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A note on A-splitting and A-admissible topologies on function spaces

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Abstract. In [3] some characterizations of **F**-splitting and **F**-admissible topologies on function spaces are given, where **F** is a finite space. Here, we note that these results remain true if we consider Alexandroff spaces (that is, spaces with the property that the intersection of any number of open sets is open) instead of finite spaces.

Keywords: function spaces, **A**-splitting topology, **A**-admissible topology

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1 Preliminaries

Let Y and Z be two fixed topological spaces. By $C(Y,Z)$ we denote the set of all continuous maps of Y into Z. If t is a topology on the set $C(Y,Z)$, then the corresponding topological space is denoted by $C_t(Y,Z)$.

Let X be a space and $F : X \times Y \to Z$ a continuous map. By F_x, where $x \in X$, we denote the continuous map of Y into Z, for which $F_x(y) = F(x,y)$, for every $y \in Y$. By $\widehat{F}$ we denote the map of X into the set $C(Y,Z)$, for which $\widehat{F}(x) = F_x$, for every $x \in X$.

Let G be a map of the space X into the set $C(Y,Z)$. By $\widetilde{G}$ we denote the map of the space $X \times Y$ into the space Z, for which $\widetilde{G}(x,y) = G(x)(y)$, for every $(x,y) \in X \times Y$.

A topology t on $C(Y,Z)$ is called *splitting* if for every space X, the continuity of a map $F : X \times Y \to Z$ implies that of the map $\widehat{F} : X \to C_t(Y,Z)$. A topology

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t on $C(Y,Z)$ is called *admissible* if for every space X, the continuity of a map $G : X \to C_t(Y,Z)$ implies that of the map $\widetilde{G} : X \times Y \to Z$ (see [1]).

If in the above definitions it is assumed that the space X belongs to a given family $\mathcal{A}$ of spaces, then the topology τ is called $\mathcal{A}-$*splitting* (respectively, $\mathcal{A}-$*admissible*). If $\mathcal{A} = \{X\}$, where X is a space, then instead of $\mathcal{A}$-splitting and $\mathcal{A}$-admissible we write X-splitting and X-admissible, respectively (see [2]).

2 On A-splitting and A-admissible topologies

1 Notation. In the present paper, an ordinal is identified with the set of all ordinals less than this ordinal and a cardinal is identified with the least ordinal of this cardinality.

By $\mathbf{A}$ we denote a space such that each its point has a minimal open neighbourhood, that is $\mathbf{A}$ is an *Alexandroff space.* Clearly, this minimal open neighbourhood is the intersection of all open neighbourhoods of the point. In what follows, we identify the set $\mathbf{A}$ with the set ν, where ν is the cardinality of $\mathbf{A}$.

Below, the results of [3] concerning $\mathbf{F}$-splitting and $\mathbf{F}$-admissible topologies on $C(Y,Z)$, where $\mathbf{F}$ is a finite space, are generalized to $\mathbf{A}$-splitting and $\mathbf{A}$-admissible topologies, where $\mathbf{A}$ is an Alexandroff space (which may be infinite). We give only formulation of the results since the proofs are similar to that of the corresponding results of [3].

2 Theorem. *The trivial topology and, hence, every topology on $C(Y,Z)$ is $\mathbf{A}$-admissible if and only if the topology of Z is trivial.*

3 Theorem. *Let Z be a T_1 space. Then, the discrete topology and, hence, every topology on $C(Y,Z)$ is $\mathbf{A}$-splitting.*

4 Theorem. *If the discrete topology and, hence, every topology on $C(Y,Z)$ is $\mathbf{A}$-splitting, then Z is a T_0 space.*

5 Theorem. *The topology $\tau_{\mathcal{Q}(\mathcal{U})}$ on the set $C(Y,Z)$ (see [4]), generated by a quasi-uniformity $\mathcal{U}$ on the space Z, is $\mathbf{A}$-splitting and $\mathbf{A}$-admissible.*

6 Definition. Let t be a topology on $C(Y,Z)$. We define a subset $\Re_t$ of the product $(C(Y,Z))^\nu$ as follows: an element $(f_i)_{i\in\nu}$ of $(C(Y,Z))^\nu$ belongs to $\Re_t$ if for every $i,j \in \mathbf{A}$, $f_i \in \mathrm{Cl}(\{f_j\})$ (in the space $C_t(Y,Z)$) provided that $i \in \mathrm{Cl}_{\mathbf{A}}(\{j\})$.

We observe that if t_1, t_2 are two topologies on $C(Y,Z)$ such that $t_1 \subseteq t_2$, then $\Re_{t_2} \subseteq \Re_{t_1}$.

7 Definition. On the product $(C(Y,Z))^\nu$ we define a subset $\Re$ as follows: an element $(f_i)_{i\in\nu}$ of $(C(Y,Z))^\nu$ belongs to $\Re$ if for every $y \in Y$, $f_i(y) \in \mathrm{Cl}_Z(\{f_j(y)\})$ provided that $i \in \mathrm{Cl}_{\mathbf{A}}(\{j\})$.

Below we give necessary and sufficient conditions for an arbitrary topology t on $C(Y,Z)$ to be **A**-splitting and **A**-admissible.

8 Theorem. *A topology t on $C(Y,Z)$ is **A**-splitting if and only if $\Re \subseteq \Re_t$.*

9 Theorem. *A topology t on $C(Y,Z)$ is **A**-admissible if and only if $\Re_t \subseteq \Re$.*

10 Corollary. *A topology t on $C(Y,Z)$ is simultaneously **A**-splitting and **A**-admissible if and only if $\Re_t = \Re$.*

11 Remark. Theorems 2, 3, 4 and 5 can be obtained also by Theorems 6 and 7 using the following facts (which are easily proved):

(1) For the trivial topology and, hence, for every topology t on the set $C(Y,Z)$ we have, $\Re_t \subseteq \Re$ if and only if the topology of Z is trivial.

(2) Let Z be a T_1 space. Then, for the discrete topology and, hence, every topology t on $C(Y,Z)$ we have, $\Re \subseteq \Re_t$.

(3) If for the discrete topology and, hence, for every topology t on $C(Y,Z)$ we have $\Re \subseteq \Re_t$, then Z is T_0 space.

(4) For the topology $\tau_{\mathcal{Q}(\mathcal{U})}$ on the set $C(Y,Z)$ generated by a quasi-uniformity $\mathcal{U}$, we have $\Re \subseteq \Re_{\tau_{\mathcal{Q}(\mathcal{U})}}$.

Theorems 6 and 7 imply also the following result.

12 Theorem. *The pointwise topology and, therefore, the compact open and the Isbell topologies are **A**-splitting and **A**-admissible.*

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