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BI-IDEALS AND GENERALIZED BI-IDEALS IN SEMIGROUPS

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Sommario. *Il concetto di bi-ideale e quello più recente di bi-ideale generalizzato di un semigruppo sono variamenti usati da differenti autori nella teoria algebrica dei semigruppi. In questa nota vengono risolti alcuni problemi proposti da S.Lajos.*

Let S be a semigroup. A non-empty subset A of S is called a *generalized bi-ideal* of S if the condition

(1) $$ASA \subseteq A$$

holds (see [1]). If A is a subsemigroup of S which satisfies (1), A is a *bi-ideal* of S.

In [2] Lajos gave an example of a semigroup for which certain generalized bi-ideals differ from the bi-ideals; he also posed the problem of characterizing those semigroups whose generalized bi-ideals are bi-ideals. This problem is solved by the following

THEOREM 1. *Let S be a semigroup. Every generalized bi-ideal of S is a bi-ideal if and only if the condition*

(2) $$ab \in \{a,b\} S \{a,b\}$$

holds for every couple a,b *in* S.

Proof. Let S be a semigroup in which every generalized bi-ideal is a bi-ideal. Then, for every $a,b \in S$, the generalized bi-ideal generated by subset $\{a,b\}$ is

$$\{a,b\} \cup \{a,b\} S \{a,b\}$$

wich is a bi-ideal of S, thus $ab \in \{a,b\} S \{a,b\}$.

Conversely, let a,b be elements of a generalized bi-ideal A of S. Then by the assumption (2), $ab \in ASA \subseteq A$, whence A is a bi-ideal of S.

Let S be a semigroup. In what follows we denote by P(S) the multiplicative semigroup of all non-empty subsets of S, we denote by GB(S) the semigroup of the generalized bi-ideals of S and we denote by B(S) the semigroup of the bi-ideals of S under set product. S.Lajos proposed the question to determine the semigroups wich have the following property:

$$P(S)/GB(S) \simeq GB(S)/B(S) \tag{3}$$

THEOREM 2. *A semigroup* S *has the property* (3) *if and only if either* S *is a left zero semigroup or* S *is a right zero semigroup.*

Proof. If S is a semigroup that satisfies the property (3) then P(S)/GB(S) is a zero semigroup by the Corollary 2 of [1]. Hence it follows that ab=abSab, thus by our Theorem 1 and by the assumption (3), $P(S)/GB(S) = \{0\}$.

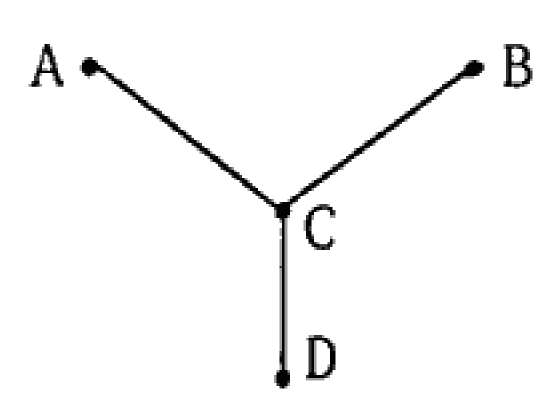
It follows that every non-empty subset of S is a bi-ideal, so aSa = a (i.e. S is a rectangular band) and $ab \in \{a,b\}$ for every a,b in S. Now, assume that there exist distinct elements a,b in S such that ab=a. Then, for every element $c \in S$ ca=cab= $= cb \in \{c,b\} \cap \{c,a\}$ hence ca=c and ac=aca=a. Thus xy=xay=xa=x, for every couple x,y in S. Therefore S is a left zero semigroup. Likewise, if there exist distinct elements a,b in S such that ab=a then S is a right zero semigroup.

Conversely, let A be a non-empty subset of S, where S is a left or right zero semigroup. Then ASA=A, thus by the Theorem 1, the semigroup S has the property (3).

A non-empty subsemigroup F of a semigroup S is called a *filter* if the implication $ab\in F \Longrightarrow a\in F$ and $b\in F$ holds for every couple a,b in S.

We denote the set of all filter of S by F(S). Lajos [3] posed the following question: is B(S) ∪ F(S) respect to set product a semigroup?

The answer is negative. In fact, let S be the semilattice of groups



such that C=AB. It is evident that A and B are filter and C does not either filter or bi-ideal.

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