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THE METHOD OF SIGNORINI IN THE ELASTOSTATIC OF A DIELECTRIC (*)

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Summary. *In this paper we propose a generalization of Signorini's method which allows ist application to equilibrium problems, of non-linear elastic dielectrics, with mixed boundary conditions.*

1. INTRODUCTION.

F. Stoppelli [1] proved a theorem of existence and uniqueness for the problem of finite elastostatic with traction boundary conditions, when the loads do not have an axis of equilibrium. Moreover this author proves that, upon suitable hypotheses, the solution of the above mentioned problem is analytic, in a convenient region [2]. These analyses were extended in [3],[4] to the case of loads exibhiting an axis of equilibrium. Later on, Van Buren [5] extablished the corresponding results for the problem of non linear elastostatics with position boundary conditions. It is obvious that the preceding methodology can be applied to the case of mixed boundary con-

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ditions. In this last problem, an exemplification will occur, due to the circumstance that one does not need to deal with the condition of compatibility since the constraints act in such a way that it is always possible to balance the action of external forces on the whole system(1).

All these results lead us to the possibility to attain an exact formulation of Signorini's method [7],[8]. As it is well known, this method is based on the hypothesis that the solution of the problems of equilibrium can be expanded in power series, with respect to a parameter λ which depends on the nature of the problem. Formally, if $\underset{\sim}{u}$ is the desplacement vector with respect to a reference configuration, we shall assume the following representation for $\underset{\sim}{u}$:

$$\underset{\sim}{u} = \sum_{n=1}^{\infty} \lambda^n \underset{\sim}{u}_n \quad , \tag{1.1}$$

where the displacements $\underset{\sim}{u}_n$ represent the solutions of successive boundary problems of linear elasticity and are such that each of them can be solved as soon as the preceeding ones have been solved.

In this paper we shall deal with the possibility of generalizing Signorini's method to the problem of elastostatics of a non linear dielectric, with mixed boundary conditions. In principle, no difficulty should arise on extending Signorini's method to the dielectric problem, provided that, one proves first a local theorem of existence and uniqueness of the solution, for the system of the equilibrium, of a non linear dielectric with mixed boundary conditions. Since such a theorem has been previously proved [9], we are assured that the method can be applied to the dielectric. In so doing, we shall

(1) We refer the reader for extra details on this problems to the work of G. Capriz & P. Podio Guidugli [6].

first need to recall (sect. 2) the electroelastostatic system of equations in the *Lagrangian form* [10], considering that now we are going to make use of the Maxwell equations along with the usual elastostatics equations. At the present moment it is worth to make some remarks about the changes, we must introduce, in order to apply Signorini's method to the present problem. Clearly, in this case we shall not need to take into account the compatibility conditions because of the boundary conditions which we have set.

Moreover, the solution, which now is represented by the displacement vector field $\underset{\sim}{u}$ and the Lagrangian electric induction field $\underset{\sim}{\mathscr{D}}$, is related (section 3) to two parameters λ and μ, the first being connected with the mechanical actions only and the second, with the electrical boundary conditions. So it appears natural to assume for $\underset{\sim}{u}$ and $\underset{\sim}{\mathscr{D}}$, the following expansions

$$(1.2) \qquad \begin{cases} \underset{\sim}{u} = \sum_{0}^{\infty}{}_{n,m} \lambda^n \mu^m \underset{\sim}{u}_{nm} \\ \underset{\sim}{\mathscr{D}} = \sum_{0}^{\infty}{}_{n,m} \lambda^n \mu^m \underset{\sim}{\mathscr{D}}_{nm} . \end{cases}$$

Whence, whenever we shall suppose $n \neq 0$ and $m = 0$, we shall be dealing with electromechanical effects, caused by *mechanical actions* only, and when $n = 0$ and $m \neq 0$, we shall be in the presence of electromechanical effects, produced by *electric actions* only.

We shall see in the last section that all the terms $\underset{\sim}{u}_{nm}$ and $\underset{\sim}{\mathscr{D}}_{nm}$, in the series, are solutions in an iterative process of specific problems of linear electroelasticity, so that Signorini's method is generalized to a non-linear elastic dielectric.

