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Centralizers in Hall's universal and direct limits of finitary symmetric groups

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Abstract. We study the centralizers of finite subgroups in Hall's universal group. We describe the structure of the centralizers of arbitrary finite subgroups in the groups $S(\xi)$ and $FSym(\kappa)(\xi)$ where $S(\xi)$'s are obtained as direct limits of finite symmetric groups and $FSym(\kappa)(\xi)$'s are obtained as direct limits of finitary symmetric groups on the set of infinite cardinality κ.

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1 Introduction

A locally finite group U satisfying,

(i) Every finite group can be embedded into U,

(ii) Any two isomorphic finite subgroups of U are conjugate in U

is called a universal locally finite group.

Philip Hall in [3] constructed the first example of universal locally finite group of countably infinite order. A countable group satisfying these two properties is called Hall's universal group, as Hall proved that such a countable locally finite group is unique up to isomorphism. But if we allow that the cardinality of the group could be of arbitrary, uncountable cardinal κ, then A. Macintyre and S. Shelah proved in [20] that, there are 2^κ pairwise non-isomorphic universal locally finite groups of cardinality κ.

The structure of Hall's universal group has been studied in the past from different points of view. Hall's universal group is countable and existentially closed in the class of locally finite groups such properties are surveyed by Leinen in [12]. Not only the direct limits of finite groups but also the direct limits of finite dimensional algebras and lie algebras are discussed in a very clear and detailed way by A. Zalesskii in [21].

The generalization of the universal locally finite groups was due to K. Hickin in [7]. Let A be a periodic abelian group. A group G is called a universal locally finite central extension of A provided that the following conditions are satisfied.

(i) $A \leq Z(G)$ (the centre of G)

(ii) G is locally finite

(iii) (A-injectivity). Suppose that $A \leq B \leq D$ with $A \leq Z(D)$, that D/A is finite and that $\psi : B \to G$ is an A-isomorphism (that is $\psi(a) = a$ for all $a \in A$). Then there exists an extension $\overline{\psi} : D \to G$ of ψ to an isomorphism of D into G.

Let $ULF(A)$ denote the class of all groups G satisfying (i)-(iii). The countable universal locally finite group of Hall is in $ULF(1)$. By [7, Theorem 1] if $G \in ULF(A)$, then $A = Z(G)$ and G/A is simple.

Although U is a direct limit of finite simple groups, if for some prime p, the group $\mathbb{Z}_p \times \mathbb{Z}_p \times \mathbb{Z}_p \leq A$, then G/A is not a direct limit of finite simple subgroups. Therefore one may construct uncountably many pairwise non-isomorphic simple countably infinite locally finite groups by this method; see [7, Corollary 1]. Universal locally finite central extensions of A has numerous interesting structural properties but we will say no more than consulting the paper of Hickin; [7] see also; [18] and [10].

2 Hall's universal group

Hall constructed his group as a union of a tower of finite symmetric groups;

$$G_1 \leq G_2 \leq \dots$$

where G_1 is a symmetric group of order greater than 2 and if G_n is given, then G_{n+1} is the symmetric group on G_n and G_n is embedded into G_{n+1} by right regular representation. One can see easily that in the regular representation of G_i into G_{i+1}, actually all the permutations in the image of the elements of G_i in G_{i+1} will be an even permutation. Hence we embed G_i into $Alt(G_i)$. So in fact, U is a direct limit of finite simple alternating groups. Then the question of whether Hall's universal group can be written as a direct limit of other families of finite simple groups is answered by F. Leinen in [11]. He proved that Hall's universal group can be constructed as a direct limit of simple linear groups $\{PSL(n_i, \mathbf{F}_q)\}$, $\{PSU(n_i, \mathbf{F}_q)\}$, $\{PSp(2n_i, \mathbf{F}_q)\}$, $\{P\Omega^+(2n_i, \mathbf{F}_q)\}$, $\{P\Omega(2n_i + 1, \mathbf{F}_q)\}$, $\{P\Omega^-(2n_i + 2, \mathbf{F}_q)\}$.

