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C-totally real pseudo-parallel submanifolds of S-space forms

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Abstract. Let $\overline{M}(c)$ be a $(2n+s)-$dimensional S-space form of constant $f-$sectional curvature c and M be an n-dimensional C-totally real, minimal submanifold of $\overline{M}(c)$. We prove that if M is pseudo parallel and $Ln - \frac{1}{4}(n(c+3s)+c-s) \geq 0$, then M is totally geodesic.

Keywords: S-manifolds, Sasakian manifolds, contact manifolds, pseudo-parallel submanifolds.

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1 Introduction

Given an isometric immersion $F : M \to \overline{M}$, let h be the second fundamental form and $\overline{\nabla}$ the Van der Weather- Bortolloti connection of M. Then Deprez defined the immersion to be semi-parallel if

$$\overline{R}(X,Y).h = (\overline{\nabla}_X \overline{\nabla}_Y - \overline{\nabla}_Y \overline{\nabla}_X - \overline{\nabla}_{[X,Y]})\, h = 0, \tag{1}$$

holds for any vectors X, Y tangent to M. Deprez mainly paid attention to the case of semi-parallel immersion in a real space form [see [9], [10]]. Later, Lumiste showed that a semi-parallel submanifold is the second order envelop of the family of parallel submanifolds [14].

In [11], authors obtained some results on hypersurfaces in 4-dimension space form $N^4(c)$ satisfying the curvature condition

$$\overline{R}.h = LQ(g,h). \tag{2}$$

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The submanifolds satisfying (1.2) are called pseudo-parallel [[1], [2]].

In [1], authors have shown that if F is a pseudo-parallel immersion with $H(p) = 0$ and $L(p) - c \geq 0$, then the point p is a geodesic point of M.

In the present paper, we generalize their results for the case of M, that is a submanifold of S-space form $\overline{M}(c)$ of constant $f-$sectional curvature c.

We prove the following result:

Theorem 1. *Let $\overline{M}(c)$ be a $(2n+s)-$dimensional S-space form of constant $f-$sectional curvature c and M be an n-dimensional C-totally real, minimal submanifold of $\overline{M}(c)$. If M is pseudo-parallel and $Ln - \frac{1}{4}(n(c+3s)+c-s) \geq 0$, then M is totally geodesic.*

2 Preliminaries

Let (M, g) be an n-dimensional $(n \geq 3)$ connected semi-Riemannian manifold of class C^∞. We denote by ∇, R and S the Levi-Civita connection, Riemannian curvature tensor, and Ricci tensor of (M, g), respectively. The Ricci operator Q is defined by $g(QX, Y) = S(X, Y)$, where $X, Y \in \chi(M)$, $\chi(M)$ being the Lie algebra of vector fields on M. Now we define endomorphisms $R(X, Y)$ and $X\Lambda_A Y$ of $\chi(M)$ by

$$R(X, Y)Z = \nabla_X \nabla_Y Z - \nabla_Y \nabla_X Z - \nabla_{[X,Y]} Z, \tag{3}$$

$$(X\Lambda_A Y)Z = A(Y, Z)X - A(X, Z)Y, \tag{4}$$

where X, Y, $Z \in \chi(M)$ and A is a symmetric (0,2)-tensor.

A concircular curvature tensor Z is defined by

$$Z(X, Y) = R(X, Y) - \frac{\kappa}{n(n-1)}(X\Lambda_A Y),$$

where κ is scalar curvature of M.

Let $F : M \to \overline{M}(c)$ be an isometric immersion of an n-dimensional Riemannian manifold M into a $(2n+1)-$dimensional real space form $\overline{M}(c)$. We denote by ∇ and $\overline{\nabla}$ the Levi-Civita connection of M and $\overline{M}(c)$, respectively. Also, we denote by $N(M)$ its normal bundle. Then for vector fields X, Y which are tangent to M, the second fundamental form h is given by the formula $h(X, Y) = \overline{\nabla}_X Y - \nabla_X Y$. Furthermore, for $\xi \in N(M)$, $A_\xi : TM \to TM$ denotes the Weingarten operator in the $\xi-$direction, $A_\xi X = \nabla^\perp_X \xi - \overline{\nabla}_X \xi$, where $\nabla^\perp$ denotes normal connection on M. The second fundamental form h and A_ξ are related by $\overline{g}(h(X, Y), \xi) = g(A_\xi X, Y)$, where g is the induced metric of $\overline{g}$ for any vector fields X, Y tangent to M. The mean curvature vector H of M is defined as

$$H = \frac{1}{n} tr(h).$$

The covariant derivative $\overline{\nabla}h$ of h is defined by

$$(\overline{\nabla}_X h)(Y,Z) = \nabla^{\perp}_X(h(Y,Z)) - h(\nabla_X Y, Z) - h(Y, \nabla_X Z), \tag{5}$$

where $\overline{\nabla}h$ is a normal bundle valued tensor of type (0, 3) and is called the third fundamental form of M. The equation of Codazzi implies that $\overline{\nabla}h$ is symmetric and hence

$$(\overline{\nabla}_X h)(Y,Z) = (\overline{\nabla}_Y h)(X,Z) = (\overline{\nabla}_Z h)(X,Y). \tag{6}$$

