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On $\Pi g\beta$-closed sets in topological spaces

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Abstract. A new class of sets called $\Pi g\beta$-closed sets is introduced and its properties are studied. Moreover the notions of $\Pi g\beta - T_{1/2}$ spaces and $\Pi g\beta$-continuity are introduced.

Keywords: Π-open set, Πg-closed set, $\Pi g\beta$-continuous, $\Pi g\beta$-irresolute.

MSC 2000 classification: 54C08, 54D10

1 Introduction and Preliminaries

In 1970, Levine [16] initiated the study of so called g-closed sets, that is, a subset A of a topological space (X, τ) is said to be g-closed if the closure of A is included in every open superset of A and defined a $T_{1/2}$ space to be one in which the closed sets and g-closed sets coicide. The notion has been studied extensively in recent years by many topologists because g-closed sets plays not only important role in generalization of closed sets but also they suggested some new separation axioms. Some of these have been found to be useful in computer science and digital topology [8], [11], [12], [13], [14] and quantum physics [10].

As generalization of β-closed sets, gsp-closed sets or $g\beta$-closed sets were introduced and studied by Dontchev [7] and Tahiliani [24]. Also Tahiliani [24] introduced some characterizations of β-normal and β-regular spaces.

On the other hand, Zaitsav [25] introduced the concepts of Π-closed sets and a class of topological spaces called quasi normal spaces.

Recently, Dontchev and Noiri [9] introduced the notion of Πg-closed sets and used this notion to obtain a characterizations and some preservation theorems for quasi normal spaces. More recently, Park [22] has introduced and studied the notion of Πgp-closed sets which is implied by that of gp-closed sets. Park and Park [23] continued the study of Πgp-closed sets and associated functions and introduced the concepts of ΠGP-compactness and ΠGP-connectedness. Also Aslim, Guler and Noiri [5] introduced the concept of Πgs-closed sets and studied it's basic properties. Moreover they also introduced the notions of Πgs-$T_{1/2}$ spaces and Πgs-continuity in topological spaces. In this paper, we introduce and study the notion of $\Pi g\beta$-closed sets which is implied by that of $g\beta$-closed sets. The notion of $\Pi g\beta$-open sets, $\Pi g\beta$-$T_{1/2}$ spaces, $\Pi g\beta$-continuity and $\Pi g\beta$-irresoluteness are also introduced.

Throughout this paper, all spaces X,Y and Z (or (X, τ), (Y, σ) and (Z, ζ)) are always topological spaces. Let A be a subset of X. We denote the interior and closure of a set A by $Int(A)$ and $Cl(A)$ respectively. The complement of a subset A of X is denoted by $X \sim A$. A subset A of a topological space X is called semi-open [6], preopen [19], α-open [20] (resp. β-open [1] or semipre-open [3] if $A \subseteq Cl(Int(A))$, $A \subseteq Int(Cl((A)))$, $A \subseteq Int(Cl(Int(A)))$

(resp. $A \subseteq Cl(Int(Cl(A)))$). Complements of semi-open, preopen (resp. β-open) sets are called semi-closed, preclosed (resp. β-closed).

The family of all β-open (resp. β-closed) subsets of X is denoted by $\beta O(X)$ (resp. $\beta C(X)$). Also $SCl(A)$, $PCl(A)$, (resp. $\beta Cl(A)$) denotes the semiclosure [6], pre-closure [19] (resp. β-closure [2]) of A and it is the intersection of all semi-closed, pre-closed, (resp. β-closed) supersets of A. Further A is said to be regular open (resp. regular closed) if $Int(Cl(A)) = A$ (resp. $Cl(Int(A)) = A$). The finite union of regular open sets is said to be Π-open and its complement is Π-closed.

2 $\Pi g\beta$-closed sets

Definition 1. A subset A of a topological space (X, τ) is said to be:

(i). g-closed [16] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(ii). Πg-closed [9] if $Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is Π-open in X.

