Abstract

This dissertion is devoted, mainly, to the study of copulas. Specifically, a copula is the restriction on the n–cube $[0,1]^n$ ($n \geq 2$) of an n–dimensional distribution function (=d.f.) with marginal d.f.'s uniformly distributed on $[0,1]$. A copula is uniquely associated with an n–dimensional vector of continuous random variables and describes its dependence properties. One of the main reasons of the interest of statisticians to copulas consists in the fact that the construction of multivariate d.f.'s (and, hence, of models describing random phaenomena) can be divided into two easier steps: the construction of the marginal d.f.'s and the construction of a suitable family of copulas.

The major part of the presented results is devoted to the construction of bivariate d.f.'s by means both of the construction of new families of copulas (depending on one or more parameters) and of the introduction of new construction methods that allow to associate with two d.f.'s (or copulas) already known another function in the same class.

In particular, three families of copulas are introduced. The first family is suitable to describe bivariate systems with positive dependence. The second one is connected to two random variables X and Y such that the behaviour of their maximum, $\max\{X,Y\}$, is known. The third one, which does not have a probabilistic interpretation, generalizes the family of Archimedean copulas that are largely used in applications thanks to their great flexibility.

Moreover, for all d.f.'s A and B and for every binary operation H on $[0,1]$, we characterize the d.f.'s F pointwise induced by A and B, viz. $F(x,y) = H(A(x,y), B(x,y))$. Such characterization needs of the definition of a new property of bivariate functions, called "P–increasing", which generalizes the concept of supermodularity. A slight modified form of this operation is, hence, considered in the class of copulas, where we give a method for adding parameters to a copula in order to transform it into another copula suitable to describe also multivariate models whith non–exchangeable random variables.

Recently, the copulas have been also used in the definition of the concepts of reliability and aging for exchangeable bivariate system. In particular, Bassan and Spizzichino (2005) introduced the so–called "bivariate aging function", which allows to define in the bivariate case some (univariate) concepts of reliability already known,

like, for instance, IFR, DFR and NBU. Such a function is called "semicopula" because it verifies some, but not all, properties of a copula. The class of semicopulas and its properties are studied in details by underlining the rôle that such functions have also in multivalued logic and in the theory of fuzzy measures (also called capacities). A method of transforming a (semi–) copula into another one is then considered and its use in extreme value theory is underlined. In connection with reliability theory, we study also the Schur–concavity in the class of copulas.

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