It is natural to ask whether U can be expressed as a union (direct limit) of the infinite simple locally finite groups $PSL(n_i, \overline{\mathbf{F}}_p)$, where $\overline{\mathbf{F}}_p$ is the algebraic closure of the field $\mathbf{F}_p$ with p elements. It is proved in [16, Theorem 1] that the answer is positive.

Theorem 2.1. *(Kuzucuoğlu-Zaleskii) Let p be any fixed prime. The Hall's universal group is a direct limit of some groups $PSL(n_i, \overline{\mathbf{F}}_p)$, $i = 1, 2, \ldots$ such that all the sequent embeddings are rational maps (morphisms of algebraic groups).*

Theorem 2.1 shows, among other things, that the characteristic of the ground field is not an invariant of a direct limit of algebraic groups. In fact, one can prove a slightly more general result [16, Theorem 2].

Theorem 2.2. *(Kuzucuoğlu-Zaleskii) Let F be a finite or an infinite locally finite field of characteristic p and G_n be one of the classical simple groups of rank n over F. Then any infinite sequence of the groups G_n contains a subsequence $G_{n_i}, i = 1, 2, \ldots$ such that the Hall's universal group U is a union of subgroups H_i, where $H_i \subset H_{i+1}$, $H_i \cong G_{n_i}$ and the embeddings $G_{n_i} \to G_{n_{i+1}}$ induced by the inclusions $H_i \subset H_{i+1}$ extend to rational embeddings (morphisms) of algebraic groups (over $\overline{\mathbf{F}}_p$) associated with the groups G_{n_i}.*

To have Theorem 2.1, one should take $\overline{\mathbf{F}}_p$ for F and $G_n = PSL(n, \overline{\mathbf{F}}_p)$.

One of the main characteristic of the universal locally finite groups which can be obtained from the properties **(i)** and **(ii)** is the following: If A is a subgroup of the finite group B, then every embedding of A into U can be extended to an embedding of B into U [13, Theorem 6.1 (b)]. It follows from this property that, as every countable locally finite group can be written as a union of an increasing sequence of finite groups, every countable locally finite group has an isomorphic copy in U. In particular, a copy of every simple countable locally finite group is contained in U.

Hall's universal group U satisfies the following properties for which some of them are quite unusual; for the proofs see [13, Chapter 6].

Proposition 2.3.

(a) *Let C_m denote the set of all elements of order $m > 1$ of U. Then C_m is a single class of conjugate elements and $U = C_m C_m$. In particular U is simple.*
The automorphism α of the group G is called locally inner if for every finite set F of elements of G there is an element $g = g_F$ of G such that $f^\alpha = f^g$ for every element $f \in F$.

(b) *If G is any locally finite universal group, then every automorphism of G is locally inner. In particular, the automorphism group of Hall's universal group satisfies $|Aut(U)| = 2^{\aleph_0}$.*

(c) *If G is any universal locally finite group and H is any countably infinite locally finite group, then there exist at least $2^{\aleph_0}$ distinct subgroups of G isomorphic to H.*

(d) *Every infinite locally finite group G can be embedded into a universal locally finite group of cardinality $|G|$. In particular there exist universal locally finite groups of arbitrary infinite cardinal.*

Perhaps, one of the most striking one, in contrast to Sylow theory for finite groups was discovered by Hickin who proved in [7, Theorem 4] that, for every prime p, every countably infinite locally finite p-group can be embedded into U as a maximal p-subgroup. Therefore there are uncountably many pairwise non-isomorphic maximal p-subgroups in U.

Could it be possible to have a maximal p-subgroup in U which is a maximal subgroup of U? M. D. Molle in [17] shows that the answer is positive.

Theorem 2.4. *The countable universal locally finite group U contains, for each prime p, a maximal subgroup that is a p-group.*

One may ask whether U can be written as a direct limit of infinite simple finitary alternating groups? The answer is negative; see [16].

Theorem 2.5. *The Hall's universal group is not a direct limit of infinite finitary alternating groups.*

About the centralizers of elements (subgroups) in Hall's universal group, the following results were announced by Hartley in [6, Proposition 1.8].

