Data Envelopment Analysis (DEA) for the evaluation of Public Healthcare Structures

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As stated in the Economist Intelligence Unit report (2011), the failure to develop a coherent plan of action in health care systems is linked to deep-rooted problems related to the organization of health care structures. These problems form the starting point of this work: the aim of the present study is to develop the potential of a Data Envelopment Analysis (DEA) Model creating a non-parametric pattern for evaluating the efficiency of healthcare structures in a south-eastern region of Italy. A non-parametric pattern allows for weighting different variables even in the presence of a particularly complex structure, such as that of healthcare systems. The results will provide some indications for health decision and policy makers, in order to improve accountability in the field of Social Responsibility, of particular relevance in this moment.

keywords: Data Envelopment Analysis, performance evaluation, Healthcare structures.

1 Introduction

Starting from the considerations made by the Economist Intelligence Unit (Unit, 2011) the failure to develop a coherent plan of action in health care system is linked to deep-rooted problems related to the organization of health care structures, we attempted to

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understand which factors may have influenced the efficiency of a hospital.

Having identified and aggregated data, they were introduced to a system as input with the output of a mathematical model of Data Envelopment Analysis, perfected in order to evaluate the relative efficiency of a healthcare facility, providing, to relevant authorities and decision-makers, an additional tool of use in assisting the administration of a sensitive area such as public health. A sector occupying a considerable and substantial portion of public spending in recent times: in OECD countries, according to the 2012 OECD report entitled Eco-Health, 6.48 % of GDP was spent on health care.

The still ongoing economic crisis in Italy has nevertheless witnessed significant and alarming increases in terms of the cost of containment of health services: in less than a decade (1999-2009), the percentage of health care spending increased from 5.7 % to 7.2 % . Although this may not be considered as an exponential increase, it results as highly effective in the planning of public activities in a country in which GDP does not register significant signs of growth and, indeed, often registers decreasing values. The identification of a leaner method for assessing the efficiency of a healthcare facility would thus serve as an important tool for ensuring both the sustainability of the health care system and the preservation of public health (Zuckerman et al., 1994). It is therefore critical, based on these assumptions, to solve those problems relating to the use of resources in order to face those issues which may be defined as more closely related to the clinical field.

If more resources are provided for health care, it becomes crucial to be able to measure the results achieved in relation to both inputs and outputs, thus measuring the impact on outcomes (Sengupta, 1990). The methodologies used in order to calculate relative efficiency of these units can be parametric, such as Deterministic Frontier Analysis DFA, Stochastic Frontier Analysis SFA (Aigner et al., 1977), or non-parametric, such as Free Disposal Hull FDH, Data Envelopment Analysis DEA. By definition, parametric analysis requires an a priori explanation of the production process needed in the unit for which the efficiency is to be calculated. Non-parametric analysis determines the relative efficiency of similar decision-making units through linear programming techniques (Coelli et al., 2005). A non-parametric method of particular interest due to its application in the efficiency measurement of hospitals is DEA (Data Envelopment Analysis) (Grosskopf and Valdmanis, 1987). This model allows for determining, through linear programming, the relative efficiency of similar decision-making units, defined as DMU, without a detailed description of the production process. It may therefore be stated that the DEA can be defined as a mathematical programming technique designed to evaluate the relative efficiency of a group of comparable units (DMU) (Puig-Junoy, 2000). This technique of mathematical programming can be declined in various forms and uses and may thus be designed to minimize or maximize values of input and output (Seiford and Thrall, 1990). Indeed, the DEA method is able to compute an efficiency frontier for a set of DMUs and relative radial distances for each DMU in the sample from the border. In input-oriented models, the distance, the index of the score of efficiency, between the DMU observed and the border represents the measurement of radial reduction of the inputs needed in order to make the single DMU efficient (Ganley and Cubbin, 1992). It is a widely employed model in the efficiency evaluation of activities in the public sector

