church donations, social voluntary activities, production costs to labour wage. We shall see which relation may be inferred between the old law and the tax reform regarding these issues.

\(^{19}\) The tax liability could be differentiated among family also by using tax credits.
One of the goals of abatement (better if scaled up according to family features) it is to increase the degree of progression of the income tax\textsuperscript{20} (vertical equity purpose). In contrast other a’s are related directly to the family size and have Horizontal Equity objectives, while HI consequences derive usually from income related deductions, reducing the progressive effects of lump sum components and the rate structure\textsuperscript{21}.

6 – In the actual legislation there are also tax credits. They can be fully redeemed by, in the new fiscal year, the possibility to sum not exploited c\textsubscript{i}’s with the new tax credits, according to:

\[ s\left( y_t \right) - c_{l,t} - c_{l-1,t} \]

In the actual law proposal, and also at the end of the transition period, they still continue to vanish becoming income deductions. The goal should be to simplify the procedure.

To conclude it has only to remind that greater (but not total) certainty regarding the design of tax reform has been obtained during time, constraining some researchers to choose different admissible scenarios from known documents. Even if many of the feasible interpretations of estimates may be judged with a range of uncertainty, nevertheless some points (e.g. the extent of revenue forgone, the fall of total tax ratio \( g \)) are apparent. This seems consistent with a law article relevant point that specifies: nobody has to lose from the tax reform; all must have a better or equal position relatively to the old income tax.

### 3.2 The Electoral Tax Reform Plan

In the electoral plan of Right Wing parties, before the vote for the new Assembly in 2001, a radical tax reform was proposed consistent with the purpose to cut the total tax burden as well as to simplify the tax schedule. With the available information it was possible to state that:

- From the old rate structure with 5 marginal rates (18% the lowest, 45% the highest) they should become two;
- A flexible exemption was introduced quote according to the family size and equal for any income source types (for a single adult 7746 euro, for a couple 10330 euro, a couple plus one, two or more children around 11362 euro).

\textsuperscript{20} - Also with a simple flat rate tax we can have a progression of the average rates. For instance, where social homogeneity exists, if \( T_i = t \cdot x_i - a \rightarrow AT_i = T_i / x_i = (t \cdot x_i - a) / x_i = (t - a / x_i) \), where \( T_i = \) tax burden for individual i; \( t = \) tax rate; \( x_i = \) pre tax income; \( a = \) lump sum allowance.

\textsuperscript{21} - Usually these deductions (and expenditure-related deductions too) rises along the income scale (Lambert, 2001, p. 182).
In the old legislation this abatement was lower as well as differentiated among different people with dissimilar income sources.

Doubts still survived about the income tax unit types (the plan gave the impression that the preference were for a household income tax) as well as the way by which the abatements applied (universal or selective, that is, always applicable - or not - on income levels higher than the tax free level, equal for any income level or decreasing along the income parade, etc).

Moreover these uncertainties were linked: if the personal income tax is operating, was the exemption for the family unique or depending on the number of the workers in the household? In the latter case which decisions about the economies of scale issue?

Furthermore the tax credits and income related deductions behaviour was not well thought-out totally.

Without this crucial information, different degrees of progression and distributional effects are possible, as well as large differences in the total tax ratio.

Hence, regarding the simulation, some choices among the potential range was compulsory:

- The household as tax unit; selective abatement (following the equivalence scale in accordance with the family size) decreasing above the tax-free income until 10000 Euro when it becomes 0 for a single adult (more for larger family); old c; and d (x_i) structure; finally there applied the new rate structure.

According to Baldini an Bosi’s results: the total tax burden decreases considerably, with revenue forgone of 19579 million of euro approximately, corresponding to 16.5% of the total old tax burden.

By the growth of the exemption level, equal for any income source, the population share with 0 tax liability increases.

The income tax reform is tax augmenting only for middle-income recipients (the average tax rate increasing slightly for the 4th decile, with a tax cut for other deciles).

Hence it is a double-crossing tax reform, with a higher average tax rate fall for the higher income group.

As a consequence the Gini index for the equivalent post tax income rises from 34.9 to 36.49, the Reynolds – Smolensky index decreases, consistent with the total tax ratio fall, and the Π^K index reduction (less disproportionality in the distribution of the tax burden).22

---

22 - The researchers proposed no decomposition of vertical equity, classical horizontal inequity and reranking.
According to Baldini and Bosi, the reduced amount of redistributive effect is due to the highest part of total tax burden cut (a percentage share of 80%) charging on the richest last two deciles.

Immediately a question arises: how to get back the revenue forgone? Two ways are possible if the public expenditure deficit has to remain stable: either the growth of revenue from other types of tax or public expenditures reduction (in such a case, which should be expenditures to cut?).