Here, $\overline{\nabla}$ is called the Van der Weather - Bortolloti connection of M. If $\overline{\nabla}h = 0$, then F is called parallel [13].

The basic equations of Gauss and Ricci are

$$\overline{g}(R(X,Y)Z,W) = cg(X\Lambda Y(Z)) + g(h(X,W),h(Y,Z)) - g(h(X,Z),h(Y,W)), \tag{7}$$

$$\overline{g}(R^{\perp}(X,Y)\xi,\eta) = g([A_{\xi},A_{\eta}]X,Y), \qquad \xi,\eta \in N(M), \tag{8}$$

respectively. Here $R^{\perp}$ is the curvature operator of the normal connection defined by

$$R^{\perp}(X,Y) = \nabla^{\perp}_X\nabla^{\perp}_Y Z - \nabla^{\perp}_Y\nabla^{\perp}_X Z - \nabla^{\perp}_{[X,Y]}Z.$$

An isometric immersion F is said to have flat normal connection if $R^{\perp} = 0$. If M has flat normal connection, then it called normally flat.

The second covariant derivative $\overline{\nabla}^2 h$ of h is defined by

$$(\overline{\nabla}^2 h)(Z,W,X,Y) = (\overline{\nabla}_X\overline{\nabla}_Y h)(Z,W)$$

$$= \nabla^{\perp}_X(\overline{\nabla}_X h)(Z,W) - (\overline{\nabla}_Y h)(\nabla_X Z,W) - (\overline{\nabla}_X h)(Z,\nabla_Y W) - (\overline{\nabla}_{\nabla_X Y} h)(Z,W). \tag{9}$$

Then we have

$$(\overline{\nabla}_X\overline{\nabla}_Y h)(Z,W) - (\overline{\nabla}_Y\overline{\nabla}_X h)(Z,W) = (\overline{R}(X,Y).h)(Z,W),$$

$$= R^{\perp}(X,Y)h(Z,W) - h(R(X,Y)Z,W) - h(Z,R(X,Y)W), \tag{10}$$

where $\overline{R}$ is curvature tensor belonging to the connection $\overline{\nabla}$.

3 S-space forms

Let $\overline{M}$ be a $(2m+s)-$dimensional framed metric manifold [18] (or almost r-contact metric manifolds [17]) with a framed metric structure $(f,\ \xi_{\alpha},\ \eta^{\alpha},\ \overline{g})$, $\alpha \in \{1,2,\ldots,s\}$, where f is a (1, 1) tensor field defining an $f-$structure of rank 2m, $\xi_1, \xi_2, \ldots, \xi_s$ are vector fields, η^1, η^2, $\ldots$, η^s are 1-forms and $\overline{g}$ is a Riemannian metric on $\overline{M}$ such that for all X, $Y \in T\overline{M}$ and α, $\beta \in \{1, 2, \ldots, s\}$,

$$f^2 = -I + \eta^\alpha \otimes \xi_\alpha, \quad \eta^\alpha(\xi_\alpha) = \delta^\alpha_\beta, \quad f(\xi_\alpha) = 0, \quad \eta^\alpha o f = 0, \tag{11}$$

$$\overline{g}(fX, fY) = \overline{g}(X, Y) - \sum_\alpha \eta^\alpha(X)\eta^\alpha(Y), \tag{12}$$

$$\Omega(X, Y) \equiv \overline{g}(X, fY) = -\Omega(Y, X), \qquad \overline{g}(X, \xi_\alpha) = \eta^\alpha(X). \tag{13}$$

A framed metric structure is an S-structure [3] if $[f, f] + 2d\eta^\alpha \otimes \xi_\alpha = 0$ and $\Omega = d\eta^\alpha$ for all $\alpha \in \{1, 2, \ldots, s\}$. When $s = 1$, a framed metric structure is an almost contact metric structure, while an S-structure is a Sasakian-structure. When $s = 0$, a framed metric structure is an almost Hermitian structure, while an S-structure is Käehler structure. If a framed metric structure on $\overline{M}$ is an S-structure, then it is known [3] that

$$(\overline{\nabla}_X f)Y = \sum_\alpha (\overline{g}(fX, fY)\xi_\alpha + \eta^\alpha(Y)f^2X), \tag{14}$$

$$\overline{\nabla}\xi_\alpha = -f, \qquad\qquad \alpha \in \{1, 2, \ldots, s\}. \tag{15}$$

The converse may also be proved. In case of Sasakian structure (i.e. $s = 1$) (3.4) implies (3.5). In Käehler case (i.e. $s = 0$), we get $\overline{\nabla} f = 0$. For $s > 1$, examples of S-structure are given in [3] [4] [5].