2. THE FUNDAMENTAL ELECTROELASTOSTATIC SYSTEM.

Let $\mathscr{B}$ be a dielectric continuous system. If $\mathscr{C}_*$ and $\mathscr{C}$ are respectively a reference configuration and the equilibrium configuration of $\mathscr{B}$, the deformation that $\mathscr{B}$ will experience from $\mathscr{C}_*$ to $\mathscr{C}$, will be expressed by the set of scalar functions

$$x^i = x^i(X^L), \qquad i,L = 1,2,3 \tag{2.1}$$

where X^L are the Lagrangian coordinates of the point $\underset{\sim}{X} \in \mathscr{C}_*$ and x^i represent the coordinates of $\underset{\sim}{x} \in \mathscr{C}$.

Furthermore, we assume that the portion $\partial \mathscr{C}'_*$ of $\partial \mathscr{C}_*$ is constrained, whereas the portion $\partial \mathscr{C}''_* = \partial \mathscr{C}_* - \partial \mathscr{C}'_*$ is free to move, under the action of assigned superficial traction, whose superficial density is $\underline{t}_*$. Finally, we suppose the whole boundary at a sectionally constant potential $\bar{\phi}$.

The Maxwell equations which we need to consider at the present physical situation are

$$\begin{cases} \int_{\partial\sigma} \underline{E} \cdot d\underline{l} = 0 \\ \\ \int_{s} \underline{D} \cdot \underline{n} d\sigma = 0, \end{cases} \tag{2.2}$$

where σ is an arbitrary open surface of $\mathscr{B}$, both contained in $\mathscr{C}$, while s is an arbitrary closed surface of $\mathscr{B}$. In (2.2) $\underline{E}$ is the electric field, $\underline{D}$ the electric induction fieled and $\underset{\sim}{n}$ is the unit exterior normal vector to the surface s.

Let us denote by $\underset{\sim}{\mathscr{E}}$ and $\underset{\sim}{\mathscr{D}}$ the lagrangian fields corresponding to $\underline{E}$ and $\underline{D}$ respectively, by means of the relations:

$$(2.3)\qquad \begin{cases} \mathscr{E}_L = F^i_L\, E_i \\ \mathscr{D}^L = J(F^{-1})^L_i\, D^i \end{cases}$$

were $F^i_L = (\frac{\partial x^i}{\partial x^L})$ is the deformation gradient and $J = \det \| F^i_L \|$.

It is easy to prove (2) that taking into account (2.3), equations (2.2) become

$$(2.4)\qquad \begin{cases} \mathrm{Rot}\, \underset{\sim}{\mathscr{E}} = 0 \\ \mathrm{Div}\, \underset{\sim}{\mathscr{D}} = 0 \end{cases}$$

Here Rot and Div denote derivative operators with respect to X^L.

Equation $(2.4)_1$ is equivalent to the other one

$$(2.5)\qquad \underset{\sim}{\mathscr{E}} = -\mathrm{Grad}\, \phi\,, \qquad (\mathscr{E}_L = -\phi_{,L})$$

where ϕ is the electric potential.

In order to obtain the fundamental system for the actual problem, we need to consider the equilibrium equation for the continuum $\mathscr{B}$:

$$(2.6)\qquad \mathrm{Div}\, \underset{\sim}{T}_* + \rho_*\, \underset{\sim}{b} = 0,$$

where $\underset{\sim}{T}_*$ is the Piola-Kirchhoff stress tensor, ρ_* is the Lagrangian mass density and $\underset{\sim}{b}$ is the specific force.

We suppose, moreover, that the material under consideration is such that,its constitutive equations are of the following form:

(2) See [9].

$$(2.7)\qquad \begin{cases} \underset{\sim}{T}_* = \underset{\sim}{A}(H_{iL}, \mathscr{E}_L) \\ \\ \underset{\sim}{\mathscr{D}} = \underset{\sim}{B}(H_{iL}, \mathscr{E}_L) \end{cases}$$

H_{iL} being the gradient of displacement $u^i = x^i - X^i$, i.e., $H_{iL} = u_{i,L}$

All this considered, we are able now to set the following fundamental boundary problem, for the equilibrium of a dielectric $\mathscr{B}$, which is described by the constitutive equations (2.7):

$$(2.8)\qquad \begin{cases} \mathrm{Div}\ \underset{\sim}{A}(u_{i,L};\phi_{,L}) + \rho_* \underset{\sim}{b} = 0 & \\ \mathrm{Div}\ \underset{\sim}{B}(u_{i,L};\phi_{,L}) = 0 & \\ \underset{\sim}{u} = \underset{\sim}{0} & \text{on } \partial\mathscr{C}'_* \\ \phi = \bar{\phi} & \text{on } \partial\mathscr{C}_* \\ \underset{\sim}{T}_* \cdot \underset{\sim}{n}_* = \underset{\sim}{t}_* & \text{on } \partial\mathscr{C}''_* \end{cases}$$

So we have to determine a solution $\underset{\sim}{u}(X)$, $\phi(X)$ of the above boundary problem.