### 3.3 The Executive Law Proposal

On 21 December 2001 the executive presented a Law proposal about a general fiscal system reform. It can find the fundamental principle of the personal income tax reform within the 3rd article exclusively.

Only now we are sure that tax credits are substituted by income related deductions although without to specify in which way they have to act (no indication on the precise form of d(x)). Note that they have to be concentrated on the low and middle-income group, according to the law.

Furthermore the income tax unit still continues to be uncertain as well as the concept of income. I still go on by using a range of parameters and choices that are compatible with the information summarized in the law article.

Arachi and Zanardi (2002) assumes for the new deduction system:

- A reduction of personal tax liability similar to that which is determined by the actual tax credits system (assumption of equivalent deduction, AED);
- The exemption works for all individuals in the same way;

### MAPP98 model

Sources: Bosi and Baldini 2000

<table>
<thead>
<tr>
<th></th>
<th>Income tax 2000</th>
<th>Estimated Income tax 2001</th>
<th>Tax reform electoral plan</th>
</tr>
</thead>
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<tr>
<td>Total tax burden variation</td>
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<td>-19579</td>
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<td>Average tax rate</td>
<td>0.2093</td>
<td>0.1884</td>
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<td>Gini equivalent pre tax income</td>
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<tr>
<td>Gini equivalent post tax income</td>
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<td>34.86</td>
<td>36.49</td>
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<td>0.0503</td>
<td>0.0340</td>
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<td>Kakwani’s index</td>
<td>0.1914</td>
<td>0.2202</td>
<td>0.1635</td>
</tr>
</tbody>
</table>
There are 3 case studies with 3 distinct abatement amount: 0, 6200 Euro (actual level of exemption), 7750 Euro (considered as the poverty line).

The form of the tax schedule could be: \( s(x_i - a) - c_i \)

where \( c_i \) continues to hold by the AED.

Arachi and Zanardi leave out the income related deductions, paying more attention to the evaluation of tax reform effects concerning the new rate structure and the different level of exemption consequences.

The results are\(^{23}\):

- with \( a = 0 \)
  
  The tax burden goes down by 14000 million of euro (~ 11.5 % of the old total tax burden: this revenue forgone increases sharply together with the growth of the abatement level).

  There is an increase of average rates for the low and middle-income recipients, only the last 3 deciles gain (the top on gaining a 26% tax liability reduction).

- with \( a \neq 0 \)
  
  The tax cut is generalized; tax liabilities diminish for all the income groups.

  The higher the exemptions level the higher this reduction for all the groups. In both cases the highest reduction of average tax rates is assigned to the top decile.

  Neither redistributive effect indexes nor the Gini index are on hand. Furthermore it seems that no equivalence scales are used to deflate pre tax incomes of different family sizes.

  The results are unambiguous only in terms of the tax burden loss (and then the certain course of public spending deficit); in the same time reporting the pattern of the gainers and losers is not sufficient to evaluate the possible social welfare level.

  Also Padoa Schioppa (2002) specifies it has to pay attention to the non-definiteness tax reform as well as the assessment of micro and macro effects resultant from the case studies.

  Concerning the micro appraisal this study focuses on individual and households aspects. The former is distinguished according to the source of incomes, if from employment or self employment: in the old income tax, different tax free level are enjoined from distinct sources, an exemption of 6200 Euro for \( x_E \) and 3100 Euro for \( x_{SE} \).

  The hypothesis is to make this allowance equal to 10300 Euro for everyone: this should lead to higher Horizontal equity.

  The author proposes 3 case studies according to different way of defining abatements:

\(^{23}\) - Tab. 1 and Charts 1 in the Appendix
- Universal: for any individual, for any income level;
- Selective: for any individual, but not above the tax-free quantity;
- Selective moderately: above the tax-free amount decreasing until 30900 euro where it becomes 0.

Also here the equivalent deduction assumption holds: the actual $c_i$ (with old $d(x_i)$) continue to be suitable.

In the first case study the tax liability remains 0 until the new exemption level (favoring actually more the self-employer relatively to the old tax schedule).

The average tax rates decrease for all the population. This variation could be represented by a convex function nearly everywhere: along the income parade the gain is increasing and only for the richest 0.31% population share is it concave.

Total tax burden decreases by 43 %, changing from 117 billions to 67 billions of Euro. More than 85% of households gain by the tax reform and 15 % are indifferent (the lower incomes that already have not tax liabilities).

For selective case, for incomes immediately above the tax-free quota, it has the poverty trap problem (and reranking). For the middle-income group, the average tax rates increase, while for other income levels there is a gain: consistent with these results this should be a double-crossing reform.

