The University of Iowa
Iowa City, Iowa 52242

Division of Mathematical Sciences
Department of Computer Science
Department of Mathematics
Department of Statistics and Actuarial Science
Program in Applied Mathematical Sciences

1847

2/4/88

Dear Stan,

I thought that you might get a kick out of seeing what your result with Thas (Concerning Deficiency one partial flocks of quadratic cones) says in the plane

— See Section 4 of the enclosed paper

Regards,

Norm

P.S. I may be interested in the July conf. So, if ∃ funds to defray costs — keep me in mind —

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1847

3/11/87

Dear Stanley:

I'm sending you some papers on flocks & cones relating to Gen. Quads.
You know about the first one —
— the 2nd one says: If a gen. quad. of order (q^2, q) q <u>even</u> exists with Defining matrices $\begin{bmatrix} t, & g(t) \\ 0, & -f(t) \end{bmatrix}$ and g, f are both <u>additive</u> then the quad. is classical — i.e. no nontrivial (non-linear) examples exist if q is even ——

Regards,

Norm

P.S. There is another paper joint with Gevaert & Thas which discusses Reguli in the translation planes associated with flocks — see Ref. [3] in Semifield paper —

THE UNIVERSITY OF IOWA
IOWA CITY, IOWA 52240

DIVISION OF MATHEMATICAL SCIENCES
Department of Computer Science
Department of Mathematics
Department of Statistics

Dear Professor Payne;

If Re-(or Pre) prints of any of your articles on "Generalized Quadrangles" are available, I would very much appreciate receiving them.

Sincerely Yours,

Norman L. Johnson

Early 1971

Editorial

Preface

These proceedings are dedicated to Professor Norm Johnson on the occasion of his 70th birthday, and bring together articles based on talks at the Norm-Fest conference presented by leading experts in incidence geometry. The conference was held at the University of Texas at San Antonio (UTSA), May 2009 . The proceedings and the conference itself were brought to fruition by the tireless efforts of Professors Minerva Cordero (UT, Arlington), Alessandro Montinaro (University of Lecce), Oscar Vega (California State University, Fresno) and Greg Wene (UTSA), supported by dedicated and efficient staff from UT and UTSA.

The papers published in the proceedings largely reflect a range of diverse areas in incidence geometries to which Professor Johnson has made immense contributions. Indeed, several participants at this conference, as well as many others worldwide, have collaborated with Norm or been inspired by his work in an eclectic range of topics including flocks, ovoids, unitals, nets, derivation, planes, free constructions, generalized quadrangles, Galois geometry, Sperner spaces and, above all, translation planes. Norm's numerous collaborations in these fields reflect his stimulating influence, his pleasure in working with others, and especially his extraordinary generosity with his ideas. His legacy (still in the making!) includes 365++ papers, as well as many books, and formal lecture notes series based on his talks, and published by various universities in Brazil, Chile, and Italy.

Professor Johnson did not enter research mathematics following the conventional route, from high school to PhD. Instead, as he playfully boasts[1], he was a high school dropout who drifted into the army. It was there, that his gift for mathematics became patently obvious, leading to Oregon State University, followed by graduate school at Washington State University, where his PhD. advisor was Ted Ostrom.

When Norm's thesis was concerned with certain chains of semi-translation planes (those derived from duals of two-dimensional translation planes), under

[1] Sorry, Norm!

sequences of the plane-changing operations associated with derivation, dualization, and transposition: he showed that such planar chains repeat after at most eight such operations. This project developed in him a life-long fascination for translation planes and derivation, and the various related geometric and combinatorial structures.

He began his post-doc career at the University of Iowa[2], Professor Johnson embarked on a deep long-term investigation of finite 2-dimensional translation planes[3]. At the time, in the early 1970's, there were few known processes for constructing translation planes, and the known types of translation planes, of any dimension, were rather restricted[4]. Nor were there many known general theorems concerning the structure of translation planes[5] and there collineation groups, with the striking exception of the celebrated Hering-Ostrom theorem, on groups generated by elations, and, somewhat later, analogous and equally fundamental work on Baer subplanes due to Foulser[6]. However, the 1970's became a period of rapid development for finite translation planes, under the influence and inspiration of the great contributions of Ostrom throughout the decade. Johnson's contribution to this activity was to develop a coherent theory for the most fundamental case: the two-dimensional finite translation planes, equivalently spreads in $PG(3, q)$.