3. STATEMENT OF THE PROBLEM AND ITS SOLUTION.

With respect to the system (2.7), let us assume that the following decompositions existe

$$
(3.1)\quad \begin{cases} \underset{\sim}{b} = \sum_{n=1}^{\infty} \lambda^n \underset{\sim}{b}_n = \lambda \underset{\sim}{b}_1 + \mathscr{B}(\lambda) \\ \underset{\sim}{t}_* = \sum_{n=1}^{\infty} \lambda^n \underset{\sim}{t}_{*n} = \lambda t_{*1} + \mathscr{I}(\lambda) \\ \phi = \sum_{n=1}^{\infty} \mu^n \bar{\phi}_n = \mu \bar{\phi}_1 + G(\mu), \end{cases}
$$

were λ and μ are characteristic parameters of the problem and $\mathscr{B}(\lambda)$, $\mathscr{I}(\lambda)$, $G(\mu)$ are analytic with respect to λ and μ and infinitesimal with them both and $\bar{\phi}_n$ are sectionally cônstant on $\partial\mathscr{C}_*$.

Moreover, let us assume that $\mathscr{C}_*$ rapresents a natural state of the system, that is

$$
(3.2)\quad \begin{cases} \underset{\sim}{T}_* = \underset{\sim}{A}(\underset{\sim}{0},\underset{\sim}{0}) = \underset{\sim}{0} \\ \underset{\sim}{\mathscr{D}} = \underset{\sim}{B}(\underset{\sim}{0},\underset{\sim}{0}) = \underset{\sim}{0} \end{cases}
$$

and along with (3.2) we suppose that $\underset{\sim}{T}_*$ and $\underset{\sim}{\mathscr{D}}$ are analytic with respect to $\underset{\sim}{H}$ and $\underset{\sim}{\mathscr{E}}$. Then, it follows that [1],[2], the solutions $\underset{\sim}{u}(X)$ and $\phi(\underset{\sim}{X})$ of the problem will be decomposable in the following double infinite summations of the parameters λ and μ, that is

$$
(3.3)\quad \begin{cases} \underset{\sim}{u} = \sum_{i,j=0}^{\infty} \lambda^i \mu^j \underset{\sim}{u}_{ij} = \lambda \underset{\sim}{u}_{10} + \mu \underset{\sim}{u}_{01} + \mathscr{U}(\lambda,\mu); \quad (\underset{\sim}{u}_{00} = \underset{\sim}{0}) \\ \phi = \sum_{i,j=0}^{\infty} \lambda^i \mu^j \phi_{ij} = \lambda\phi_{10} + \mu\phi_{01} + \mathscr{F}(\lambda,\mu); \quad (\phi_{00} = 0) \end{cases}
$$

Recalling the hypoteses on $\underset{\sim}{T}_*$ and $\mathscr{D}$ we have

$$(3.4)\quad \begin{cases} \underset{\sim}{T}_* = \underset{\sim}{A}(\underset{\sim}{H},\underset{\sim}{\mathscr{E}}) = \sum_{i,j=0}^{\infty} \underset{\sim}{T}_{*ij}(\underset{\sim}{H},\underset{\sim}{\mathscr{E}}) \\ \\ \underset{\sim}{\mathscr{D}} = \underset{\sim}{B}(\underset{\sim}{H},\underset{\sim}{\mathscr{E}}) = \sum_{i,j=0}^{\infty} \underset{\sim}{\mathscr{D}}_{ij}(\underset{\sim}{H},\underset{\sim}{\mathscr{E}}) \end{cases}$$

Clearly because of (3.2) $\underset{\sim}{T}_{*00} = \underset{\sim}{\mathscr{D}}_{00} = \underset{\sim}{0}$.