(iii). gp-closed [18] if $PCl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(iv). Πgp-closed [22] if $PCl(A) \subseteq U$ whenever $A \subseteq U$ and U is Π-open in X.

(v). gs-closed [4] if $SCl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(vi). Πgs-closed [5] if $SCl(A) \subseteq U$ whenever $A \subseteq U$ and U is Π-open in X.

(vii). $g\beta$-closed [24] if $\beta Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is open in X.

(viii). $\Pi g\beta$-closed if $\beta Cl(A) \subseteq U$ whenever $A \subseteq U$ and U is Π-open in X.

(ix). $\Pi g\beta$-open (resp. g-open, Πg-open, gp-open, Πgp-open, gs-open, Πgs-open, $g\beta$-open) if the complement of A is $\Pi g\beta$-closed (resp. g-closed, Πg-closed, gp-closed, Πgp-closed, gs-closed, Πgs-closed, $g\beta$-closed).

Definition 2. A subset A of a topological space (X, τ) is said to be:

(i). $T_{1/2}$space [16] if every g-closed set is closed.

(ii). $\Pi g\beta$-$T_{1/2}$space if every $\Pi g\beta$-closed set is β-closed.

Proposition 1. *Let (X, τ) be a topological space and $A \subseteq X$. The following properties hold:*

(i). If A is $g\beta$-closed, then it is $\Pi g\beta$-closed.

(ii). If A is Πg-closed, then it is $\Pi g\beta$-closed.

Proof. Obvious.

Remark 1. From Proposition 1, we have the following diagram:

$$\begin{array}{c} \Pi gp\textbf{-closed} \\ \downarrow \\ g\beta\textbf{-closed} \rightarrow \Pi g\beta\textbf{-closed} \\ \uparrow \\ \Pi gs\textbf{-closed} \end{array}$$

Remark 2. None of the implications are reversible as the concept of Πgp-closed sets and Πgs-closed sets are independent of each other as can be seen in Example 2.1 and 2.2 of [5].

Example 1. Let $X = \{a, b, c, d\}$ and $\tau = \{\emptyset, X, \{b\}, \{c\}, \{b, c\}, \{b, d\}, \{a, b, c\}, \{b, c, d\}\}$ be the topology on X. Then $\{b\}$ is $\Pi g\beta$-closed but not $g\beta$-closed.

Theorem 1. *For a subset $A \subseteq X$, the following conditions are equivalent:*

(i). A is Π-open and $\Pi g\beta$-closed.

(ii). A is regular open.

Proof.
$(i) \Rightarrow (ii)$. By (i), $\beta Cl(A) \subseteq A$, since A is Π-open and $\Pi g\beta$-closed. Thus $Int(Cl(Int(A))) \subseteq A$, since $\beta Cl(A) = A \cup Int(Cl(Int(A)))$. As A is open, then A is clearly α-open and so $A \subseteq Int(Cl(Int(A)))$. Thus $Int(Cl(Int(A))) \subseteq A \subseteq Int(Cl(Int(A)))$ or equivalently $A = Int(Cl(A))$. This shows that A is regular open.

$(ii) \Rightarrow (i)$. Every regular open set is Π-open and it is even β-closed.

Corollary 1. *If A is open and $\Pi g\beta$-closed set of (X, τ), then A is β-closed and hence $g\beta$-closed.*

Proof. By assumption and Theorem 1, A is regular open. Thus A is β-closed and hance $g\beta$-closed.

Levine [15] defines a set A in a topological space to be a Q-set if $Int(Cl(A)) = Cl(Int(A))$.

Theorem 2. *For a subset $A \subseteq X$, the following properties are equivalent:*

(i). A is Π-clopen.

(ii). A is Π-open, Q-set and $\Pi g\beta$-closed.

Proof.
$(i) \Rightarrow (ii)$. Obvious.

$(ii) \Rightarrow (i)$. By Theorem 1, A is regular open. Since A is a Q-set, $A = Int(Cl(A)) = Cl(Int(A))$. So A is regular closed and Π-closed and hence A is Π-clopen.