After Johnson set to work, the entire field of finite two-dimensional translation planes underwent radical changes, mainly through his own pioneering efforts and the results of Ostrom, Foulser and others, especially the Schaeffer-Walker classification of the 2-dimensional translation planes of order q^2 admit-

[2]In fact, UI has become his permanent base and a mecca for finite geometers, his visitors ranging from Yukata Hirmamine in Japan to Rolando Pomareda in Chile.

[3]His work, however, was by no means restricted to this field. Not only did his work include semifields and other areas of translation planes, but he also explored derivation and other problems in the infinite case. For instance, his fundamental contributions to the study of free planes, shortly after his PhD, led to a visiting professorship at the university of Oslo, when it came to the attention of Odvar Iden.

[4]They were mostly among the (generalized) André planes, the subregular planes of Bruck, the Lüneburg-Tits planes, semifield planes, and various planes obtained from these classes by derivation and net replacement. Throughout his career, Norm has investigated all these planes with meticulous thoroughness, and made many significant and surprising contributions to their study.

[5]Apart from André's classical characterizations of translation planes and their collineation groups from the early 1950's, and their projective analogues, due to Bruck and Bose; other results were mainly concerned with characterizations of translation planes among wider classes of projective planes. However, the systematic exploration of finite translation planes began only in the late 1960's, mainly under the influence of Ted Ostrom and his pioneering investigations based on his discovery of the geometric meaning of derivation and the related theory of net replacements.

[6]All these results arose, very roughly, in the early 1970's shortly after Norm's own post-doc work got started.

ting $SL(2,q)$ (in their linear translation complements), equivalently the classification of spreads in $PG(3,q)$ that admit $SL(2,q)$.

By the end of the 1970's Johnson had become the preeminent specialist in the field of 2-dmensional planes; he had established an enormous range of theorems, planes, construction methods, and powerful characterizations of both classical planes, such as the Hall planes, and the interesting new planes, such as the Ott-Schaeffer and the Lorimer-Rahilly planes that were emerging at the time; there was also his work on affine central collineations.

He established his elegant and powerful results using a mixture of geometric insights combined with brutal and punishing matrix calculations[7] that often mask their underlying elegance: they deserve careful attention, as buried within them are various ingenious computational tricks that are reusable in other contexts.

However, although Norm revels in complexities, it is worth stressing that he is not just a 'street-fighter'. Throughout his career, he has proved elegant and profound theorems using geometric and group-theoretic arguments. His complete and amazing, yes amazing, classification of derivation nets in the 1990's illustrates this perfectly. (We consider this ahead).

A peculiar feature of his originality is his strange ability to conjure up a profusion of planes or other geometric objects by methods that seem to almost totally ignore the underlying geometry. We can't resist sketching a charming example that arose almost at the start of his career and still defies analysis.

Recall that a spread set $\tau \subset GL(n,q)$ is a set of q^n matrices that contains $0_n, 1_n$ and such that if $A, B \in \tau$ are distinct then $A - B \in GL(n,q)$. If the associated translation plane π_τ includes a derivation set then τ contains a field of matrices $\mathcal{F} \cong GF\left(q^{n/2}\right)$: actually the existence of $\mathcal{F}$ is in itself sufficient, but the two conditions are equivalent by Johnson's fundamental theorem on derivation, indicated above. Now for any $T \in \tau \setminus \mathcal{F}$ the additive group of matrices $\delta = \mathcal{F} + \mathcal{F}T$ is also a spread set, which coordinatizes a semifield plane π_δ with essentially no geometric connection with π_τ! Moreover, the plane π_δ (or even its dual) may also be derived to a new plane ψ — again with no apparent geometric connection between ψ and $\pi_\mathcal{T}$. This process may be endlessly repeated, with numerous added frills along the way: dualize sometimes; transpose sometimes; even change the 'T', etc. The result is a chaotic mess of derivable planes with nothing more in common — just the sort of thing that Norm delights in!

The 1970's may be crudely summarized as Johnson's 'two-dimensional phase' during which his main concern were 2-dimensional planes and derivation. In the 1980's, although continuing to contribute vigorously to these areas and 'low-

[7] Or, more appropriately, 'heroic' as a reviewer in Zentralblatt remarked.

dimensional' planes, he began taking an ever growing interest in exploring finite translation planes without further assumptions. His fundamental contributions to this area changed the entire field of finite translation planes by extending many major 2-dimensional theorems to arbitrary finite translation planes.