The households’ share that has higher tax liabilities is approximately equal to the share with lower tax burdens and the lower incomes continue to be quite indifferent to the tax reform. The revenue forgone is very much less relatively to the first case, only 6.5 % of the old total tax burden.

Finally Padoa Schioppa notes that losses are not compatible with the law article (which declare that no one may have a loss). Thus the last third case could be considered an average between previous cases, in the same time more consistent to the objective to concentrate exemptions on the low and middle-income group.

Now the loss in terms of total tax burden is around 23.5 % giving a value of 23 billion of Euro. No households lose, 14 % are indifferent, 86 % clearly not (especially the higher income level).

In all cases:
- the low-income individuals below the tax-free quota are not able to use it fully, because the allowance is not a lump sum transfer and, observing the different results for different

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24 - Tab. 2.
income groups, we may infer that the degree of inequality of post tax incomes increases probably. No indexes of redistributive effect are presented.

Finally, the paper defines the macroeconomic effects positively, according to the demand effect (by Kahn’s multiplier) and supply side effect: they should be able to cover a large part of revenue forgone as well as expand the economy. I agree with regard to the word “should”, for the supply side effect especially: the well-known reservations about the current position on the Laffer curve, the sign (and the absolute value) of the derivative in the Slutsky’s equation (the income effect) or the sign of elasticity of labour supply don’t allow to be sure.

The last paper (Baldini and Bosi 2002) of this section can be considered the one that pay more attention on the redistributive characteristics researching to encapsulate the tax schedule, together with the pre tax distribution, in a single number, a progressivity index.

The case studies work by using the deductions rather than tax credits. The former are divided in three typologies looking at

- Household size (there is an exemption for a single adult as a function of the poverty level - 6000 Euro - equal for everyone, scaled up by the ISE equivalent scale for increasing family size);
- Income related deductions (the old tax credits become d (x₁) by the rule: $c_i \times 23\%$);
- Production costs for xₑ and xₛₑ: 500 Euro for all income sources.

Eight cases are presented according to dissimilar tax-free quota assumptions: the first (no deductions at all) and the last (AED) as limiting cases, selective, universal, selective decreasing (with different level where it becomes 0), 80% of aₖ level, etc.

Table 4 and Chart 2 show the average marginal rate for deciles for some of the case studies. In contrast with the other papers, there are positive variations especially for the middle-income group. This is not feasible in accordance with the tax reform objectives: as a consequence the 9th article says that in case of reduction of household post incomes, the Budget Ministry may modify the rate structure or other tax types to avoid those losses. The researchers note that this discretionarity is really not common, with no similar cases in the history of our fiscal system. They also note the risk that tax-free amount may be taken up partly by the lower income group.

All case studies show an augment Gini’s index, then a higher inequality in the post reform income distribution and, excluding the no deduction and selective cases, a total tax burden cut.

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25 - Tab. 3
26 - The reason could be the way by which $c_i$’s are transformed in $d (x_i)$: $d (x_i) = c_i \times 23\%$
The same reservations about the macro effect are proposed by Baldini and Bosi, especially about the supposed higher labour supply propensity.

3.4 Falsitta Case Studies

On 1 May 2002 the member of Assembly V. Falsitta presented some case studies (Tab. 5) where promising scenarios for 5 different household typologies are illustrated (different incomes, different size, one or two incomes into the household).

By using them it can infer which may be the parameters of the functional forms of the tax schedule for the three marginal rates, the implicit 0, 23% and 33%.

Baldini et al. (2002) do it and it obtains higher certainty (whether the case studies will be real at the end of transition process), avoiding the larger range of choices compatible with 3rd article explicit statements. Some points are still open to discussions and give no absolute confidence to conclusions.

However, now only one case study is defined.

To summarize, by Falsitta’s relation the income tax continues to be personal and two kinds of deductions work.

The abatement \( (d_1) \), with purposes of vertical equity, may be considered equal for all income sources\(^{27}\) and decreasing along the income line (note that this exemption is not scaled down according to the economies of scale existing usually in a household\(^{28}\)).

The second \( (d_2) \), coming from old tax credits, is equal to \( c_i / 23\% \) where the \( c_i \)'s are of two types, the first, \( (c_{i1}) \) fully operating for family with more than one individual (according to the family size) and not relating with the income level, the second, \( (c_{i2} = x_i \times 19\%) \) for individual expenditures or donations and so on.

Until that time the family size matter was considered by using the tax credits: now they are substituted by equivalent deductions, with similar impact on the individual tax liability. Thus the change could be considered only formal, being \( d_2 = c_i / 23\% \).

\(^{27}\) - By the fact that the Executive case studies do not specify the income source of family type.