Proposition 2.6.

(a) *If F is a finite subgroup of U with trivial center, then $C_U(F)$ is isomorphic to U.*

(b) *If F is a subgroup of U of prime order and M is a subgroup of U with $C_U(F) \leq M < U$, then $M \leq N_U(F)$ is a maximal subgroup of U.*

(c) *If A is a finite abelian subgroup of U, then $C_U(A)/A$ is an infinite simple group.*

For the centralizers of subgroups in algebraically closed groups see; [8] and [9, Chapter 2].

As Hall's universal group is a union of increasing sequence of finite symmetric groups, every finite subgroup F of U is contained in one of the symmetric groups and the ones containing it. The structure of centralizers of subgroups in finite symmetric groups is well known; see [1, Chapter 4] and [19, Chapter 6].

Is it possible to find the structure of centralizers of finite subgroups in U by using basic group theory?

One may use the well known information about the centralizers of finite subgroups in symmetric groups to answer the above question. For this we spell out some of the salient facts.

Let F be a subgroup of the symmetric group $Sym(\Omega)$ where Ω is a finite set. Then $C_{Sym(\Omega)}(F)$ acts on the set of orbits Σ of F on Ω. One may define a relation on Σ: If Δ_1 and Δ_2 in Σ, then $\Delta_1 \sim \Delta_2$ if and only if Δ_1 and Δ_2 are permutationally isomorphic F-sets. i.e. there exists a bijection $\vartheta : \Delta_1 \to \Delta_2$ such that, for any $\delta \in \Delta_1$ and $h \in F$ we have $\vartheta(\delta.h) = \vartheta(\delta).h$

Clearly this defines an equivalence relation on Σ. If ϑ is a bijection on the isomorphic orbits Δ_1 and Δ_2, then $\vartheta \cup \vartheta^{-1} : \Delta_1 \cup \Delta_2 \to \Delta_1 \cup \Delta_2$, and acting trivially on $\Omega \setminus (\Delta_1 \cup \Delta_2)$ defines an element in $C_{Sym(\Omega)}(F)$. Therefore $C_{Sym(\Omega)}(F)$ acts transitively on the isomorphic orbits. If $\mathcal{C}_1,\ \mathcal{C}_2, \ldots, \mathcal{C}_r$ are the equivalence classes of orbits of F on Ω, then

$$C_{Sym(\Omega)}(F) \cong \underset{i=1}{\overset{r}{Dr}}\, C_{Sym(\Gamma_i)}(F) \wr Sym(n_i)$$

where Γ_i is a representative of an orbit of F in $\mathcal{C}_i$ and n_i is the number of orbits in the class $\mathcal{C}_i$.

Proposition 2.7. *Let F be a finite subgroup of the Hall's universal group U and Γ be an orbit of F in U. Then $C_U(F) = \bigcup_{i=1}^{\infty} C_i$ where $C_i = C_{G_i}(F) \cong C_{Sym(\Gamma)}(F) \wr Sym(k_i)$, $\quad k_i = \frac{|n_i|}{|F|}$ and $G_i \cong Sym(n_i)$.*

Proof. Let F be a finite subgroup of U. Then there exists $i \in \mathbb{N}$ such that $F \leq G_{i-1} \cong Sym(n_{i-1})$ where G_{i-1} is acting on a set with n_{i-1} elements and $n_i = (n_{i-1})!$. Then by assumption F has an orbit Γ on the set with n_i elements. Since F acts by the right regular representation, the orbits of F are all isomorphic F-sets namely left cosets of F in G_{i-1}. We may write $C_U(F) = \bigcup_{i=1}^{\infty} C_{Sym(G_i)}(F)$. Then by the above observation for all $j \geq i$, we have

$$C_{Sym(G_j)}(F) \cong C_{Sym(\Gamma)}(F|_\Gamma) \wr Sym(k_j)$$

where $k_j = \frac{n_j}{|F|}$.