A plane section in $T_p\overline{M}$ is called a $f-$section if there exists a vector $X \in T_p\overline{M}$ orthogonal to ξ_1, ξ_2, $\ldots$, ξ_s such that $\{X, fX\}$ span the section. The sectional curvature of a $f-$section is called a $f-$sectional curvature. It is known that [14] in an S-manifold of constant $f-$sectional curvature c

$$\begin{aligned} \overline{R}(X, Y)Z = &\sum_{\alpha,\beta} \{\eta^\alpha(X)\eta^\beta(Z)f^2Y - \eta^\alpha(Y)\eta^\beta(Z)f^2X \\ &- \overline{g}(fX, fY)\eta^\alpha(Y)\xi_\beta + \overline{g}(fY, fZ)\eta^\alpha(X)\xi_\beta\} \\ &+ \frac{(c + 3s)}{4}\{-\overline{g}(fY, fZ)f^2X + \overline{g}(fX, fZ)f^2Y\} \\ &+ \frac{(c - s)}{4}\{\overline{g}(X, fZ)fY - \overline{g}(Y, fZ)fX + 2\overline{g}(X, fY)fZ\}, \end{aligned} \tag{16}$$

for all $X, Y, Z \in T\overline{M}$, where $\overline{R}$ is curvature tensor of $\overline{M}$. An S-manifold of constant $f-$sectional curvature c is called an S-space form $\overline{M}(c)$.

A submanifold M of an S-space form $\overline{M}(c)$ is called a C-totally real submanifold if and only if $f(T_xM) \subset T_x^\perp M$, for all $x \in M(T_xM$ and $T_x^\perp M$ are respectively the tangent space and normal space of M at $x)$. When ξ_α is tangent to M, M is a C-totally real submanifold if and only if $\nabla_X\xi_\alpha = 0$, for all

$X \in M$, $\alpha \in \{1, 2, \ldots, s\}$, where ∇ is the connection on M induced from Levi-Civita connection $\overline{\nabla}$ on $\overline{M}$. It is to see that the C-totally real submanifolds M of $\overline{M}$ are submanifolds with $\xi_\alpha \in T^\perp M$.

We already know that [1] if M is an n-dimensional C-totally real submanifold of a $(2m+s)-$dimensional S-space form $\overline{M}(c)$, then following statements are equivalent:

(i) M is totally geodesic.

(ii) M is of constant curvature $K = \frac{1}{4}(c+3s)$.

(iii) The Ricci tensor $S = \frac{1}{4}(n-1)(c+3s)g$.

(iv) The scalar curvature $\kappa = \frac{1}{4}n(n-1)(c+3s)$.

Following the argument as in [11], we can prove

Theorem 2. *Let M be a minimal, C-totally real submanifold of an S-space form $\overline{M}(c)$, then*

$$\kappa > \frac{n^2(n-2)}{2(2n-1)}(c+3s),$$

implies that M is totally geodesic.

Following the argument as in [10], we can prove:

Proposition 1. *If M is an n-dimensional C-totally real submanifold of an S-space form $\overline{M}(c)$. Then the following conditions are equivalent:*

(i) M is minimal.

(ii) The mean curvature vector H of M is parallel.

4 Main Results

Theorem 3. *Let $\overline{M}(c)$ be a $(2n+s)-$dimensional S-space form of constant f-sectional curvature c and M be an n-dimensional C-totally real, minimal submanifold of $\overline{M}(c)$. If M is pseudo parallel and $Ln - \frac{1}{4}(n(c+3s)+c-s) \geq 0$, then M is totally geodesic.*