\(^{28}\) - If two workers are in a family, without any children, the scale coefficient simply doubles. No applied equivalence scale in any country has so high coefficient.

\(^{29}\) - To check if this interpretation is correct, it may proceed by using the case studies presented by the Executive, by following \( \text{taxable income } = x_t = \text{(pre tax income } - d_1 - c_i / 23\% \text{)} \); \( \text{tax liability } = x_t \times \text{marginal rate} \) e.g. for the first case study (the values are in old millions of £):

\[
(35 - 9.48 - 2.05 \div 0.23) = 16.5742668 \text{ (taxable income)}; 16.5742668 \times 0.23 = 3.812081 \text{ (tax liability)}.
\]

This tax liability amount is exactly the amount indicated by the 1st case study. To find the exemption level \( d_1 \) see the next footnote.
A little misunderstanding remains about the $c_{ij}$ term because no clear indications are given in Falsitta’s case study. Hence the treatment is assumed equal for both $c_{ii}$ ($n = 1, 2$).

The explicit formula is obtained, following Baldini et al. (2002), by using two examples. With a simple proportion

$$\text{tax liability : marginal rate } = \frac{x_r}{100}$$

I derive the taxable income ($= x_T$) and subtracting this from the pre tax income it can infer the exemption amount\(^{30}\). Making it for two levels of incomes and using two resulting exemption amount (together with the two pre tax incomes) it may identify the negative slope of the corner of the decreasing linear function $d_1$. To derive the two intercepts, it can use the pre tax income, the magnitude and the obtained tax-free quota for the $y$ line intercept and the consequent formula for the income level where exemption is 0.

Finally it can find the income level below which the tax liability is 0, by the cross between the derived line and the $45^\circ$ line. Thus (the values are in old millions £):

$$d_1 = x \quad \text{if } x \leq 12.214$$

$$d_1 = 13.680 - 0.12 \times x \quad \text{if } 12.214 < x \leq 114$$

$$d_1 = 0 \quad \text{if } x > 114$$

The resulting form of $d_1$

\[d_1\]

Thus tax liability ($T_i$) should be (the values are in old millions £):

$$T_i = 0 \quad \text{if } x \leq 12.214$$

\(^{30}\) - e.g. for the first case study (the values are in old millions of £Lire)

\[5,8696; 23 = x_T : 100 \quad ; \quad x_T = 25.2;\]

\[35 \text{ (gross pre tax income)} - 25.2 \text{ (taxable income)} = 9.48 \text{ = exemption quota}.\]
\[ T_1 = 0.23 \left( x - d_1 - d_2 \right) \quad \text{if} \quad 12.214 < x \leq 114 \]
\[ T_1 = 0.23 \left( x - d_2 \right) \quad \text{if} \quad 114 < x \leq 193.627 \]
\[ T_1 = 0.23 \left( 193.627 - d_2 \right) + 0.33 \left( x - 193.627 \right) \quad \text{if} \quad x > 193.627 \]

From the result of Baldini et al. the total tax burden loss is 30.47 billion of Euro (approximately 100% more than the Executive estimates).

Already looking at the charts 3 and 4 by means of the *equivalent* household post tax income notion, it notes that all population shares gain by the fall in the tax liabilities and the average tax rates behavior along the income parade continues to depart from proportionality, but in a different way, probably becoming structurally less progressive (post tax income gains higher for the top deciles).

This is confirmed by the Reynolds-Smolensky index.

\[
G_{\text{pre}} - G_{\text{post}} = \Pi^{RS} = \Pi^K * \frac{g}{(1-g)} - R
\]

<table>
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<tr>
<th>Old Tax</th>
<th>Gini pre</th>
<th>- Gini post</th>
<th>( \Pi^{RS} )</th>
<th>( \Pi^K )</th>
<th>* ( \frac{g}{(1-g)} )</th>
<th>- R</th>
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<td>Tax Ref.</td>
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<td>0.0374</td>
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<td>0.0247</td>
<td>0.1992</td>
<td>0.1283</td>
<td>0.000857</td>
</tr>
</tbody>
</table>

Sources: Bosi et al. 2002

These results show a fall in \( \Pi^{RS} \) index according to the decrease of \( g \) (total tax ratio) and \( \Pi^K \). The lower reranking effect is not sufficient to overcome these reductions.

In conclusion even if the Executive objective is to pay more attention to the low and middle-income group, the researchers affirm that 55% of the total tax burden loss goes to incomes higher than 50000 Euro (not more than 5% of the total population). Together with the negative variation of \( \Pi^{RS} \) index, this allows to affirm that the redistribution continues to go from the richer group to the poorer, but with a reduction of intensity\(^{31} \).