QED

Corollary 2.8. *Let F be a finite subgroup of the Hall's universal group U and Γ be an orbit of F in U with $C_{Sym(\Gamma)}(F) = 1$. Then $C_U(F) \cong U$.*

Proof. By Proposition 2.7 we have $C_U(F) = \bigcup_{i=1}^{\infty} C_i$ where $C_i \cong C_{Sym(\Gamma)}(F) \wr Sym(k_i)$. Since each $C_{Sym(\Gamma)}(F) = 1$ we have $C_i \cong Sym(k_i)$. Moreover, C_i is embedded into C_{i+1} by right regular representation and k_i's is an increasing sequence of integers. Now it is clear that every finite group can be embedded into $C_U(F)$ as it is the union of increasing sequence of finite symmetric groups. Let A and B be two finite isomorphic subgroups of $C_U(F)$. Then A and B are contained in C_i for some i. Since C_i is embedded into C_{i+1} by right regular representation (probably it has more than one orbit), then by [13, Lemma 6.3] A and B are conjugate in C_{i+1}. Hence by definition of universal group and uniqueness of U we have $C_U(F) \cong U$. QED

There are infinitely many subgroups satisfying $C_{Sym(\Gamma)}(F) = 1$. For example one may take an element α of maximum cycle length n, an odd integer, in the symmetric group and a permutation β of order 2 which inverts α and let $F = \langle \alpha, \beta \rangle$. Then $C_{Sym(\Gamma)}F = 1$.

3 Centralizers of finite subgroups in groups of type $S(\xi)$

Recall that direct limit of the groups $G_1, G_2, \dots, G_n, \dots$ with the embeddings $\varphi_{ij} : G_i \to G_j$ where $i \leq j$ depends not only the groups G_i but also the embedding φ_{ij}'s, see [15, §7.]. Observe that, one can obtain the additive group of rational numbers as a direct limit of infinite cyclic groups $G_n = \langle \frac{1}{n!} \rangle$ and also the dyadic rational numbers as a direct limit of infinite cyclic groups $K_n = \langle \frac{1}{2^n} \rangle$. Clearly dyadic rational numbers are not isomorphic to the additive group of rational numbers. In this sense we may construct non-isomorphic groups by using different embeddings of finite symmetric groups.

Let $\alpha \in Sym(n)$. For a natural number $p \in \mathbb{N}$, a permutation $d^p(\alpha) \in Sym(pn)$ defined by $(kn+i)^{d^p(\alpha)} = kn + i^{\alpha}, \;\; 0 \leq k \leq (p-1)$ and $1 \leq i \leq n$ is called a **homogenous p-spreading** of the permutation α.

Let ξ be an infinite sequence of not-necessarily distinct primes. By using homogenous p_i-spreadings as embeddings in the following diagram where p_i is the i^{th} prime in the sequence ξ we have the following direct systems

$$\{1\} \overset{d^{p_1}}{\to} Sym(n_1) \overset{d^{p_2}}{\to} Sym(n_2) \overset{d^{p_3}}{\to} Sym(n_3) \overset{d^{p_4}}{\to} \dots$$

and

$$\{1\} \stackrel{d^{p_1}}{\rightarrow} A_{n_1} \stackrel{d^{p_2}}{\rightarrow} A_{n_2} \stackrel{d^{p_3}}{\rightarrow} A_{n_3} \stackrel{d^{p_4}}{\rightarrow} \dots$$

where $n_i = n_{i-1}p_i, \quad i = 1, 2, 3 \dots$ and $Sym(n_i)$ is the symmetric group on n_i letters, A_{n_i} is the alternating group on n_i letters and $n_0 = 1$. The direct limit groups obtained from the above direct systems are denoted by $S(\xi)$ and $A(\xi)$, respectively. Observe that $S(\xi) \leq Sym(\mathbb{N})$.

Recall that the formal product $n = 2^{r_2}3^{r_3}5^{r_5}\dots$ of prime powers with $0 \leq r_k \leq \infty$ for all primes k is called a **Steinitz number** (supernatural number).

