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C-α-Compact Spaces

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Abstract. Viglino[13], introduced the family of C-compact spaces, showing that every continuous function from a C-compact space into a Hausdorff space is a closed function and that this class of spaces properly contains the class of compact spaces. In the present paper, we study these spaces by considering α-open sets introduced by Njastad [11]. We also characterize their fundamental properties.

Keywords: α-open sets, α-regular space, α-compact space, C-compact spaces

MSC 2000 classification: 54A05, 54A20, 54C05, 54C10, 54D10, 54D30

Introduction

It is well known that the image of a compact space under a continuous function into a Hausdorff space is closed. If we denote by $\mathcal{P}$ the property that every continuous function from a topological space into a Hausdorff space is closed then the problem is whether underlying topological space having the property $\mathcal{P}$ is always compact. Viglino [13] resolved this problem in 1969 in the negative and substantiated his argument with an example. He simultaneously introduced a new class of topological space for which property $\mathcal{P}$ held. He called these spaces as C-compact. Since then, a tremendous number of papers such as Viglino[13], Sakai[12], Herringaton et.al.[7], Viglino[14], Goss & Viglino[6] and Kim[8] have appeared on C-compact spaces. The notion of α-open set was introduced by Njasted [11] in 1965. Since then, these sets are being used in investigating separation covering and connectivity properties such as Njasted[11], Biswas[2], Andrijevic[1], Caldas et.al.[3], Devi et.al.[4], Mashour et.al.[10], Maheshwari & Thakur[9] and Goss & Viglino[5].

In the present paper, we venture to generalize C-compact spaces by using α-open set and shall term them as C-α-compact spaces.

1 Preliminaries

Throughout this paper X and Y represents non-empty topological spaces on which no separation axioms are assumed, unless otherwise stated. For any subset A of X, cl(A) and int(A) respectively represents the closure and interior of A. Now we recall some definitions and results, which we have used in the sequel.

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Definition 1. Viglino[13] A topological space X is called C-compact if for each closed subset $A \subset X$ and for each open cover $\mathcal{U} = \{U_\lambda | \lambda \in \Lambda\}$ of A, there exists a finite subcollection $\{U_{\lambda_i} | 1 \leq i \leq n\}$ of $\mathcal{U}$ such that $A \subset \bigcup_{i=1}^{n} \mathrm{cl}(U_{\lambda_i})$.

Definition 2. Njastad[11] A subset A of a topological space X is called α-open if $A \subseteq \mathrm{int}(\mathrm{cl}(\mathrm{int}(A)))$. The complement of an α-open set is called an α-closed. Equivalently, a set F is α-closed in X if $\mathrm{cl}(\mathrm{int}(\mathrm{cl}(F))) \subseteq F$. The family of all α-open (respectively α-closed) sets in X is denoted by $\alpha O(X)$ (respectively $\alpha C(X)$.

Definition 3. Caldas et.al.[3] The intersection of all α-closed sets containing a subset $A \subset X$ is called the α-closure of A and is denoted by $\mathrm{cl}_\alpha(A)$.

Definition 4. Devi et.al.[4] A topological space X is said to be α-regular if for every closed set F and a point $x \notin F$, there exists disjoint α-open sets A and B such that $x \in A$ and $F \subset B$. A set U is α-regular open if $\mathrm{int}_\alpha(\mathrm{cl}_\alpha(U)) = U$.

Definition 5. Mashour et.al.[10] A map $\boldsymbol{f}\colon X \to Y$ is said to be α-continuous if the inverse image of every open subset of Y is α-open in X.

Remark 1. Mashour et.al.[10] Continuity implies α-continuity but not conversely.

Remark 2. Mashour et.al.[10] Every open mapping (closed mapping) is α-open (α-closed) but the converse is not true.

Definition 6. Maheshwari & Thakur[9] A topological space X is called α-compact if every α-cover of X has a finite subcover.

2 C-α-compact spaces

Definition 7. A topological space X is said to be C-α-compact if for each closed subset $A \subset X$ and for each α-open cover $\mathcal{U} = \{U_\lambda | \lambda \in \Lambda\}$ of A, there exists a finite sub collection $\{U_{\lambda_i} | 1 \leq i \leq n\}$ of $\mathcal{U}$, such that $A \subset \bigcup_{i=1}^{n} \mathrm{cl}_\alpha (U_{\lambda_i})$.