One of the most fundamental of these results is the Foulser-Johnson theorem, which established the Schaeffer-Walker classification without the imposition of any condition on the kernel; thus the Foulser-Johnson theorem classifies all translation planes of order n^2 that admit $SL(2, n)$ in their translation complements. This theorem has proven to be an indispensable tool for studying finite translation planes, especially when used in conjunction with other dimension-free theorems, many of which Johnson established, alone or in collaboration with others, and the various consequences of the classification of finite simple groups — which was completed around this time. The proof the Foulser-Johnson theorem required a mixture of deep results in modular representation theory together with cunning but messy matrix calculations — a hallmark of Johnson[8].

The year 1987 marked a watershed in Norm's career and, one might say, in the career of finite translation planes. These events were triggered off by a visit to Iowa by Hans Gevaert, who was a student of Jeff Thas in Gent. While talking to Hans, Norm realized at once that a rather technical characterization of the spread set of a certain type, among the vast inventory of such technical characterizations in his head, provided a two-way bridge between a class of 2-dimensional translation planes (that admit a geometric characterization in terms of an elation group fixing a regulus) and flocks of quadratic cones in $PG(3, q)$. This led to a joint paper involving Thas, Gevaert and Johnson that had a revolutionary impact on 2-dimensional plane theory and the corresponding flock theory, hence also on the class of generalized quadrangles equivalent to conical flocks. These connections led to a surge of interrelated activity in all these areas, yielding many significant results in finite translation planes by finite geometers from other areas — and vice versa.

From then on, Johnson began contributing at an ever-increasing rate to a multitude of diverse branches of finite geometries; his work became known and widely admired by the community of finite geometers rather than just by specialists in translation planes where he was already universally recognized as the leading international expert.

However, even as he expanded his contributions to translation planes and finite geometries, he developed a growing interest in infinite incidence geometries. Especially worthy of interest, is his complete solution to one of the most

[8]This illustrates Johnson's essential pragmatism: his willingness, and ability, to learn to use any tool however far removed from his previous practice and background — from modular representation theory to Italian.

fundamental questions in net theory:

When is a net derivable?

Johnson beautiful and surprising answer to this question is that any derivable net $\mathcal{N}$ may be identified within $PG(3, K)$, K any skew field, such that the lines and the points of the net correspond to the points and lines of a projective space that are non-incident with an (arbitrary) fixed line ℓ, and any parallel class of net-lines consists of the projective points that lie on a plane containing ℓ: the Baer subplanes of the net $\mathcal{N}$, that become the lines of the derived net $\mathcal{N}'$, are the planes of $PG(3, K)$ that do not contain ℓ. This theorem should surely be an essential part of any standard text on nets or projective planes.

In the finite case, a plane is derivable relative to a subnet if and only if the subnet is derivable: the result follows virtually by the definition of a derivable net. However, shortly after he proved the above fundamental theorem of derivable nets, Norm gave an example of an infinite affine plane which is *not* derivable with respect to a derivable net that it contains!

The sketchy glimpses we have provided of Johnson's work in the 20-th century cannot, of course, hope to do justice to even this subset of his nachlass. Describing his work in the 21-century provides even greater challenges —he has already produced a torrential volume of results in an ever expanding list of fields, while he continues to contribute to all fields that he has worked on in the previous century. However, fortunately for us, his 21-century work is well-represented by the articles in these proceedings, so we may end by saluting Norm as a worthy heir to Ted Ostrom, and thank all the participants at the Norm-Fest, and especially the referees and contributors to the proceedings, as well as a beautiful tribute to Professor Johnson from Professor Stan Payne. We end by saluting for their efforts in making the Norm-Fest such a success.

Minerva Cordero; Vikram Jha; Alessandro Montinaro; Oscar Vega.

Postscript

Recently we were able to contact Mike Kallaher, who has been a close colleague of Ted Ostrom for many decades at Washington State University. Mike emailed us the following summary of Ostrom's judgement concerning Norm Johnson:

> *"I know Ted believes Norm is the most productive American of his generation in the field of projective and affine planes. He is very proud of Norm and his accomplishments."*

Postscript

The organizers of Norm-Fest are especially grateful to Professor Mauro Biliotti, University of Salento, for his invaluable support in fostering the link between the conference and Note di Matematica.