Characterization of the groups $S(\xi)$ using Steinitz numbers is done by Kroshko-Sushchansky in [14]. They proved that there are uncountably many pairwise non-isomorphic simple locally finite groups of type $S(\xi)$. Now we describe the structure of the centralizers of arbitrary finite subgroups in $S(\xi)$.

Let F be a finite subgroup of $S(\xi) \leq Sym(\mathbb{N})$. Then F acts on $\mathbb{N}$. The type of F is defined by $t(F) = ((n_{j_1}, r_1), (n_{j_2}, r_2), \dots, (n_{j_k}, r_k))$ where n_{j_i} is the smallest positive integer in which F has an orbit Ω_i on the set with n_{j_i} elements and there are r_i orbits giving equivalent actions of F and n_{j_i}'s are not necessarily distinct. We say that the i^{th} representation of F appears and appears as r_i times in $Sym(n_{j_i})$. For the centralizer of an arbitrary finite subgroup F of $S(\xi)$, we prove the following.

Theorem 3.1. *(Güven, Kegel, Kuzucuoğlu [2]) Let F be a finite subgroup of the infinite group $S(\xi)$ and $\Gamma_1, \dots, \Gamma_k$ be the set of orbits of F such that the action of F on any two orbits in Γ_i is equivalent. Let the type of F be $t(F) = ((n_{j_1}, r_1), (n_{j_2}, r_2), \dots, (n_{j_k}, r_k))$. Then*

$$C_{S(\xi)}(F) \cong \underset{i=1}{\overset{k}{Dr}} \, (C_{Sym(\Omega_i)}(F|_{\Omega_i})(C_{Sym(\Omega_i)}(F|_{\Omega_i})\bar{\wr}\; S(\xi_i)))$$

where $Char(\xi_i) = \frac{Char(\xi)}{n_{j_i}} r_i$ and Ω_i is a representative of an orbit in the equivalence class Γ_i for $i = 1, \dots, k$.

By Proposition 2.3 (c) Hall's universal group U contains an isomorphic copy of $S(\xi)$ and when $Char(\xi) = \Pi\; 2^{\infty}3^{\infty}5^{\infty}\dots$ the group $S(\xi)$ contains isomorphic copy of U; see [6, Proposition 1.17]. But they are non-isomorphic as the structure of centralizers of elements are non-isomorphic; see [2].

For the direct limits of finite alternating groups, the following proposition is of interest.

Proposition 3.2. *(Hartley [6, Proposition 1.22]) Let G be the union of a tower of alternating groups, $G_1 \leq G_2 \leq G_3 \dots$ and the sequence $t_i \geq 2$ for infinitely* *many i where t_i is the natural representation of G_{i-1} repeated t_i times diagonally as above. Then the direct limit group G is not isomorphic to* $Alt(\mathbb{N})$.

One can see from [14], that if we take the prime decomposition of the sequence $(t_1, t_2, \ldots, t_i, \ldots)$, then the above group G will be isomorphic to $A(\xi)$ where ξ is the sequence obtained from t_i.

One may use our results about the structure of the centralizers of elements or centralizers of subgroups in $S(\xi)$ to decide easily that, such direct limit groups cannot be isomorphic to $Alt(\mathbb{N})$, as the structure of the centralizers of elements are completely different in direct limit group $S(\xi)$ and $Alt(\mathbb{N})$; see [2].

4 Centralizers of finite subgroups in FSym$(\kappa)(\xi)$

By using similar technique as in [14], we may construct uncountably many simple locally finite groups for any infinite cardinal κ. Let $FSym(\kappa)$ denote the finitary symmetric group and $Alt(\kappa)$ denote the alternating group on the set κ. Let Π be the set of sequences of prime numbers and $\xi \in \Pi$. Then ξ is a sequence of not necessarily distinct primes.