Lemma 1. *A topological space X is C-α-compact iff for each closed subset $A \subset X$ and for each α-regular open cover $\{U_\lambda | \lambda \in \Lambda\}$ of A, there exists a finite subcollection $\{U_{\lambda_i} | 1 \leq i \leq n\}$ such that $A \subset \bigcup_{i=1}^{n} cl_\alpha(U_{\lambda_i})$.*

Proof: Let X be C-α-compact and let $\{U_\lambda | \lambda \in \Lambda\}$ be any cover of A by α-open sets. Then $\mathcal{V} = \{\mathrm{int}_\alpha(\mathrm{cl}_\alpha(U_\lambda))\}$ is a α-regular open cover of A and so there exists a finite subcollection $\{\mathrm{int}_\alpha (\mathrm{cl}_\alpha(U_{\lambda_i})) : 1 \leq i \leq n\}$ of $\mathcal{V}$ such that $A \subset \bigcup_{i=1}^{n} \mathrm{cl}_\alpha\{\mathrm{int}_\alpha(\mathrm{cl}_\alpha(U_{\lambda_i}))\}$. But for each i, we have $\mathrm{cl}_\alpha \{\mathrm{int}_\alpha(\mathrm{cl}_\alpha(U_{\lambda_i}))\} = \mathrm{cl}_\alpha(U_{\lambda_i})$. Therefore, $A \subset \bigcup_{i=1}^{n} \mathrm{cl}_\alpha(U_{\lambda_i})$ implying that X is C-α-compact.

Theorem 1. *A α-continuous image of a C-α-compact space is C-α-compact.*

Proof: Let A be a closed subset of Y and let $\mathcal{V}$ be an α-open cover of A. By α-continuity of $\boldsymbol{f}$, $\boldsymbol{f}^{-1}(A)$ is an α-closed subset of X and is such that $P = \{\boldsymbol{f}^{-1} (V)\colon V \in \mathcal{V}\}$ is a cover of $\boldsymbol{f}^{-1}(A)$ by α-open sets. By C-α-compactness of X, there exists finite collection say; $\{P_i\colon 1 \leq i \leq n\}$ of P such that $\boldsymbol{f}^{-1} (A) \subset \bigcup_{i=1}^{n} \{\mathrm{cl}_\alpha (\boldsymbol{f}^{-1} (V_i)\colon 1 \leq i \leq n\}$. Now by α-continuity of $\boldsymbol{f}$, $A \subset \bigcup_{i=1}^{n} \{\mathrm{cl}_\alpha (V_i)\colon 1 \leq i \leq n\}$. Thus Y is a C-α-compact space.

3 α-Hausdorff and C-α-compact spaces

X is said to be an α-Hausdorff space if for any pair of distinct points x and y in X, there exists α-open set a U and V in X such that $x \in U$, $y \in V$ and $U \cap V = \phi$.

Definition 8. A set U in a topological space X is an α-neighborhood of a point x if U contains an α-open set V, such that $x \in V$.

Definition 9. Let X be a topological space and A be a subset of X then an element $x \in X$ is called α-adherent point of A if every α-open set G containing x contains at least one point of A, that is, $G \cap A \neq \phi$.

Definition 10. Let X is a non-empty set. A non-empty collection B of non-empty subset of X is called a basis for some filter on X if

(1) $\phi \notin \mathcal{B}$

(2) If B_1, $B_2 \in \mathcal{B}$ then there exist a $B \in \mathcal{B}$ such that $B \subset B_1 \cap B_2$.

Definition 11. Let $\mathcal{B}$ be a filter base on a set X then the filter $\mathcal{F}$ is said to be generated by $\mathcal{B}$ if $\mathcal{F} = \{ A : A \subset B, B \in \mathcal{B} \}$.

Definition 12. A filter base $\mathcal{F}$ is said to be α-adherent convergent if every neighborhood of the α-adherent set of $\mathcal{F}$ contains an element of $\mathcal{F}$.

Theorem 2. *For any α-Hausdorff space X. The following properties are equivalent.*

(1) X is C-α-compact.

(2) For each closed subset $A \subset X$ and for each family $\boldsymbol{f}$ of closed set of X with $\cap\{F \cap A\colon F \in \boldsymbol{f}\} = \phi$, there exists finite collection say; $\{F_i : 1 \leq i \leq n\}$ of $\boldsymbol{f}$ with $\bigcap_{i=1}^{n} \{(int_\alpha F_i) \cap A\colon 1 \leq i \leq n\} = \phi$.