From (3.3) in view of (2.1) it follows that:

$$(3.5)\quad \begin{cases} \underset{\sim}{H} = \sum_{i,j=0}^{\infty} \lambda^i \mu^j \underset{\sim}{H}_{ij} \\ \\ \underset{\sim}{\mathscr{E}} = \sum_{i,j=0}^{\infty} \lambda^i \mu^j \underset{\sim}{\mathscr{E}}_{ij} \end{cases}$$

Expanding $\underset{\sim}{T}_*$ and $\underset{\sim}{\mathscr{D}}$ in power series of $\underset{\sim}{H}$ and $\underset{\sim}{\mathscr{E}}$ with initial point $(\underset{\sim}{0},\underset{\sim}{0})$ and in view of (3.2), we have

$$(3.6)\quad \begin{cases} \underset{\sim}{T}_* = \underset{\sim}{A}_{10}\cdot\underset{\sim}{H} + \underset{\sim}{A}_{01}\cdot\underset{\sim}{\mathscr{E}} + \underset{\sim}{H}^T\cdot\underset{\sim}{A}_{20}\cdot\underset{\sim}{H} + \\ \qquad + \underset{\sim}{\mathscr{E}}^T \cdot \underset{\sim}{A}_{20}\cdot\underset{\sim}{\mathscr{E}} + \underset{\sim}{H}^T \cdot \underset{\sim}{A}_{11} \cdot\underset{\sim}{\mathscr{E}} + \ldots \\ \\ \underset{\sim}{\mathscr{D}} = \underset{\sim}{B}_{10}\cdot\underset{\sim}{H} + \underset{\sim}{B}_{01}\cdot\underset{\sim}{\mathscr{E}} + \underset{\sim}{H}^T\cdot\underset{\sim}{B}_{20} \cdot \underset{\sim}{H} + \\ \qquad + \underset{\sim}{\mathscr{E}}^T \cdot \underset{\sim}{B}_{02} \cdot \underset{\sim}{H}^T \cdot \underset{\sim}{B}_{11}\cdot\underset{\sim}{\mathscr{E}} + \ldots \end{cases}$$

By substituting $\underset{\sim}{H}, \underset{\sim}{\mathscr{E}}$ given by (3.6), into (3.7), we obtain:

$$(3.7)\quad \begin{cases} \underset{\sim}{T}_* = \sum_{n,m=0}^{\infty} \lambda^n \mu^m (\underset{\sim}{A}_{10} \cdot \underset{\sim}{u}_{nm} + \underset{\sim}{A}_{01} \underset{\sim}{\mathscr{E}}_{nm} + \underset{\sim}{\mathscr{A}}_{nm}) \\ \underset{\sim}{\mathscr{D}} = \sum_{n,m=0}^{\infty} \lambda^n \mu^m (\underset{\sim}{B}_{10} \underset{\sim}{u}_{nm} + \underset{\sim}{B}_{01} \cdot \underset{\sim}{\mathscr{E}}_{nm} + \underset{\sim}{\mathscr{B}}_{nm}) \end{cases}$$

where $\underset{\sim}{A}_{10}$ and $\underset{\sim}{A}_{01}$ must be identified with the elasticity and polarization tensors of the linear elasticity, and $\underset{\sim}{B}_{01}$ is the matching term of the dielectric tensor of the material.

Moreover, we have

$\underset{\sim}{\mathscr{A}}_{n0}$ ≡ polinomial function of the variables

$$\underset{\sim}{u}_{10}, \dots, \underset{\sim}{u}_{n-10}, \quad \underset{\sim}{\mathscr{E}}_{10}, \dots \underset{\sim}{\mathscr{E}}_{n-10}$$

$\underset{\sim}{\mathscr{A}}_{0m}$ ≡ polinomial function of the variables

$$\underset{\sim}{u}_{01}, \dots, \underset{\sim}{u}_{0m-1}, \underset{\sim}{\mathscr{E}}_{01}, \dots, \underset{\sim}{\mathscr{E}}_{0m-1}$$

$\underset{\sim}{\mathscr{A}}_{nm}$ ≡ polinomial function of the variables

$$\underset{\sim}{u}_{01}, \underset{\sim}{u}_{10}, \dots, \underset{\sim}{u}_{(n-1)0}, \underset{\sim}{u}_{0(m-1)}, \underset{\sim}{\mathscr{E}}_{10}, \underset{\sim}{\mathscr{E}}_{01}, \dots \underset{\sim}{\mathscr{E}}_{(n-1)0}, \underset{\sim}{\mathscr{E}}_{0(m-1)}$$

and the same result holds for $\underset{\sim}{\mathscr{B}}_{0m}$, $\underset{\sim}{\mathscr{B}}_{n0}$, $\underset{\sim}{\mathscr{B}}_{nm}$.