Proof. Let M be an n-dimensional C-totally real submanifold of a $(2n+s)-$dimensional S-space form $\overline{M}(c)$ of constant $f-$sectional curvature c. We choose an orthonormal basis $\{e_1, e_2, \ldots, e_n, fe_1 = e_1^*, \ldots, fe_n = e_n^*, e_{n+1}^* = \xi_{1,\ldots,}$ $e_{n+s}^* = \xi_s\}$. Then for $1 \leq i,\ j \leq n$, $n+1 \leq \alpha \leq 2n+s$, the components of second fundamental form h are given by

$$h_{ij}^\alpha = g(h(e_i, e_j), e_\alpha). \tag{17}$$

Similarly, the components of first and second covariant derivative of h are given by

$$h^{\alpha}_{ijk} = g((\overline{\nabla}_{e_k} h)(e_i, e_j), e_\alpha) = \overline{\nabla}_{e_k} h^{\alpha}_{ij} \tag{18}$$

and

$$h^{\alpha}_{ijkl} = g((\overline{\nabla}_{e_l}\overline{\nabla}_{e_k} h)(e_i, e_j), e_\alpha) = \overline{\nabla}_{e_l} h^{\alpha}_{ijk} = \overline{\nabla}_{e_l}\overline{\nabla}_{e_k} h^{\alpha}_{ij} \tag{19}$$

respectively. It is well known that

$$h^{k^*}_{ij} = h^{i^*}_{kj} = h^{j^*}_{ik}, \qquad h^{(n+1)^*}_{ij} = 0.$$

If F is pseudo-parallel, then by definition, the condition

$$\overline{R}(e_l, e_k).h = L[(e_l \Lambda_g e_k)]h \tag{20}$$

is fulfilled where

$$[(e_l \Lambda_g e_k)h](e_i, e_j) = -h((e_l \Lambda_g e_k)e_i, e_j) - h(e_i, (e_l \Lambda_g e_k)e_j), \tag{21}$$

for $1 \leq i, j, k, l \leq n$.

Now using (2.2) in (4.5), we get

$$\begin{aligned}(e_i, e_j) = &- g(e_k, e_i)h(e_l, e_j) + g(e_l, e_i)h(e_k, e_j)\\ &- g(e_k, e_j)h(e_l, e_i) + g(e_l, e_j)h(e_k, e_i).\end{aligned} \tag{22}$$

By (2.9) we have

$$(\overline{R}(e_l, e_k).h)(e_i, e_j) = (\overline{\nabla}_{e_l}\overline{\nabla}_{e_k} h)(e_i, e_j) - (\overline{\nabla}_{e_k}\overline{\nabla}_{e_l} h)(e_i, e_j). \tag{23}$$

Making use of (4.1), (4.3), (4.6) and (4.7), the pseudo-parallelity condition (4.4) gives us

$$h^{\alpha}_{ijkl} = h^{\alpha}_{ijlk} - L\{\delta_{ki} h^{\alpha}_{lj} - \delta_{li} h^{\alpha}_{kj} + \delta_{kj} h^{\alpha}_{il} - \delta_{lj} h^{\alpha}_{ki}\}, \tag{24}$$

where $g(e_i, e_j) = \delta_{ij}$ and $1 \leq i, j, k, l \leq n$, $n + 1 \leq \alpha \leq 2n + s$.

Recall that the Laplacian $\triangle h^{\alpha}_{lj}$ of h^{α}_{lj} is defined by

$$\triangle h^{\alpha}_{lj} = \sum_{i,j,k=1}^{n} h^{\alpha}_{ijkk}. \tag{25}$$

Then we obtain

$$\frac{1}{2}\triangle(||h||^2) = \sum_{i,j,k,l=1}^{n} \sum_{\alpha=n+1}^{2n+s)} h^{\alpha}_{lj} h^{\alpha}_{ljkl} + ||\overline{\nabla} h||^2, \tag{26}$$

where

$$||h||^2 = \sum_{i,j,k,=1}^{n} \sum_{\alpha=n+1}^{2n+s} (h^{\alpha}_{lj})^2, \tag{27}$$

$$||\overline{\nabla} h||^2 = \sum_{i,j,k,l=1}^{n} \sum_{\alpha=n+1}^{2n+s} (h^{\alpha}_{ljkl})^2, \tag{28}$$

are the square of the length of second and third fundamental forms of M, respectively. In view of (4.1) and (4.3), we obtain

$$\begin{aligned} h^{\alpha}_{lj} h^{\alpha}_{ijkk} &= g(h(e_i, e_j), e_\alpha) g((\overline{\nabla}_{e_k} \overline{\nabla}_{e_k} h)(e_i, e_j), e_\alpha) \\ &= g((\overline{\nabla}_{e_k} \overline{\nabla}_{e_k} h)(e_i, e_j), g(h(e_i, e_j), e_\alpha), e_\alpha) \\ &= g((\overline{\nabla}_{e_k} \overline{\nabla}_{e_k} h)(e_i, e_j), h(e_i, e_j)). \end{aligned} \tag{29}$$