(3) If A is a closed subset of X and $\boldsymbol{f}$ is an open filter base on X whose element have non-empty with A, then $\boldsymbol{f}$ has a α-adherent point in X.

Proof (i) $\Rightarrow$ (ii) Let A be a closed subset of a C-α-compact space X and $\boldsymbol{f}$ be a family of closed subset of X with $\cap\{F \cap A\colon F \in \boldsymbol{f}\} = \phi \Rightarrow A \subset X \sim \cap\{F\colon F \in \boldsymbol{f}\}$ or $A \subset \cup\{X \sim F\colon F \in \boldsymbol{f}\}$. Therefore $\{X \sim F\colon F \in \boldsymbol{f}\}$ is an open cover of A. Since every open set is a α-open set, therefore $\mathcal{G} = \{X \sim F\colon F \in \boldsymbol{f}\}$ is a α-open cover of A and so by C-α-compactness of X, there exists a finite subfamily say; $\{X \sim F_i : 1 \leq i \leq n$ and $F_i \in \boldsymbol{f}\}$ of $\mathcal{G}$ such that $A \subset \bigcup_{i=1}^{n} \{\mathrm{cl}_\alpha (X \sim F_i) : 1 \leq i \leq n\} \Rightarrow A \subset X \sim \bigcap_{i=1}^{n} \{(\mathrm{int}_\alpha F_i)\colon 1 \leq i \leq n\}$. Therefore $\bigcap_{i=1}^{n} \{(\mathrm{int}_\alpha F_i) \cap A\colon 1 \leq i \leq n\} = \phi$.

(ii) $\Rightarrow$ (iii) Suppose that there exist a closed set A and let $\mathcal{G}$ be an open filter base having non-empty trace with A such that $\mathcal{G}$ has no α-adherent point. Now $\boldsymbol{f} = \{(\mathrm{cl}_\alpha G)\colon G \in \mathcal{G}\}$ is a family of closed set such that $\cap\{(\mathrm{cl}_\alpha G)\colon G \in \mathcal{G}\} \cap A = \phi$ (But $\mathrm{cl}_\alpha G$ is a superset of G so $\mathrm{cl}_\alpha G \in \mathcal{G}$. Therefore $\cap\{(\mathrm{cl}_\alpha G) \cap A\colon G \in \mathcal{G}\}$ is a trace of A and by hypothesis $\mathcal{G}$ has no α-adherent point, therefore $\cap\{(\mathrm{cl}_\alpha G) \cap A\colon G \in \mathcal{G}\} = \phi$) so there is a finite subfamily of $\boldsymbol{f}$, say $\{F_i = \mathrm{cl}_\alpha G_i\colon 1 \leq i \leq n\}$ with $\bigcap_{i=1}^{n} \{(\mathrm{int}_\alpha F_i) \cap A\colon 1 \leq i \leq n\} = \phi$ or $A \subset \bigcup_{i=1}^{n} \{ X \sim \mathrm{int}_\alpha (\mathrm{cl}_\alpha G_i)\colon 1 \leq i \leq n\}$. Therefore $\bigcap_{i=1}^{n} \{ G_i \cap A\colon 1 \leq i \leq n\} = \phi$. Since $\mathcal{G}$ is a filter base therefore there must exist a $G \in \mathcal{G}$ such that $G \subset \bigcap_{i=1}^{n} \{G_i\colon 1 \leq i \leq n\}$. So $G \cap A = \phi$, a contradiction.

(iii) $\Rightarrow$ (i) Assume that X is not C-α-compact then there is a closed subset A and a covering $\mathcal{U}$ of A consisting of α-open subset of Xsuch that for any finite subfamily $\{U_i\colon 1 \leq i \leq n\}$ of $\mathcal{U}$, $A \not\subset \bigcup_{i=1}^{n} \{\mathrm{cl}_\alpha U_i\colon 1 \leq i \leq n\}$. Now $\mathcal{G} = [X \sim \bigcup_{i=1}^{n} \{\mathrm{cl}_\alpha U_i\colon 1 \leq i \leq n$ and $U_i \in \mathcal{U}\}]$

is an open filter base having non-empty trace with A, so by (iii) there is an α-adherent point of G in A, let it be x. Thus $x \in \mathrm{cl}_\alpha\ [X \sim \bigcup_{i=1}^{n} \{\mathrm{cl}_\alpha U_i\colon 1 \le i \le n\}]$ say; for each $\mathcal{G} \in G$ or $x \in [X \sim \bigcup_{i=1}^{n} \{U_i\colon 1 \le i \le n\}]$. Therefore, $\mathcal{U}$ is not a covering of A, a contradiction .