Remark 3. The union of two $\Pi g\beta$-closed sets may not be $\Pi g\beta$-closed as can be seen from the following example:

Example 2. Let $X = \{a, b, c\}$ and let $\tau = \{\oslash, X, \{a\}, \{b\}, \{a, b\}\}$. Then $\{a\}$ and $\{b\}$ are $\Pi g\beta$-closed but $\{a, b\}$ is not $\Pi g\beta$-closed.

Remark 4. The intersection of two $\Pi g\beta$-closed sets need not be $\Pi g\beta$-closed as it can be seen from the following example:

Example 3. Let $X = \{a, b, c, d\}$ and let $\tau = \{\oslash, X, \{c\}, \{d\}, \{c, d\}, \{a, c, d\}, \{b, c, d\}\}$. Then $\{a, c, d\}$ and $\{b, c, d\}$ are $\Pi g\beta$-closed but $\{c, d\}$ is not $\Pi g\beta$-closed.

Theorem 3. *Let A be a $\Pi g\beta$-closed in (X, τ). Then $\beta Cl(A) \sim A$ does not contain non empty Π-closed set.*

Proof. Let F be a Π-closed set such that $F \subseteq \beta Cl(A) \sim A$. Then $F \subseteq X \sim A$ implies $A \subseteq X \sim F$. Therefore $\beta Cl(A) \subseteq X \sim F$. That is $F \subseteq X \sim \beta Cl(A)$. Hence $F \subseteq (\beta Cl(A)) \cap (X \cap \beta Cl(A)) = \oslash$. This shows that $F = \oslash$.

Theorem 4. *If A is $\Pi g\beta$-closed and $A \subseteq B \subseteq \beta Cl(A)$, then B is $\Pi g\beta$-closed.*

Proof. Let A be a $\Pi g\beta$-closed and $B \subseteq U$, where U is Π-open. Then $A \subseteq B$ implies $A \subseteq U$. Since A is $\Pi g\beta$-closed, $\beta Cl(A) \subseteq U$. Also $B \subseteq \beta Cl(A)$ implies $\beta Cl(B) \subseteq \beta Cl(A)$. Therefore $\beta Cl(B) \subseteq U$ and B is $\Pi g\beta$-closed.

Definition 3. Let (X, τ) be a topological space and $A \subseteq X$. A point $x \in X$ is called a β-limit poin of A [17] if every β-open set containing x contains a point of A different from x.

Definition 4. Let (X, τ) be a topological space and $A \subseteq X$. The set of all β-limit points of A is said to be β-derived set of A and is denoted by $D_\beta [A]$.

Similarly, $x \in D[A]$ iff any open set U such that $x \in U$ meets $A \sim \{x\}$.

Theorem 5. *Let A and B be $\Pi g\beta$-closed sets in (X,τ) such that $D[A] \subseteq D_\beta[A]$ and $D[B] \subseteq D_\beta[B]$. Then $A \cup B$ is $\Pi g\beta$-closed.*

Proof. For any set $E \subseteq X$, $D_\beta[E] \subseteq D[E]$. Therefore $D[A] = D_\beta[A]$ and so $D[B] = D_\beta[B]$. Then $\beta Cl(A) = Cl(A)$ and $\beta Cl(B) = Cl(B)$. Let $A \cup B \subseteq U$ where U is Π-open. Then $A \subseteq U$ and $B \subseteq U$. Since A and B are $\Pi g\beta$-closed, $\beta Cl(A) \subseteq U$ and $\beta Cl(B) \subseteq U$. Then $\beta Cl(A \cup B) \subseteq Cl(A \cup B) = Cl(A) \cup Cl(B) = \beta Cl(A) \cup \beta Cl(B) \subseteq U$. So $\beta Cl(A \cup B) \subseteq U$ and hence $A \cup B$ is $\Pi g\beta$-closed.