(Sherman, 1984), experiencing, inter alia, different levels of success relating to its utility. A utility that fundamentally depends on various characteristics of this model, above all, while not requiring a system of a priori assumptions, the technique allows for analysis and, consequently, assessment of multi-input and multi-output production systems; secondly, it does not require an a priori weighting of inputs and outputs; thirdly, the model is capable of providing, for each DMU, a synthetic and straightforward interpretation measurement of efficiency and, lastly, the model is capable of putting into relief, for each DMU, sources and levels of relative inefficiency (Harold et al., 1993). Evidently, despite its non-negligible advantages, it should not be considered as a perfect model. Indeed, it shows various signs of weakness: being an estimation procedure based on extreme points, it shows sensitivity in the selection process of inputs, i.e. aggregation and specification of variables of the sample data. Ordinarily, the selection of the input and the output variables is one of most relevant issues in correctly estimating relative efficiency. Having detected inputs for analysis, it can be assumed that these represent critical success factors: the strengths of DEA can be fully exploited only if the considered input and output variables are effectively relevant for all the DMU examined. This factor may be particularly relevant in the healthcare sector where the definition of resources employed and of products obtained is a particularly complex process (Puig-Junoy, 2000). A common measure of efficiency is thus:

$$Efficiency = \frac{Weighted\ sum\ of\ outputs}{Weighted\ sum\ of\ inputs} \tag{1}$$

Introducing notation:
K = number of considered outputs;
I = number of considered inputs;
ykj = amount of the output of type k (K = 1, ..., K) produced by the j unit;
xij = amount of the input of type i (i=1,.,I) produced by the j unit;
ukj = weight assigned to the output type k of j unit;
vij = weight assigned to the input type i from j unit.
The efficiency of the j production unit can be described as:

$$E_j = \frac{\sum_{k-1}^{K} u_{kj} y_{kj}}{\sum_{i-1}^{I} v_{ij} x_{ij}} \tag{2}$$

Having assigned values to the weights of the outputs and inputs, it is possible to calculate the value of the efficiency for the j production unit. Charnes, Cooper and Rhodes (Charnes et al., 1978) recognized the difficulty in identifying a set of weights to determine relative efficiency. They admitted, moreover, that it would not be inaccurate to evaluate inputs and outputs for each production unit differently: each production unit had the right to adopt the set of weights that could place it in a good light in comparison with other production units. Given these considerations, the efficiency of the j organizational unit can be obtained as the solution of the following problem:
maximize the efficiency of the j unit, placing as a constraint that the efficiency of the

other units is $< = 1$. The variables of the problem are the weights, and the solution provides the weights of inputs and outputs and the efficiency measurement.The algebraic model is:

$$maxE_j = \frac{\sum_{k-1}^{K} u_{kj}y_{kj}}{\sum_{i-1}^{I} v_{ij}x_{ij}} \tag{3}$$

placing as constraits:

1)$\frac{\sum_{k-1}^{K} u_{kj}y_{kj}}{\sum_{i-1}^{I} v_{ij}x_{ij}} \geq 1 with 1 = 1.2...N$

2) $u_{kj}, v_{ij} \geq 0$

The problem of finding the maximum efficiency weights can be solved in two ways: given the available levels of input, by attempting to maximize the amount of output (INPUT-ORIENTED MODEL), or minimizing the input quantities used in order to target the levels of output (OUTPUT-ORIENTED MODEL).
In the case of the input-oriented model the additional constraint to the algebraic model is: $\sum_{i-1}^{I} v_{ij}x_{ij} = 1$. In the case of the output-oriented model the additional constraint to the algebraic model is:$\sum_{k-1}^{K} u_{kj}y_{kj}$.
The problem expressed by the algebraic model must be solved for all hospitals compared. The weights obtained represent the best ever for the hospital in question and any other vector of weights would lead to lower efficiency ratios. Identifying a set of weights for which the efficiency of the j unit is equal to 1, hospital j thus results as efficient; if the efficiency is less than 1, the unit is inefficient as there are other units capable of producing the same output with a lower use of input or greater output with the same amount of input (Daraio and Simar, 2007). If a unit is inefficient even when most favourable weights were assigned to measuring its efficiency, judgment of the inefficiency appears to be reasonably well-founded; Indeed, despite the best weights being chosen to maximize efficiency, an index of $Ej < = 1$ shows there to be a linear combination of the other more efficient units. The weights provide important information regarding the choices (implicitly) made by each production unit in order to appear as efficient as possible in comparing itself to the other: a high weight (low) associated with a given output indicates a strength (weakness) of the production unit considered, while a high weight (low), associated with a given input signals a relative scarcity (abundance) to that input with other factors used by the production unit in question and by the others of the sample.