Theorem 3. *A α-Hausdorff space X is C-α-compact iff every open filter base $\mathcal{F}$ is α-adherent convergent.*

Proof: Let $\mathcal{F}$ be an open filter base of the C-α-compact space X. Let A be α-adherent set of $\mathcal{F}$. Let G be an open-neighborhood of A. Since A is the α-adherent set of $\mathcal{F}$, we have $A = \cap\{\mathrm{cl}_\alpha F\colon F \in \mathcal{F}\}$. Since G is an open-neighborhood of A, we have $A \subset G$ and $X \sim G$ is closed. Clearly $\{X \sim \mathrm{cl}_\alpha F\colon F \in \mathcal{F}\}$ is an α-open cover of $X \sim G$ and so $X \sim G \subset \bigcup_{i=1}^{n}\{\mathrm{cl}_\alpha (X \sim \mathrm{cl}_\alpha\ F_i)\colon 1 \le i \le n\}$. This implies $\bigcap_{i=1}^{n}\{\mathrm{cl}_\alpha\ (X \sim \mathrm{cl}_\alpha F_i)\colon 1 \le i \le n\} \subset G$. Further $X \sim \mathrm{cl}_\alpha F_i \subset X \sim F_i$ or $\bigcap_{i=1}^{n}\{F_i\colon 1 \le i \le n\} \subset \bigcap_{i=1}^{n}\{\mathrm{cl}_\alpha\ (X \sim \mathrm{cl}_\alpha F_i)\colon 1 \le i \le n\}$. Thus $\bigcap_{i=1}^{n}\{F_i\colon 1 \le i \le n\} \subset G$, that is, open-neighborhood G of A contains a point of $\mathcal{F}$.

Conversely, let X be a non C-α-compact space and let A be any closed subset of X. Choose an α-open cover $\mathcal{U}$ of A such that A is not contained in the α-closure of any finite union of elements in $\mathcal{U}$. Without loss of generality we may consider $\mathcal{U}$ to be closed under finite unions. Obviously then $\mathcal{F} = \{X \sim \mathrm{cl}_\alpha G\colon G \in \mathcal{U}\}$ is an open filter base in X. Let x be an α-adherent point of $\mathcal{F}$. This clearly implies that $x \notin A$. So the α-adherent set of the open filter base $\mathcal{F}$ is contained in $X \sim A$, but no element of $\mathcal{F}$ is contained in $X \sim A$.

Theorem 4. *A α-Hausdorff space X is C-α-compact iff for each closed subset C of X and α-open cover $\mathcal{C}$ of $X \sim C$ and a open-neighborhood U of C, there exists a finite collection $\{G_i \in \mathcal{C}\colon 1 \le i \le n\}$ such that $X = U \cup \bigcup_{i=1}^{n}\{cl_\alpha G_i\colon 1 \le i \le n\}$.*

Proof: Since U is an open-neighborhood of C, therefore $C \subset U \subset \mathrm{cl}(C)$, or $X \sim U \subset X \sim C$ where $X \sim U$ is a α-closed set. Further, as $\mathcal{C}$ is a α-open cover of $X \sim C$. Therefore $\mathcal{C}$ is a α-open cover of the α-closed set $X \sim U$ too. Now by C-α-compactness of X, there exists a finite subfamily $\{G_i\colon 1 \le i \le n\}$ of $\mathcal{C}$ such that $X \sim U \subset \bigcup_{i=1}^{n}\{\mathrm{cl}_\alpha G_i\colon 1 \le i \le n\}$. Which implies $X = U \cup \bigcup_{i=1}^{n}\{\mathrm{cl}_\alpha G_i\colon 1 \le i \le n\}$.

Conversely; Let A be a closed subset of X, $\mathcal{G}$ be an α-open cover of A, Therefore $A \subset \cup\{G\colon G \in \mathcal{G}\} = H$ (say), obviously H is α-open, therefore $X \sim H = C$ (say), is α-closed and $C \subset X \sim A$. since $X \sim A$ is α-open. Therefore we can take $X \sim A = U$ is an open-neighborhood of C, thus by the given statement $X = U \cup \bigcup_{i=1}^{n}\{\mathrm{cl}_\alpha G_i\colon 1 \le i \le n\}$. Hence X is C-α-compact.