3 $\Pi g\beta$-open sets

Remark 5. $\beta Cl(X \sim A) = X \sim \beta Int(A)$, for any subset A of X.

Theorem 6. *Let (X,τ) be a topological space. $A \subseteq X$ is $\Pi g\beta$-open if and only if whenever $F \subseteq \beta Int(A)$ whenever F is Π-closed and $F \subseteq A$.*

Proof.

Necessity. Let A be $\Pi g\beta$-open. Let F be Π-closed and $F \subseteq A$. Then $X \sim A \subseteq X \sim F$ where $X \sim F$ is Π-open. $\Pi g\beta$-closedness of $X \sim A$ implies $\beta Cl(X \sim A) \subseteq X \sim F$. By Remark 5, $\beta Cl(X \sim A) = X \sim \beta Int(A)$. So $F \subseteq \beta Int(A)$.

Sufficency. Suppose that F is Π-closed and $F \subseteq A$ implies $F \subseteq \beta Int(A)$. Let $X \sim A \subseteq U$ where U is Π-open. Then $X \sim U \subseteq A$ where $X \sim U$ is Π-closed. By hypothesis, $X \sim U \subseteq \beta Int(A)$. That is $X \sim \beta Int(A) \subseteq U$. By Remark 5, $\beta Cl(X \sim A) \subseteq U$. So $X \sim A$ is $\Pi g\beta$-closed and hence A is $\Pi g\beta$-open.

Theorem 7. *If A is $\Pi g\beta$-open and $\beta Int(A) \subseteq B \subseteq A$, then B is $\Pi g\beta$-closed.*

Proof. This is an immediate consequence of Theorem 4.

4 $\Pi g\beta$-continuity and $\Pi g\beta$-irresoluteness

Definition 5. A function $f : X \to Y$ is called $\Pi g\beta$-continuous if $f^{-1}(V)$ is $\Pi g\beta$-closed in X for every closed set V of Y.

Definition 6. A function $f : X \to Y$ is said to be

(i). Πgs-continuous [5], (resp. Πgp-continuous [23]) if $f^{-1}(V)$ is Πgs-closed (resp. Πgp-closed) in X for every closed set V of Y.

(ii). gsp-continuous [7] or $g\beta$-continuous [24] if $f^{-1}(V)$ is $g\beta$-closed in X for every closed set V of Y.

(iii). Π-irresolute [5] (resp. β-irresolute [17]) if $f^{-1}(V)$ is Π-closed (resp. β-closed) in X for every Π-closed (resp. β-closed) set V of Y.

(iv). pre-β-closed [17] if $f(V)$ is β-closed in Y for every β-closed set V of X.

(v). regular open [21] if $f(V)$ is regular open in Y for every open set V of X.

Definition 7. A function $f : X \to Y$ is said to be $\Pi g\beta$-irresolute if $f^{-1}(V)$ is $\Pi g\beta$-closed in X for every $\Pi g\beta$-closed set V of Y.

Theorem 8. *For a function $f : X \to Y$, the following implications hold:*

$$\begin{array}{c} \Pi gs\text{-}\textit{continuous} \\ \downarrow \\ g\beta\text{-}\textit{continuous} \rightarrow \Pi g\beta\text{-}\textit{continuous} \\ \uparrow \\ \Pi gp\text{-}\textit{continuous} \end{array}$$

Proof. Obvious by Remark 1. Reverse implications of above diagram may not be true as can be seen from Remark 2 and Example 1.

Theorem 9. *If $f : X \to Y$ is a Π-irresolute and pre-β-closed function, then $f(A)$ is $\Pi g\beta$-closed in Y for every $\Pi g\beta$-closed set A of X.*

Proof. Let A be any $\Pi g\beta$-closed set of X and U be any Π-open set of Y containing $f(A)$. Since f is Π-irresolute, $f^{-1}(U)$ is Π-open in X and $A \subseteq f^{-1}(U)$. Therefore, we have $\beta Cl(A) \subseteq f^{-1}(U)$ and hence $f(\beta Cl(A)) \subseteq U$. Since f is pre-β-closed, $\beta Cl(A) \subseteq \beta Cl(\beta Cl(A)) = f(\beta Cl(A)) \subseteq U$. Hence $f(A)$ is $\Pi g\beta$-closed in Y.