2 Analysis of the performance of various healthcare structures in a south-eastern region of Italy.

Analysis was thus carried out of the relative efficiency of several healthcare structures in south-eastern Italy. This process considered as output:
- y1: the total number of ordinary releases adjusted with the weight of the DRG;

- y2: the total number of day hospital releases adjusted with the weight of the DRG;
as input of each hospital STRUCTURE:
- x1: the number of doctors;
- x2: healthcare staff;
- x3: the number of other nonmedical employees;
- x4: total bed numbers.
Please note that the number of doctors, healthcare staff and other staff does not include employees assigned to the emergency department as evaluation of the first aid performance was not considered.An input-oriented model and an output-oriented model have been applied.Summary tables on the inputs and outputs are shown in the Table 1 and in the Table 2.

Table 1: Table 1. Input of examined Healthcare Structures.

	Doctors	Healthcare Staff	Other Staff	Beds
Structure 1	74	188	43	170
Structure 2	50	176	52	160
Structure 3	71	233	65	188
Structure 4	86	249	63	157
Structure 5	19	72	34	32
Structure 6	11	57	26	68
Structure 7	87	226	31	187
Structure 8	242	788	140	452

Table 2: Table 2. Output of examinated Healtcare Structures.

	WEIGHT OF ORDINARY HOSPITALIZATION (/00)	WEIGHT OF DAY HOSPITAL HOSPITALIZATION (/00)
Structure 1	46.0016	28.31736
Structure 2	51.31935	8.99806
Structure 3	46.02258	15.84978
Structure 4	53.32869	21.78842
Structure 5	9.16952	5.20338
Structure 6	2.68952	6.554108
Structure 7	54.9197	24.61987
Structure 8	163.2873	33.95003

2.1 Input-oriented model Results

From a first analysis of the structures all result efficient with the exception of Structure 3 (see table 3).In the table 4 it may be noted that for structures 1, 3, 4, 6 and 7, weights of the two outputs are in the same order while for the second structure 2 weights are unbalanced: the weight of day hospital admissions is too low and, therefore, the PO of Structure 2 should increase performance in the day hospital (element confirmed by the analysis of the efficient frontier shown in the Table 5).

Table 3: Efficiency in input-oriented model.

	Efficiency (/00)
Structure 1	1
Structure 2	1
Structure 3	0.807
Structure 4	1
Structure 5	1
Structure 6	1
Structure 7	1
Structure 8	1

Table 4: Weight of Output 1-2 of examinated Healtcare Structures.

	WEIGHT OF OUTPUT 1	WEIGHT OF OUTPUT 2
Structure 1	0.0016	0.0327
Structure 2	0.0185	0.0056
Structure 3	0.0134	0.012
Structure 4	0.015	0.0092
Structure 5	0.0456	0.1118
Structure 6	0.0034	0.1512
Structure 7	0.0132	0.0112
Structure 8	0.0061	0

Table 5: Weight of Input 1-2-3-4 of examined Healthcare Structures.

|	Weight Input 1	Weight Input 2	Weight Input 3	Weight Input 4
Structure 1	0.001	0.0029	0.0063	0.0007
Structure 2	0.0169	0.0004	0.0007	0.0002
Structure 3	0.0053	0	0	0.0033
Structure 4	0.0003	0.0001	0.0009	0.0057
Structure 5	0.0002	0	0	0.0311
Structure 6	0.078	0.0004	0.0024	0.0009
Structure 7	0.0012	0.0003	0.0265	0.0001
Structure 8	0.0001	0.0001	0.0014	0.0015