Clearly, any analysis based on redistributive indexes may be accepted according to a *given* F(x), that is a given distribution of pre tax (*equivalent*) incomes.

### 4 - Non-Progression Neutral Tax Cut: Conclusive Remarks

In the literature, outcomes about the vertical distance between \( L_{x,T}^1 \) and \( L_{x,T}^2 \) for all \( p \) (where \( L_{x,T}^2 \) is the Concentration curve for the post reform post tax incomes) and then about

\(^{31} \) According to the fact that this tax reform does not lead to a negative income tax, 20% of poorest families gain only 2% of the total tax burden fall.
distributional effects, have been obtained taking into account the pre tax income distribution in two different ways.

According to Jakobsson (1976, proposition 1) it is possible to state that

\[ \text{RP}^2 \leq \text{RP}^1 \quad \forall x \quad \text{iff} \quad L^2_{x-T} \geq L^1_{x-T} \quad \text{for every pre tax distribution } F(x) \]

In order to investigate the distributional effect of different schedules only the information about magnitudes of marginal and average rates is required.

On the other hand consistent with Hemming and Keen (1983, proposition 1), when two income taxes \((T^2, T^1)\) raise the same revenue\(^{32}\)

"an income tax \((T^2)\) is more progressive than another \((T^1)\) for a given pre tax distribution \(F(x)\) if and only if the post tax income function \((x - T^2)\) single crosses from above the post tax income function \((x - T^1)\) on some interval \(Y = [x_{\text{min}}, x_{\text{max}}]\)."

In the former case it obtains a global result and it can define a comparison between any two schedules by using their specifications, verifying if the residual progression is increased or not for all \(x\).

In the latter, relative distributional implications of alternative taxes can be defined also if it verifies a single-crossing condition between the two post tax income schedules conditional on the pre tax distribution\(^{33}\). For \(g^1 = g^2\), the two curves must intersect - a pure-redistributive case - as a result the only remaining trouble is just find which is the curve that single crosses the other from above.

Hence when this distribution is such that tax schedules are equal-yield also for Hemming and Keen the only necessary information comes from the form of \(T^i\) \((i = 1, 2, \text{in our case})\).

In contrast when non-equal yield taxes have to be compared, the post tax schedules may not necessarily cross. In such a case Hemming and Keen were able to define a transformation of post tax schedules that involves a new type of single-crossing condition.

Let \([x - t^2(x)]\) be the post tax reform income schedule, \([x - t^1(x)]\) the old schedule, \(L^2_{x-T}\) and \(L^1_{x-T}\) their respective Lorenz curves\(^{34}\). The condition of Lorenz domination (LD) of new schedule over the old one, for the equal-yield case is, for all \(v\) those belong to \(Y\)

\[
\int_{x_{\text{min}}}^{y} [x - t^2(x)] f(x) \, dx / \int_{x_{\text{min}}}^{y} f(x) \, dx \geq \int_{x_{\text{min}}}^{y} [x - t^1(x)] f(x) \, dx / \int_{x_{\text{min}}}^{y} f(x) \, dx
\]

\(^{32}\) - This proposition may be adapted if it wish to follow Kakwani (1977) approach, relating progressivity to the distribution of the tax burden.

\(^{33}\) - If the elasticity condition of Jakobsson theorem holds, it is well known that this implies at most a single crossing, while the reverse is not true.

\(^{34}\) - To simplify we assume \(L^i_{x-T}\) \((i = 1, 2)\) as a Lorenz curve.
By assuming $g^1 = g^2$, 

$$\int_y [x - \hat{r}^2(x)] f(x) \, dx = \int_y [x - \hat{r}'(x)] f(x) \, dx$$

Hence only the numerators are relevant for Hemming and Keen’s proof.

By following a normalization procedure for both sides it can deal, at least for the sufficiency condition, with a non equal-yield case ($g^1 \neq g^2$) as an equal-yield one, that is acting on the post tax income function in such a way to have

$$\int_{x_{\min}}^{v} \left\{ \frac{[x - \hat{r}(x)]}{\int_y [u - \hat{r}'(u)] f(u) \, du} \right\} f(x) \, dx = \int_{x_{\min}}^{v} \left[ x - \hat{r}(x) \right]_N f(x) \, dx$$

where $\left[ x - \hat{r}(x) \right]_N = \left[ x - \hat{r}(x) \right] / \int_y [u - \hat{r}'(u)] f(u) \, du$ indicates the share of total net income associated with pre tax income $x$ under $\hat{r}(x)$ given $F(x)$.