We observe that, by setting $\mu = 0$, from $(3.7)_1$, we obtain the same result that Signorini obtained for the linear elasticity.

Taking into consideration the term in λ :

$$(3.8)\qquad \lambda(\underset{\sim}{A}_{10} \cdot \underset{\sim}{u}_{10} + \underset{\sim}{A}_{01} \cdot \underset{\sim}{\mathscr{E}}_{10})$$

it is worth noticing that it represents the stress in the case of linear elasticity for a dielectric, corresponding to the solutions $(\underset{\sim}{u}_{10}, \underset{\sim}{\mathscr{E}}_{10})$ which are equilibrium solutions in the presence of mechanical forces only, i.e. of the type $(\lambda \underset{\sim}{b}_1, \lambda \underset{\sim}{t}_{*1})$. Analogously, if we consider the therm in μ:

$$(3.9) \qquad \mu(\underset{\sim}{A}_{10} \cdot \underset{\sim}{u}_{01} + \underset{\sim}{A}_{01} \cdot \underset{\sim}{\mathscr{E}}_{01})$$

we find that it represents the stress in the case of linear elasticity for a dielectric at the presence of the equilibrium solutions $(\underset{\sim}{u}_{01}, \underset{\sim}{\mathscr{E}}_{01})$, due to the electrical actions only $(\mu, \bar{\phi}_1)$

In the linear case, in which the superposition principle holds, the stress and the induction vectors, at the presence of both mechanical $(\lambda \underset{\sim}{b}_1, \lambda \underset{\sim}{t}_{*1})$ and electrical $(\mu\bar{\phi}_1)$ actions correspond to the sum of (3.8) and (3.9), as the terms in λ and μ in (3.8) and (3.9) indicate.

The quantities in λ μ of the type

$$\lambda\mu \, \underset{\sim}{\mathscr{A}}_{11}(\underset{\sim}{u}_{10}, \underset{\sim}{u}_{01}, \underset{\sim}{\mathscr{E}}_{10}, \underset{\sim}{\mathscr{E}}_{01})$$

where $\underset{\sim}{\mathscr{A}}_{11}$ is a polynomial quadratic function with respect to $\underset{\sim}{u}_{10}, \underset{\sim}{u}_{01}, \underset{\sim}{\mathscr{E}}_{10}, \underset{\sim}{\mathscr{E}}_{01}$, represent the second order interaction between electrical and mechanical effects.

Further by substituting (3.7) into (2.7) the following system of equations is obtained:

$$(3.10)\quad \begin{cases} \sum_{n,m=0}^{\infty} \lambda^n \mu^m \operatorname{Div}(\underset{\sim}{A}_{10} \cdot \underset{\sim}{u}_{nm} + \underset{\sim}{A}_{01} \cdot \underset{\sim}{\mathscr{E}}_{nm} + \underset{\sim}{\mathscr{A}}_{nm}) + \rho_* \sum_{n=1}^{\infty} \lambda^n \underset{\sim}{b}_n = 0 \\ \sum_{n,m=0}^{\infty} \lambda^n \mu^m \operatorname{Div}(\underset{\sim}{B}_{10} \cdot \underset{\sim}{u}_{nm} + \underset{\sim}{B}_{01} \cdot \underset{\sim}{\mathscr{E}}_{nm} + \underset{\sim}{\mathscr{B}}_{nm}) = 0 \\ \sum_{n,m=0}^{\infty} \lambda^n \mu^m \underset{\sim}{u}_{nm} = \underset{\sim}{0} \qquad \text{on} \quad \partial \mathscr{C}'_* \\ \sum_{n,m=0}^{\infty} \lambda^n \mu^m \phi_{nm} = \sum_{m=1}^{\infty} \mu^m \bar{\phi}_m \qquad \text{on} \quad \partial \mathscr{C}_* \\ \sum_{n,m=0}^{\infty} \lambda^n \mu^m (\underset{\sim}{A}_{10} \cdot \underset{\sim}{u}_{nm} + \underset{\sim}{A}_{01} \cdot \underset{\sim}{\mathscr{E}}_{nm} + \underset{\sim}{\mathscr{A}}_{nm}) \cdot \underset{\sim}{n}_* = \sum_{n=1}^{\infty} \lambda^n \underset{\sim}{t}_{*n} \qquad \text{on} \quad \partial \mathscr{C}''_* \end{cases}$$