In the table 5, all weights equal to 0 indicate an abundance of that input factor in relation to the output expected. Note that, in general, the weights are not high indicating a good use of resources. The only notable weight relates to doctors (input 1) in Structure 6, indicating the possibility of reducing the levels of medical staff to further improve efficiency.
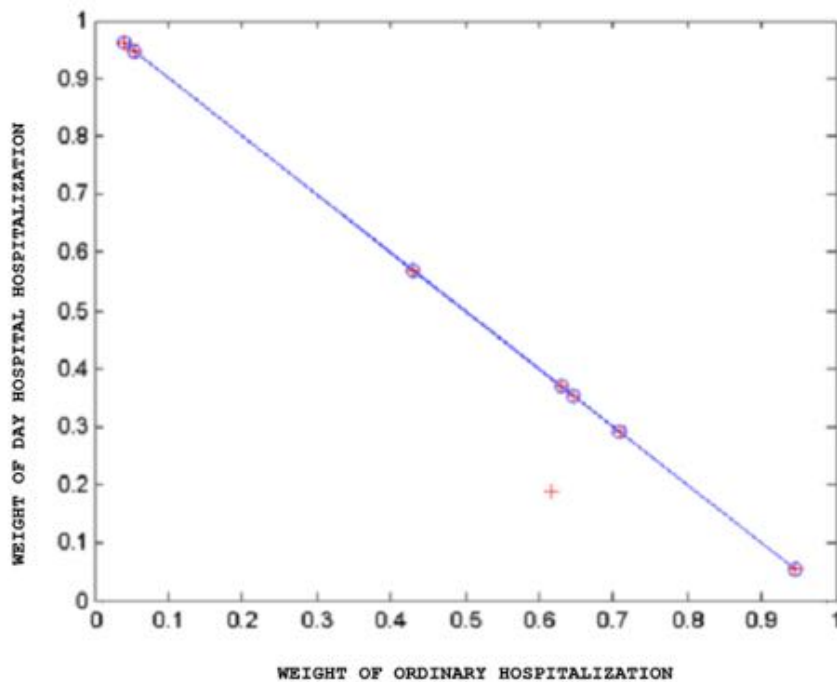
1.jpg

Note the efficiency frontier as a straight line with an inclination of -45 , demonstrating

that in order to obtain high efficiency a healthcare facility should pay equal attention to both ordinary admissions and day hospital admissions. It should also be noted that Structure 3 (red mark not surrounded by blue circle) is external to the efficiency frontier. Furthermore, the three structures demonstrate a significant difference in terms of the weighted admission values and the weighted day hospital values: for the perceptive policy maker, despite efficiency, it should not be considered acceptable if one output significantly prevails over the other. This is the case of Structure 1 (represented in the chart below with a black '+'), Structure 6 (represented by a pink '+') and Structure 2 (represented by a green '+').
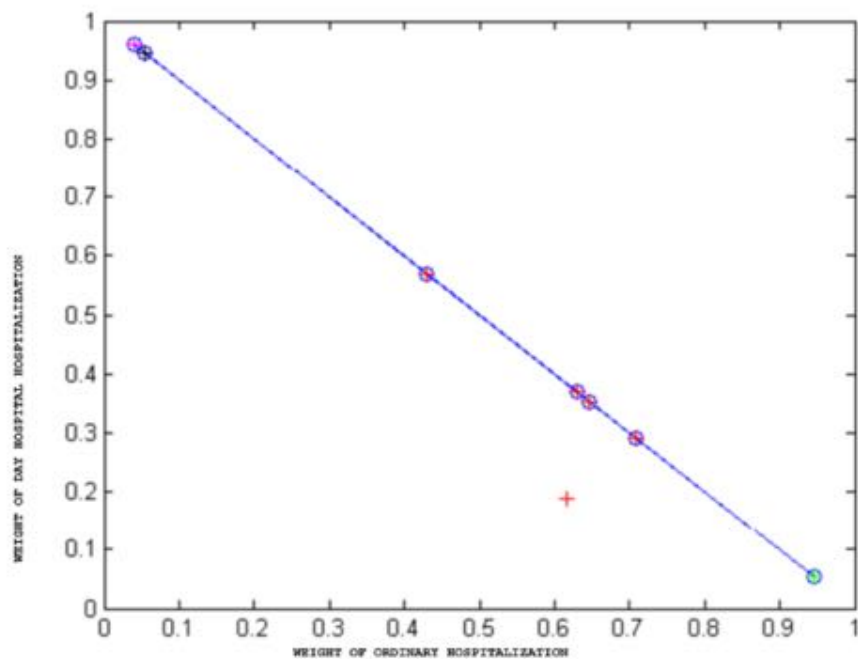
2.jpg