Making the same operation on the denominator:

$$\int_y \left\{ \frac{[x - \hat{r}(x)]}{\int_y [u - \hat{r}'(u)] f(u) \, du} \right\} f(x) \, dx = \left[ 1 / (1 - g) \right] \int_y \left[ x - \hat{r}(x) \right] f(x) \, dx = \left[ 1 / (1 - g) \right] (1 - g) = 1$$

i = 1, 2

and it can rewrite the condition for LD as:

$$\int_{x_{\min}}^{v} \left[ x - \hat{r}(x) \right]_N f(x) \, dx \geq \int_{x_{\min}}^{v} \left[ x - \hat{r}'(x) \right]_N f(x) \, dx \quad \text{for all } v \in Y$$

In this way Hemming and Keen show how also in this case it is possible to make use of the same proof for the sufficiency, as when the total tax ratios are equal. Their propositions is necessary with respect to the Italian tax reform under analysis. Even if I have not many data it may infer some interesting information just looking at this chart:

**Average tax rates for household equivalent post tax income**

![Average tax rates chart](chart.png)

*Sources: Baldini-Bosi-Matteuzzi 2002*
By the decrease of average rates for all the deciles, post tax reform household equivalent incomes may be considered higher than under the old schedule. This chart seems to involve a tax cut for all the families, increasing along the income parade: the progressivity changes.

On the other hand, for instance, with a residual progression neutral tax cut the gain in percentage terms has to be equal for every households, in accordance with:

\[ t'(x) = t'(x) - a [x - t'(x)] \quad \text{or} \quad t'(x) / x = t'(x) / x - a [1 - t'(x) / x] \quad \text{where} \quad a > 0 \]
and \[ x - t'(x) = [1 + a] [x - t'(x)] \]

This residual progression neutral tax cut would push down the old piecewise linear curve for the average rates by the term \[ a [1 - t'(x)/x] \]

The elasticity RP for every deciles would remain constant, \[ [1 - t'(x)] / [1 - t(x)/x] \], then also the slope: the vertical distance between the two curves would be equal.

If it imposes an equal-yield \( g^r = g^2 \) residual progression neutral tax cut, this leads to an RP neutral curve of average rates that single crosses once from below the tax reform schedule of average rates \( t^2(x) / x \).

Moreover the Lorenz curves with respect to the old schedule and the RP neutral tax cut are exactly superimposes.

More generally it can be shown that \[ a = (g^1 - g^r) / (1 - g^1) \]
Then \[ x - t'(x) = [x - t'(x)] + [(g^1 - g^r) / (1 - g^1)] [x - t'(x)] = [x - t'(x)] [1 + (g^1 - g^r) / (1 - g^1)] = [x - t'(x)] [(1 - g^r) / (1 - g^1)] \]

Hence, by using the fact that \( g^r = g^2 \) it can compare the two schedules

\[ x - t^2(x) \quad \text{with respect to} \quad [x - t'(x)] [(1 - g^r) / (1 - g^1)] = x - t'(x) \]

In this case the proposition changes and it can be shown that:

"If \[ x - t^2(x) / (1 - g^2) \] crosses \[ x - t'(x) / (1 - g^1) \] once from above, then \[ L^2_{X - T}(p) \geq L^1_{X - T}(p) \] for all \( p \in [0, 1] \)."\(^{35}\)

---

\(^{35}\) - Where, as before, \( [x - t^i(x)] / (1 - g^i) = [x - t'(x)]_i \quad (i = 1, 2) \), is the share of total net income associated with pre-tax income \( x \) under \( t'(x) \) given the \( F(x) \).
In our case the reality seems to contradict this proposition because, like in a mirror, if the RP
neutral curve of average rates single crosses once from below the tax reform schedule of
average rates, the RP neutral post tax income curve single crosses once from above the tax
reform schedule of post tax income, while to ensure LD for the distribution of net incomes
under the tax reform over the distribution of net incomes under the old schedule, it should be
the opposite: the Executive’s policy seems apparent.

From another point of view, it finds that the necessary condition of Latham (1988, theorem
5) may be verified.

Latham states that ($a^1 = \text{distribution of net incomes under the two schedules}$):

“given that $g^2 < g^1$, $a^2$ L.D $a^1$ only if there exists an interval $(x_{\text{min}}, x^*)$ over
which $t^1(x) < t^2(x)$, where $x_{\text{min}} < x^* < x_{\text{max}}$.”

Reasonably, observing the chart this interval seems to exist, but it is not enough to ensure
Lorenz dominance: in this case it appears that after-tax incomes are higher everywhere, as
allowed by the theorem.

Hence, apparent results can be obtained by the use of these theorems. What the practitioner
needs is either the functional form of the tax, or post tax, schedules, or these functional forms
together with the total net income, as in Hemming and Keen’s non-equal yield case.