Theorem 5. *Every α-continuous function from a C-α-compact space to a α-Hausdorff space is closed.*

Proof: Let $\boldsymbol{f}$ be α-continuous function from a C-α -compact space X to a α-Hausdorff space Y. Let C be a closed set in X and let $p \notin \boldsymbol{f}\ (C)$. Now for every $x \in \boldsymbol{f}\ (C)$, $x \neq p$ and hence choose a open-neighborhood N_x such that $p \notin \mathrm{cl}_\alpha\ (N_x)$, obviously $\{\ \boldsymbol{f}^{-1}(N_x)\colon x \in\ \boldsymbol{f}(C)\}$ is a α-open cover of C. Let $\{x_i\colon 1 \le i \le n\}$ be such that $C \subset \bigcup_{i=1}^{n}\{\mathrm{cl}_\alpha\ \boldsymbol{f}^{\ -1}\ (N_{x_i})\colon 1 \le i \le n\}$, because X is C-α-compact space. Thus by the α-continuity of $\boldsymbol{f}$, $Y \sim \bigcup_{i=1}^{n}\{\ \mathrm{cl}_\alpha (N_{x_i})\colon 1 \le i \le n\}$ is a α-neighborhood of p disjoint from Y. Hence C is closed, so α-continuous function $\boldsymbol{f}$ from C-α-compact space X to a α-Hausdorff space Y is closed.

4 Study of functionally compact spaces and C-α -compact spaces

A Hausdorff space X is said to be a functionally compact space if for every open filter base $\mathcal{U}$ in X, the intersection A of the elements of $\mathcal{U}$ is equal to the intersection of the closure of the elements of $\mathcal{U}$, then $\mathcal{U}$ is a base for the neighbourhood of A.

Theorem 6. *Every C-α-compact space is functionally compact.*

Proof: Let $\mathcal{U}$ be an α-open filter base in the C-α-compact space X. Let $A = \cap(U : U \in \mathcal{U})$ $= \cap$ $(\mathrm{cl}_\alpha U{:}U \in \mathcal{U})$. Let G be an α-open set containing A. Then $\cap$ $(\mathrm{cl}_\alpha U{:}\ U \in \mathcal{U})$ is a subset of G, that is, $\cap$ $(X \sim \mathrm{cl}_\alpha U{:}\ U \in \mathcal{U})$. Now $X \sim G$ is a α-closed subset of the C-α-compact space X. Therefore the α-open cover $(X \sim \mathrm{cl}_\alpha U{:}\ U \in \mathcal{U})$ of $X \sim G$ has a finite subfamily, say $(X \sim \mathrm{cl}_\alpha U_i{:}\ 1 \leq i \leq n)$ such that $X \sim G \subset \cup$ $(\mathrm{cl}_\alpha(X \sim \mathrm{cl}_\alpha U_i){:}\ 1 \leq i \leq n) \subset (X \sim \mathrm{cl}_\alpha U_i{:}\ 1 \leq i \leq n)$, that is, $\cap$ $(U_i{:}\ 1 \leq i \leq n) \subset G$. Since $\mathcal{U}$ is a filter base there exists a $U \in \mathcal{U}$ such that $U \subset \cap$ $(U_i{:}\ 1 \leq i \leq n)$ and hence $U \subset G$ and the space X is functionally compact.

Theorem 7. *A α-Hausdorff space X is functionally compact iff for every α-regular closed subset C of X and α-open cover $\mathcal{B}$ of $X \sim C$ and a open-neighborhood U of C, there exists a finite collection $\{B_{x_i} \in \mathcal{B}{:}\ 1 \leq i \leq n\}$ such that $X = U \cup [\bigcup_{i=1}^{n}\{cl_\alpha B_{x_i}{:}\ 1 \leq i \leq n\}]$.*