Theorem 10. *If $f : X \to Y$ is an β-irresolute regular-open bijection, then f is $\Pi g\beta$-irresolute.*

Proof. Let F any $\Pi g\beta$-closed set of Y and U be a Π-open set of X containing $f^{-1}(F)$. Since f is regular open, $f(U)$ is Π-open in Y and $F \subseteq f(U)$. Since F is $\Pi g\beta$-closed, $\beta Cl(F) \subseteq f(U)$ and hence $f^{-1}(\beta Cl(F)) \subseteq U$. Since f is β-irresolute, $f^{-1}(\beta Cl(F))$ is β-closed. Hence $\beta Cl(f^{-1}(F)) \subseteq \beta Cl(f^{-1}(\beta Cl(F))) = f^{-1}(\beta Cl(F)) \subseteq U$. Therefore, $f^{-1}(F)$ is $\Pi g\beta$-closed and f is $\Pi g\beta$-irresolute.

Remark 6. The composition of two $\Pi g\beta$-continuous functions need not be $\Pi g\beta$-continuous as can be seen from the following example:

Example 4. Let $X = \{a, b, c\}$. Let $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$, $\sigma = \{\emptyset, X, \{b, c\}\}$ and $\zeta = \{\emptyset, X, \{c\}\}$. Define $f : (X, \tau) \to (Y, \sigma)$ as identity mapping and $g : (Y, \sigma) \to (Z, \zeta)$ as identity mapping. Both f and g are $\Pi g\beta$-continuous but $(g \circ f)^{-1}(\{a, b\}) = f^{-1}\left(g^{-1}(\{a, b\})\right) = \{a, b\}$ is not $\Pi g\beta$-closed in (X, τ).

Definition 8. A space (X, τ) is called a Πgs-$T_{1/2}$ [5] (resp. $\Pi g\beta$-$T_{1/2}$) if every Πgs-closed (resp. $\Pi g\beta$-closed) set is gs-closed (resp. $g\beta$-closed).

The notion of $\Pi g\beta$-$T_{1/2}$ space and $T_{1/2}$ space are independent of each other as shown by the following examples:

Example 5. Let $X = \{a, b, c\}$ and let $\tau = \{\emptyset, X, \{a\}, \{b\}, \{a, b\}\}$. Then (X, τ) is $\Pi g\beta$-$T_{1/2}$ but not $T_{1/2}$.

Example 6. Consider Example 1. It is $T_{1/2}$ but not $\Pi g\beta$-$T_{1/2}$.

Theorem 11. *A space (X, τ) is $\Pi g\beta$-$T_{1/2}$ if and only if every singleton of X is either Π-closed or β-open.*

Proof.

Necessity. Let $x \in X$ and assume that $\{x\}$ is not Π-closed. Then clearly $X \sim \{x\}$ is not Π-open and $X \sim \{x\}$ is trivially $\Pi g\beta$-closed. Since (X, τ) is a $\Pi g\beta$-$T_{1/2}$ space, it is β-closed. Therefore, $\{x\}$ is β-open.

Sufficency. Let $A \subseteq X$ be $\Pi g\beta$-closed. Let $x \in \beta Cl(A)$. We will show that $x \in A$. Consider the following two cases:

Case (i). the set $\{x\}$ is Π-closed. Then if $x \notin A$, then $A \subseteq X \sim \{x\}$. Since A is $\Pi g\beta$-closed and $X \sim \{x\}$ is Π-open, $\beta Cl(A) \subseteq X \sim \{x\}$ and hence $x \notin \beta Cl(A)$. This is a contradiction. Therefore, $x \in A$.