To summarize:

- According to Baldini et al. (2002) the total tax burden loss is 30,47 billion of Euro
  (around 100% more than the Executive estimates);
- The inequality of the distribution of the net incomes increases with respect to the old
distribution of after-tax incomes, as a result of both total tax ratio and
disproportionality of tax burden (progression) reductions;
- Assessing by the analysis of this section, the result of a lower Reynolds-Smolensky
  index finds confirmation by the unverified Hemming-Keen’s single-crossing condition,
  useful especially to work when it is not possible to assess effects by using the
  Jakobsson’s elasticity condition;
- If one of the objectives of the Executive is to decrease the total tax ratio, a RP neutral
tax cut for all could be regarded as a possibility to increase social welfare. By Baldini
et al. (2002) there is evidence that other strategies are possible to avoid the
progressivity reduction and, at the same time, to cut in the same way the total tax
burden.
All the results of these chapters are correct in a partial equilibrium framework: the distribution of pre tax income is assumed to be independent of the tax code in operation and, of course, it has to be convincing that - using a utility function where the (equivalent) incomes are the only input - a social welfare function such as

$$W_F = \int_0^y U(x) f(x) \, dx$$

is related with the actual well-being of the society.

**REFERENCES**


APPENDIX

Tab 1.
Average tax rate according to pre tax income deciles 2001

<table>
<thead>
<tr>
<th>Reddito imponibile (euro)</th>
<th>Aliquota media</th>
</tr>
</thead>
<tbody>
<tr>
<td>Decile</td>
<td>Limite inf.</td>
</tr>
<tr>
<td>I</td>
<td>0</td>
</tr>
<tr>
<td>II</td>
<td>4070</td>
</tr>
<tr>
<td>III</td>
<td>5990</td>
</tr>
<tr>
<td>IV</td>
<td>9012</td>
</tr>
<tr>
<td>V</td>
<td>12051</td>
</tr>
<tr>
<td>VI</td>
<td>14798</td>
</tr>
<tr>
<td>VII</td>
<td>17252</td>
</tr>
<tr>
<td>VIII</td>
<td>20828</td>
</tr>
<tr>
<td>IX</td>
<td>25681</td>
</tr>
<tr>
<td>X</td>
<td>oltre 34512</td>
</tr>
</tbody>
</table>

Reddito medio 18429

Charts 1. Average tax rates pre and after tax reform

Sources: Arachi-Zanardi
Tab. 2.  
Comparison actual norm (A) Vs tax reform (with different exemption rules: universal (B), selective (C), decreasing selective (D))

<table>
<thead>
<tr>
<th></th>
<th>(A)</th>
<th>(B)</th>
<th>(C)</th>
<th>(D)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>12</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>22</td>
<td>2.65</td>
<td>18%</td>
<td>12.75%</td>
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<tr>
<td></td>
<td>30</td>
<td>4.65</td>
<td>24%</td>
<td>16.56%</td>
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<tr>
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<td>60</td>
<td>14.65</td>
<td>32%</td>
<td>24.75%</td>
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<td>150</td>
<td>44.75</td>
<td>39%</td>
<td>33.16%</td>
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<td>200</td>
<td>74.0</td>
<td>45%</td>
<td>37.06%</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>119.0</td>
<td>45%</td>
<td>59.67%</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(A)</td>
<td>(B)</td>
<td>(C)</td>
<td>(D)</td>
</tr>
<tr>
<td></td>
<td>6</td>
<td>0.0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>20</td>
<td>5.4</td>
<td>18%</td>
<td>17.00%</td>
</tr>
<tr>
<td></td>
<td>30</td>
<td>9.8</td>
<td>24%</td>
<td>19.33%</td>
</tr>
<tr>
<td></td>
<td>60</td>
<td>15.5</td>
<td>32%</td>
<td>25.93%</td>
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<tr>
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<td>150</td>
<td>44.9</td>
<td>30%</td>
<td>33.22%</td>
</tr>
<tr>
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<td>200</td>
<td>74.1</td>
<td>45%</td>
<td>37.06%</td>
</tr>
<tr>
<td></td>
<td>300</td>
<td>119.1</td>
<td>45%</td>
<td>39.70%</td>
</tr>
</tbody>
</table>

Fonte: elaborazioni IAE.
(1) Calcolata al netto delle dimissioni per redditi da lavoro.