from which it follows that all the coefficients of all the powers of $\lambda\ \mu$ in the (3.10) equations, must be equal to zero. Whence, we obtain the following set of boundary problems:

$$(3.11)\quad \begin{cases} \operatorname{Div}(\underset{\sim}{A}_{10} \cdot \underset{\sim}{u}_{nm} + \underset{\sim}{A}_{01} \cdot \underset{\sim}{\mathscr{E}}_{nm} + \underset{\sim}{\mathscr{A}}_{nm}) + \rho_* \underset{\sim}{b}_n \, \delta_{0n} = \underset{\sim}{0} \\ \operatorname{Div}(\underset{\sim}{B}_{10} \cdot \underset{\sim}{u}_{nm} + \underset{\sim}{B}_{01} \cdot \underset{\sim}{\mathscr{E}}_{nm} + \underset{\sim}{\mathscr{B}}_{nm}) = 0 \\ \underset{\sim}{u}_{nm} = \underset{\sim}{0} \qquad \text{on} \quad \partial \mathscr{C}'_* \\ \phi_{nm} = \delta_{0n} \, \bar{\phi}_n \qquad \text{on} \quad \partial \mathscr{C}_* \\ (\underset{\sim}{A}_{10} \cdot \underset{\sim}{u}_{nm} + \underset{\sim}{A}_{01} \cdot \underset{\sim}{\mathscr{E}}_{nm} + \underset{\sim}{\mathscr{A}}_{nm}) \cdot \underset{\sim}{n}_* = \underset{\sim}{t}_{*n} \, \delta_{0m} \qquad \text{on} \quad \partial \mathscr{C}''_* \end{cases}$$

By setting n=1, m=0 the foundamental elastostatic system for the dielectric is obtained with mechanical forces $\rho_* \underset{\sim}{b}_1$ and mechanical boundary conditions given by t_{*1} on $\partial \mathscr{C}''_*$ and $\phi_{10} = 0$ on

$\partial\mathscr{C}_*$. If n=0 and m=1, we have the foundamental elastostatic system for the dielectric in the absence of mechanical forces and electrical boundary conditions given by $\phi = \bar{\phi}_1$ on $\partial\mathscr{C}_*$. Wen m = 0 and n takes on any value, we again have a system concerning the linear electroelastostatic for the dieletric but, this time, the density forces would be modified into $\rho_* \underset{\sim}{b}_n + \mathrm{Div}\,\mathscr{A}_{n0}$ and a *fictitious charge density* Div $\mathscr{B}_{n0}$ would be present; in this case the boundary conditions for the electric field whould be $\phi_{n0} = 0$ on $\partial\mathscr{C}_*$.

By the way, it should be observed that the terms Div $\mathscr{A}_{n0}$ and Div $\mathscr{B}_{n0}$, which depend on the variables $\underset{\sim}{u}_{10}, \underset{\sim}{u}_{20}, \dots, \underset{\sim}{u}_{(n-1)0}$, $\underset{\sim}{\mathscr{E}}_{10}$, $\underset{\sim}{\mathscr{E}}_{20}, \dots, \underset{\sim}{\mathscr{E}}_{(n-1)0}$, are known as soon as the previous problems of the linear elastostatics have been solved by recurrence.

Of course the same kind of discussion can be carried out for the case n=0 and m taking any value; the boundary problems we would have in this situation would have no mechanical forces and a charge density given by Div $\mathscr{B}_{0m}$.

Finally the solution $\underset{\sim}{u}$, $\underset{\sim}{\mathscr{E}}$ as power series of $\lambda\cdot\mu$,can be obtained by solving the subsequent boundary problems which result by setting $n \neq 0$, $m \neq 0$. In such problems all the preceding solutions of the separated problems in λ and μ will occur.

We may conclude, then, by affirming that Signorini's method has been in such a way extended to the case of the elastostatic of a dielectric.

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