Case (ii). the set $\{x\}$ is β-open. Since $x \in \beta Cl(A)$, then $\{x\} \cap A \neq \emptyset$. Thus $x \in A$. So, in both cases, $x \in A$. This shows that A is β-closed.

Theorem 12. *Let $f : (X, \tau) \to (Y, \sigma)$ and $g : (Y, \sigma) \to (Z, \zeta)$ be any two functions. Then*

(i). $g \circ f$ is $\Pi g\beta$-continuous, if g is continuous and f is $\Pi g\beta$-continuous.

(ii). $g \circ f$ is $\Pi g\beta$-irresolute, if g is $\Pi g\beta$-irresolute and f is $\Pi g\beta$-irresolute.

(iii). $g \circ f$ is $\Pi g\beta$-continuous, if g is $\Pi g\beta$-continuous and f is $\Pi g\beta$-irresolute.

(iv). $g \circ f$ is β-continuous, if f is β-irresolute and g is $\Pi g\beta$-continuous and Y is a $\Pi g\beta$-$T_{1/2}$ space.

Proof.

(i). Let V be a closed in (Z, ζ). Then $g^{-1}(V)$ is closed in (Y, σ), since g is continuous. $\Pi g\beta$-continuity of f implies that $f^{-1}\left(g^{-1}(V)\right)$ is $\Pi g\beta$-closed in (X, τ). Hence $g \circ f$ is $\Pi g\beta$-continuous.

(ii). Let V be a $\Pi g\beta$-closed in (Z, ζ). Then $g^{-1}(V)$ is $\Pi g\beta$-closed in (Y, σ), since g is $\Pi g\beta$-irresolute. $\Pi g\beta$-irresoluteness of f implies that $f^{-1}\left(g^{-1}(V)\right)$ is $\Pi g\beta$-closed in (X, τ). Hence $g \circ f$ is $\Pi g\beta$-irresolute.

(iii). Let V be a closed in (Z, ζ). Then $g^{-1}(V)$ is $\Pi g\beta$-closed in (Y, σ), since g is $\Pi g\beta$-continuous. $\Pi g\beta$-irresoluteness of f implies that $f^{-1}\left(g^{-1}(V)\right)$ is $\Pi g\beta$-closed in (X, τ). Hence $g \circ f$ is $\Pi g\beta$-continuous.

(iv). Let V be a closed in (Z, ζ). Then $g^{-1}(V)$ is $\Pi g\beta$-closed in (Y, σ), since g is $\Pi g\beta$-continuous. As (Y, σ) is a $\Pi g\beta$-$T_{1/2}$ space, $g^{-1}(V)$ is β-closed in (Y, σ). β-irresoluteness of f implies that $f^{-1}\left(g^{-1}(V)\right)$ is β-closed in (X, τ). Hence $g \circ f$ is β-continuous.

Remark 7. For a function $f : (X, \tau) \to (Y, \sigma)$, where X is a $\Pi g\beta$-$T_{1/2}$, $\Pi g\beta$-continuity, $g\beta$-continuity and β-continuity are equivalent.

Theorem 13. *Let $f : (X, \tau) \to (Y, \sigma)$ be a pre-β-closed and $\Pi g\beta$-irresolute surjection. If (X, τ) is a $\Pi g\beta$-$T_{1/2}$ space, then (Y, σ) is also a $\Pi g\beta$-$T_{1/2}$ space.*

Proof. Let F be any $\Pi g\beta$-closed set of Y. Since f is $\Pi g\beta$-irresolute, $f^{-1}(F)$ is $\Pi g\beta$-closed in X. Since X is $\Pi g\beta$-$T_{1/2}$, $f^{-1}(F)$ is β-closed in X. As f is pre-β- closed, $f\left(f^{-1}(F)\right) = F$ is β-closed in Y. This shows that (Y, σ) is also a $\Pi g\beta$-$T_{1/2}$ sèpace.

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