Therefore, due to (4.13), equation (4.10) becomes

$$\frac{1}{2} \triangle(||h||^2) = \sum_{i,j,k,=1}^{n} g((\overline{\nabla}_{e_k} \overline{\nabla}_{e_k} h)(e_i, e_j), h(e_i, e_j)) + ||\overline{\nabla} h||^2. \tag{30}$$

Further, by the use of (4.4), (4.6) and (4.7), we get

$$\begin{aligned} g((\overline{\nabla}_{e_k} \overline{\nabla}_{e_k} h)(e_i, e_j), h(e_i, e_j)) &= g((\overline{\nabla}_{e_k} \overline{\nabla}_{e_i} h)(e_k, e_j), h(e_i, e_j)) \\ =& g((\overline{\nabla}_{e_i} \overline{\nabla}_{e_k} h)(e_j, e_k)) - L\{g(e_i, e_j) g(h(e_k, e_k), h(e_i, e_j)) \\ &- g(e_k, e_j) g(h(e_k, e_i), h(e_i, e_j)) + g(e_k, e_i) g(h(e_j, e_k), h(e_i, e_j)) \\ &- g(e_k, e_k) g(h(e_i, e_j), h(e_i, e_j))\}. \end{aligned} \tag{31}$$

From equations (4.14) and (4.15), we have

$$\begin{aligned} \frac{1}{2} \triangle(||h||^2) = \sum_{i,j,k=1}^{n} & \Big[g((\overline{\nabla}_{e_i} \overline{\nabla}_{e_j} h)(e_k, e_k), h(e_i, e_j)) \\ &- L\{g(e_i, e_j) g(h(e_k, e_k), h(e_i, e_j)) - g(e_k, e_j) g(h(e_k, e_i), h(e_i, e_j)) \\ &+ g(e_k, e_i) g(h(e_j, e_k), h(e_i, e_j)) - g(e_k, e_k) g(h(e_i, e_j), h(e_i, e_j))\}\Big] \\ &+ ||\overline{\nabla} h||^2. \end{aligned} \tag{32}$$

Further by definitions

$$||h||^2 = \sum_{i,j=1}^{n} g(h(e_i, e_j), h(e_i, e_j)),$$

$$H^{\alpha} = \sum_{k=1}^{n} h^{\alpha}_{kk},$$

$$||H||^2 = \frac{1}{n^2} \sum_{\alpha=n+1}^{2n+s} (H^{\alpha})^2,$$

and after some calculations, we get

$$\frac{1}{2}\triangle(||h||^2) = \sum_{i,j=1}^{n}\sum_{\alpha=n+1}^{2n+s} h_{ij}^{\alpha}(\overline{\nabla}_{e_i}\overline{\nabla}_{e_j}H^{\alpha}) - L\{n^2||H||^2 - n||h||^2\} + ||\overline{\nabla}h||^2. \quad (33)$$

Using minimality condition, equation (4.17) reduces to

$$\frac{1}{2}\triangle(||h||^2) = Ln||h||^2 + ||\overline{\nabla}h||^2. \quad (34)$$

Now,using the arguments as Blair has shown in [9], we have

$$\begin{aligned}\frac{1}{2}\triangle(||h||^2) = ||\overline{\nabla}h||^2 - &\sum_{\alpha,\beta=n+1}^{2n+s}\left\{[T_r(A_\alpha o A_\beta)]^2 + ||[A_\alpha, A_\beta]||^2\right\}\\ &+\frac{1}{4}(n(c+3s)+c-s)||h||^2.\end{aligned} \quad (35)$$

From (4.18) and (4.19), we have

$$0 = (Ln - \frac{1}{4}(n(c+3s)+c-s))||h||^2 + \sum_{\alpha,\beta=n+1}^{2n+s}\left\{[T_r(A_\alpha o A_\beta)]^2 + ||[A_\alpha, A_\beta]||^2\right\},$$

if $Ln - \frac{1}{4}(n(c+3s)+c-s) \geq 0$, then $T_r(A_\alpha o A_\beta) = 0$.

In particular, $||A_\alpha||^2 = T_r(A_\alpha o A_\beta) = 0$, then $h = 0$ and hence M is totally geodesic.

Corollary 1. *Let $\overline{M}(c)$ be a $(2n+s)-$dimensional S-space form of constant $f-$sectional curvature c and M be an n-dimensional C-totally real, minimal submanifold of $\overline{M}(c)$. If M is semi-parallel (i.e. $\overline{R}.h = 0$) and $n(c+3s)+c-s \leq 0$, then it is totally geodesic.*

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