Sources: Padoa Schioppa

Tab. 3
Total tax burden and redistributive effects of case studies

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Irpef 2002</td>
<td>226783</td>
<td>34.03</td>
<td>0.00</td>
</tr>
<tr>
<td>Nessun' esenzione</td>
<td>266167</td>
<td>37.63</td>
<td>3.60</td>
</tr>
<tr>
<td>Soglia esente</td>
<td>236533</td>
<td>36.52</td>
<td>2.50</td>
</tr>
<tr>
<td>Deduzione Universale</td>
<td>150880</td>
<td>-75903</td>
<td>35.17</td>
</tr>
<tr>
<td>Deduzione selettiva a 200 m.</td>
<td>160522</td>
<td>35.21</td>
<td>0.96</td>
</tr>
<tr>
<td>Deduzione selettiva a 60 m.</td>
<td>594879</td>
<td>35.21</td>
<td>0.96</td>
</tr>
<tr>
<td>Deduzione Programma Polo a 60 m.</td>
<td>203283</td>
<td>35.13</td>
<td>1.10</td>
</tr>
<tr>
<td>Detrazioni attuali trasformate in deduzioni</td>
<td>188003</td>
<td>35.30</td>
<td>1.36</td>
</tr>
</tbody>
</table>

Sources: Baldini-Bosi 2002
Tab. 4
Marginal tax rates average values for household equivalent income deciles

<table>
<thead>
<tr>
<th>Decile</th>
<th>irpef 2002</th>
<th>deduzione selettiva graduale</th>
<th>detrazioni attuali trasformate in deduzioni</th>
<th>Programma Polo</th>
<th>80% graduale</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>3.4%</td>
<td>0.8%</td>
<td>2.4%</td>
<td>2.0%</td>
<td>1.4%</td>
</tr>
<tr>
<td>2</td>
<td>17.1%</td>
<td>10.8%</td>
<td>13.9%</td>
<td>14.4%</td>
<td>18.2%</td>
</tr>
<tr>
<td>3</td>
<td>23.6%</td>
<td>21.4%</td>
<td>18.2%</td>
<td>22.3%</td>
<td>24.4%</td>
</tr>
<tr>
<td>4</td>
<td>24.1%</td>
<td>24.3%</td>
<td>19.5%</td>
<td>25.4%</td>
<td>26.7%</td>
</tr>
<tr>
<td>5</td>
<td>26.3%</td>
<td>25.8%</td>
<td>20.7%</td>
<td>26.9%</td>
<td>26.9%</td>
</tr>
<tr>
<td>6</td>
<td>27.8%</td>
<td>27.2%</td>
<td>21.0%</td>
<td>27.9%</td>
<td>27.9%</td>
</tr>
<tr>
<td>7</td>
<td>28.7%</td>
<td>27.9%</td>
<td>21.6%</td>
<td>28.5%</td>
<td>27.8%</td>
</tr>
<tr>
<td>8</td>
<td>30.4%</td>
<td>27.9%</td>
<td>21.9%</td>
<td>27.8%</td>
<td>27.7%</td>
</tr>
<tr>
<td>9</td>
<td>32.2%</td>
<td>26.1%</td>
<td>22.2%</td>
<td>26.8%</td>
<td>26.4%</td>
</tr>
<tr>
<td>10</td>
<td>36.5%</td>
<td>25.8%</td>
<td>23.3%</td>
<td>25.4%</td>
<td>24.9%</td>
</tr>
</tbody>
</table>

**Total** 24.9%  22.0%  18.5%  22.7%  23.2%

Sources: Baldini-Bosi 2002

Chart 2
Marginal tax rates average values for household equivalent income deciles

Sources: Baldini-Bosi 2002

Chart 3.
Average tax rates over individual taxable income

Sources: Baldini-Bosi-Matteuzzi 2002
Chart 4
Percentage variation of household equivalent post tax incomes as a result of tax reform, per deciles of household equivalent post tax incomes

Sources: Baldini-Bosi-Matteuzzi 2002

Tab. 5
Falsitta’s case studies

<table>
<thead>
<tr>
<th>Income 35 mil Lire – single income – 2 dependants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre tax income</td>
</tr>
<tr>
<td>35 mil</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Income 70 mil Lire – single income – 3 dependants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre tax income</td>
</tr>
<tr>
<td>70 mil</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Income 70 mil Lire – two income – 2 dependants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre tax income</td>
</tr>
<tr>
<td>35 mil</td>
</tr>
<tr>
<td>35 mil</td>
</tr>
<tr>
<td>35 mil</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Income 140 mil Lire – single income – 2 dependants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre tax income</td>
</tr>
<tr>
<td>140 mil</td>
</tr>
<tr>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Income 140 mil Lire – two income – 2 dependants</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pre tax income</td>
</tr>
<tr>
<td>70 mil</td>
</tr>
<tr>
<td>70 mil</td>
</tr>
<tr>
<td>70 mil</td>
</tr>
<tr>
<td>70 mil</td>
</tr>
</tbody>
</table>

Sources: Budget Ministry 2002