Let $\alpha \in FSym(\kappa)$, respectively $(Alt(\kappa))$. For a natural number $p \in \mathbb{N}$, a permutation $d^p(\alpha) \in FSym(\kappa p)$ defined by $(\kappa s + i)^{d^p(\alpha)} = \kappa s + i^\alpha, \quad i \in \kappa$ and $0 \leq s \leq p-1$ is called **homogeneous** p**-spreading** of the permutation α. We divide the ordinal κp into p equal parts and on each part we repeat the permutation diagonally as in the finite case. So if

$$\alpha = \begin{pmatrix} 1 \ldots \ n \\ i_1 \ldots i_n \end{pmatrix} \in FSym(\kappa),$$

then the homogeneous $p-$spreading of the permutation α is

$$d^p(\alpha) = \left(\begin{array}{ccc|ccc|c|ccc} 1 & \ldots & n & \kappa+1 & \ldots & \kappa+n & \ldots & \kappa(p-1)+1 & \ldots & \kappa(p-1)+n \\ i_1 & \ldots & i_n & \kappa+i_1 & \ldots & \kappa+i_n & \ldots & \kappa(p-1)+i_1 & \ldots & \kappa(p-1)+i_n \end{array} \right)$$

with the obvious meaning that the elements in $\kappa p \setminus supp(d^p(\alpha))$ are fixed.

We continue to take the embeddings using homogeneous p-spreadings with respect to the given sequence of primes in ξ. From the given sequence of embeddings, we have direct systems and hence direct limit groups $FSym(\kappa)(\xi)$ and $Alt(\kappa)(\xi)$ respectively. Observe that $FSym(\kappa)(\xi)$ and $Alt(\kappa)(\xi)$ are subgroups of $Sym(\kappa\omega)$ where ω is the first infinite ordinal.

Let F be a finite subgroup of $FSym(\kappa)(\xi) \leq Sym(\kappa\omega)$. Then F acts on $\kappa\omega$. The type of F is defined by $t(F) = ((n_{j_1}, r_1), (n_{j_2}, r_2), \ldots, (n_{j_k}, r_k))$ where n_{j_i} is the smallest positive integer in which F has an orbit Ω_i on the set with κn_{j_i} elements and there are r_i orbits giving equivalent representations of F and n_{j_i}'s are not necessarily distinct. We say that the i^{th} representation of F appears and appears as r_i times in $FSym(\kappa n_{j_i})$. For the centralizer of an arbitrary finite subgroup F of $FSym(\kappa)(\xi)$, we prove the following.

Theorem 4.1. *(Güven, Kegel, Kuzucuoğlu [2]) Let ξ be an infinite sequence of not necessarily distinct primes. Let F be a finite subgroup of $FSym(\kappa)(\xi)$ and $\Gamma_1, \ldots, \Gamma_k$ be the set of orbits of F such that the action of F on any two orbits in Γ_i is equivalent. Let the type of F be $t(F) = ((n_{j_1}, r_1), (n_{j_2}, r_2), \ldots, (n_{j_k}, r_k))$. Then*

$$C_{FSym(\kappa)(\xi)}(F) \cong (\underset{i=1}{\overset{k}{Dr}}\, C_{Sym(\Omega_i)}(F)(C_{Sym(\Omega_i)}(F)\bar{\wr}\ S(\xi_i)) \times FSym(\kappa)(\xi')$$

where $Char(\xi_i) = \frac{Char(\xi)}{n_{j_i}} r_i$ and $Char(\xi') = \frac{Char(\xi)}{n_{j_1}}$ and Ω_i is a representative of an orbit in the equivalence class Γ_i for $i = 1, \ldots, k$.

The following theorem gives the characterization of the groups $FSym(\kappa)(\xi)$ in terms of the lattice of Steinitz numbers. Therefore for any given infinite cardinal κ, there exists uncountably many pairwise non-isomorphic locally finite simple groups.

Theorem 4.2. *Let κ be a fixed infinite cardinal. There is a lattice isomorphism between the lattice of groups $\Sigma = \{FSym(\kappa)(\xi) \mid \xi \in \Pi\ \}$ ordered with respect to being a subgroup and the lattice $\mathcal{S}$ of Steinitz numbers ordered with respect to division in Steinitz numbers.*

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