Proof: For each $x \in$ X $\sim C$, since C is α-regular closed, there exists an α-open set A_x such that $\mathrm{cl}_\alpha A_x \subset X \sim C$. Also there exists a $B_x \in \mathcal{B}$ such that $x \in B_x$. Let $G_x = A_x \cap B_x$. Then G_x is an α-open set such that $x \in G_x$, $\mathrm{cl}_\alpha G_x \subset X \sim C$ and there exists a $B_x \in \mathcal{B}$ such that $G_x \subset B_x$. Also $X \sim C = \cup$ $\{G_x{:}\ x \in X \sim C\} = \cup$ $\{\mathrm{int}_\alpha\ \mathrm{cl}_\alpha\ G_x{:}\ x \in X \sim C\} = \cup$ $\{\mathrm{cl}_\alpha\ G_x{:}\ x \in X \sim C\}$. Suppose, if possible, that no finite collection of $\mathcal{G}$ is such that the α-closure of its members cover $X \sim U$. Now for any finite collection $\{G_{x_i}{:}\ 1 \leq i \leq n\}$ of $\mathcal{G}$, $\cap$ $\{X \sim \mathrm{cl}_\alpha\ G_{x_i}{:}\ 1 \leq i \leq n\} \neq \phi$. Let $\mathcal{V}$ be the family of the all finite intersection of the family $\{X \sim \mathrm{cl}_\alpha\ G_x{:}\ x \in X \sim C\}$. Now $\mathcal{V}$ is an α-open filter base such that $\cap$ $\{V : V \in \mathcal{V}\} = \cap$ $\{X \sim \mathrm{cl}_\alpha\ G_x{:}\ x \in X \sim C\} = X \sim \cap\{\mathrm{cl}_\alpha\ G_x{:}\ x \in X \sim C\} = C$ and $\cap$ $\{\mathrm{cl}_\alpha V : V \in \mathcal{V}\} = \cap$ $\{\mathrm{cl}\ _\alpha(X \sim \mathrm{cl}_\alpha\ G_x){:}\ x \in X \sim C\} = \cap$ $\{X \sim \mathrm{int}_\alpha\ \mathrm{cl}_\alpha\ G_x){:}\ x \in X \sim C\} = X \sim \cup$ $\{\mathrm{int}_\alpha\ \mathrm{cl}_\alpha\ G_x){:}\ x \in X \sim C\}$ $= C$. But there exists no $V \in \mathcal{V}$ such that $C \subset V \subset U$ and this is a contradiction to the fact that X is functionally compact. Hence there exists a finite collection $\{G_{x_i}{:}\ 1 \leq i \leq n\}$ and hence $\{B_{x_i}{:}\ 1 \leq i \leq n\}$ such that $X = U \cup [\bigcup_{i=1}^{n}\{\mathrm{cl}_\alpha B_{x_i}{:}\ 1 \leq i \leq n\}]$.

Conversely; Let $\mathcal{U}$ be an α-open filter base such that $A = \cap$ $\{U : U \in \mathcal{U}\} = \cap$ $\{\mathrm{cl}\ _\alpha U : U \in \mathcal{U}\}$. Let G be an α-open set containing A. Now for each $x \in X \sim A$, there exists a $U \in \mathcal{U}$ such that $x \notin \mathrm{cl}_\alpha U$. Now $x \in X \sim \mathrm{cl}_\alpha U$ and $\mathrm{cl}_\alpha(X \sim \mathrm{cl}_\alpha U) \cap A = \phi$, because $A \subset U$ for each $U \in \mathcal{U}$. Therefore, A is a α-regular closed set. $\{X \sim \mathrm{cl}_\alpha U{:}\ U \in \mathcal{U}\}$ is an α-open cover of $X \sim A$. Therefore there exists a finite collection $\{X \sim \mathrm{cl}_\alpha\ U_i{:}\ 1 \leq i \leq n\}$ of $\{X \sim \mathrm{cl}_\alpha\ U{:}\ U \in \mathcal{U}\}$ such that $X = G \cup [\cup\{\mathrm{cl}_\alpha(X \sim \mathrm{cl}_\alpha\ U_i){:}\ 1 \leq i \leq n\}]$. Thus $X \sim G \subset \cup$ $\{\mathrm{cl}_\alpha(X \sim \mathrm{cl}_\alpha\ U_i){:}\ 1 \leq i \leq n\}$, that is, $\cup\{\mathrm{int}_\alpha\ \mathrm{cl}\ _\alpha\ U_i{:}\ 1 \leq i \leq n\} \subset G$, that is, $\cup\{U_i{:}\ 1 \leq i \leq n\} \subset G$. Since $\mathcal{U}$ is an α-open filter base, there exists a $U \in \mathcal{U}$ such that $U \subset \{U_i{:}\ 1 \leq i \leq n\}$ and hence $U \subset G$. Thus X is functionally compact.

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