

Notation

Let Ω be an open set of \mathbb{R}^N , $1 \leq p < +\infty$, $k, N \in \mathbb{N}$, $0 < \alpha < 1$, $T > 0$, $a < b$.

$ x $	euclidean norm of $x \in \mathbb{R}^N$;
$\langle x, y \rangle$	euclidean inner product in \mathbb{R}^N ;
$B(x, r)$	open ball in \mathbb{R}^N centered in x with radius $r > 0$;
B_r	$B(0, r)$;
Q	$(0, T) \times \Omega$;
$\partial'Q$	$(0, T) \times \partial\Omega \cup \{0\} \times \bar{\Omega}$;
$\partial_{tx}Q$	$\{0\} \times \partial\Omega$;
$\text{card } J$	cardinality of a given set J ;
$ J $	Lebesgue measure of a given set J ;
J^c	complementary set of J ;
χ_J	characteristic function of a set J , that is the function defined as $\chi_J(x) = 1$ if $x \in J$ and $\chi_J(x) = 0$ if $x \notin J$;
1	characteristic function of \mathbb{R}^N ;
$\text{supp } u$	support of a given function u ;
D_t	partial derivative with respect to the variable t ;
D_i	partial derivative with respect to x_i ;
D_{ij}	$D_{x_i x_j}$;
Du	space gradient of a real-valued function u with norm $ Du ^2 = \sum_{i=1}^N (D_i u)^2$;
D^2u	Hessian matrix of a real-valued function u with respect to the space variables with norm $ D^2u ^2 = \sum_{i,j=1}^N (D_{ij}u)^2$;
$C_c^\infty(\Omega)$	space of real-valued C^∞ functions with compact support in Ω ;
$C_b(\bar{\Omega})$	space of bounded continuous functions in $\bar{\Omega}$;

$C_b^k(\overline{\Omega})$	space of real-valued functions with derivatives up to order k in $C_b(\overline{\Omega})$;
$C_0(\Omega)$	space of functions in $C_b(\overline{\Omega})$ vanishing at $\partial\Omega$ and at infinity;
$C_0(\mathbb{R}^N)$	space of functions in $C(\mathbb{R}^N)$ vanishing at infinity;
$C^1(\mathbb{R}^N; \mathbb{R}^N)$	space of functions $F = (F_1, \dots, F_N)$ such that $F_i \in C^1(\mathbb{R}^N)$, for every i ;
$C^{1,2}((a, b) \times \Omega)$	space of functions $u(t, x)$ which are continuous in $(a, b) \times \Omega$ with their indicated derivatives (not necessarily bounded);
$C^{k+\alpha}(\Omega) = C^{k+\alpha}(\overline{\Omega})$	space of functions such that the derivatives of order k are α -Hölder continuous in Ω ;
$C^{1+\alpha/2, 2+\alpha}((a, b) \times \Omega)$ $= C^{1+\alpha/2, 2+\alpha}([a, b] \times \overline{\Omega})$	space of functions $u = u(t, x)$ such that $D_t u$ and $D_{x_i x_j} u$ are α -Hölder continuous in $(a, b) \times \Omega$ with respect to the parabolic distance $d((t, x), (s, y)) = t - s ^{1/2} + x - y $;
$C_{\text{loc}}^{1+\alpha/2, 2+\alpha}((0, +\infty) \times \overline{\Omega})$	space of functions u such that $u \in C^{1+\alpha/2, 2+\alpha}([\varepsilon, T] \times \overline{\Omega}')$, for all $0 < \varepsilon < T$ and bounded open $\Omega' \subseteq \Omega$;
$C_{\text{loc}}^{1+\alpha}(\overline{\Omega})$	space of the functions which belong to $C^{1+\alpha}(\overline{\Omega}')$, for all bounded open set $\Omega' \subseteq \Omega$;
$C^k(\overline{\mathbb{R}})$	space of continuous functions with finite limits at $\pm\infty$ together with their derivatives up to order k ;
$\ \cdot\ _\infty$	sup-norm;
$\ u\ _{[a, b]}$	$\sup_{x \in [a, b]} u(x) $;
$\ u\ _{C^{\frac{\alpha}{2}, \alpha}(]0, T[\times \Omega)}$	$\ u\ _\infty + [u]_{C^{\frac{\alpha}{2}, \alpha}(]0, T[\times \Omega)}$;
$[u]_{C^{\frac{\alpha}{2}, \alpha}(]0, T[\times \Omega)}$	$\sup_{\substack{t \in]0, T[, \\ x, y \in \Omega, \\ x \neq y}} \frac{ u(t, x) - u(t, y) }{ x - y ^\alpha} + \sup_{\substack{t, s \in]0, T[, \\ t \neq s, \\ x \in \Omega}} \frac{ u(t, x) - u(s, x) }{ t - s ^{\frac{\alpha}{2}}}$;
$\ u\ _{1,2}$	$\ u\ _\infty + \ u_t\ _\infty + \ Du\ _\infty + \ D^2 u\ _\infty$;
$[u]_{1+\frac{\alpha}{2}, 2+\alpha}$	$[u_t]_{\frac{\alpha}{2}, \alpha} + [D^2 u]_{\frac{\alpha}{2}, \alpha}$;
$\ u\ _{1+\frac{\alpha}{2}, 2+\alpha}$	$\ u\ _{1,2} + [u]_{1+\frac{\alpha}{2}, 2+\alpha}$;
$(L^p(\Omega), \ \cdot\ _p)$	usual Lebesgue space;
$(W^{k,p}(\Omega), \ \cdot\ _{k,p})$	usual Sobolev space;
$W_{\text{loc}}^{k,p}(\Omega)$	space of functions belonging to $W^{k,p}(\Omega')$ for all bounded open set Ω' such that $\overline{\Omega'} \subset \Omega$;
$W_0^{k,p}(\Omega)$	closure of $C_c^\infty(\Omega)$ in $W^{k,p}(\Omega)$;
$\mathcal{M}(\mathbb{R}^N)$	set of all Borel probability measures in \mathbb{R}^N .