2.2 Output-oriented model results

The table of efficiency is shown below, the table of weights of the input and the table of weights of the output, in an output-oriented model.

Table 6: Efficiency in ouput-oriented model.

	Efficiency
Structure 1	1
Structure 2	1
Structure 3	0.807
Structure 4	1
Structure 5	1
Structure 6	1
Structure 7	1
Structure 8	1

Note that values of the efficiency obtained with both models are identical.

Table 7: Weight of Output 1-2 of examinated Healtcare Structures.

	WEIGHT OF OUTPUT 1	WEIGHT OF OUTPUT 2
Structure 1	0.0003	0.0349
Structure 2	0.0182	0.0072
Structure 3	0.0166	0.0148
Structure 4	0.0149	0.0095
Structure 5	0.0338	0.1327
Structure 6	0.0283	0.141
Structure 7	0.0036	0.0325
Structure 8	0.0061	0.002

Note that the weights of output 2, in the case of Structure 7 and Structure 8 are equal to 0, in contrast to that highlighted by the input-oriented model. In these cases, however, the solution offered should not be acceptable to a healthcare policy maker, as day hospital admissions cannot be completely ignored.

Table 8: Weight of Input 1-2-3-4 of examined Healthcare Structures.

	Weight Input 1	Weight Input 2	Weight Input 3	Weight Input 4
Structure 1	0.0015	0.0027	0.0055	0.0008
Structure 2	0.0081	0.0025	0.0004	0.0008
Structure 3	0.0065	0	0	0.0041
Structure 4	0.0006	0.0004	0.0005	0.0053
Structure 5	0.0001	0	0	0.0312
Structure 6	0.0802	0.0001	0.002	0.0009
Structure 7	0.0009	0.0007	0.0227	0.0003
Structure 8	0.0002	0.0001	0.0002	0.0018

2.3 Comparison between Structures with First Aid.

The first aid facilities for the region analyzed are 4, composed as follows:
A. Structure 3
B. Structure 4
C. Structure 1 Structure 5 Structure 6
D. Structure 8 Structure 7 Structure 2
Therefore, DEA has been applied in these four structures, considering the same inputs from the previous example and adding inputs for first aid staff; in terms of outputs, the output of first aid has been added to the same outputs of the previous example. Regarding the centres formed by different structures (C, D), inputs and outputs of individual structures are added (e.g. the number of doctors for C will be the sum of the doctors of Structure 1, Structure 5 and Structure 6). The output for first aid is provided by the sum of patients admitted and those not admitted multiplied by the value of performance (19.21).

Table 9: Input Healthcare Structures with First Aid.

	Doctors	Healthcare Staff	Other Staff	Beds
A-Structure 3	81	252	70	188
B-Structure 4	97	262	72	157
C- Structure 1-5-6	110	342	113	270
D- Structure 8-7-2	410	1259	260	799

Table 10: Output Healthcare Structures with First Aid.

	Weight of ordinary Hospitalization (/00)	Weight of Day Hospital Hospitalization (/00)	First Aid Performance (/000)
A-Structure 3	46.02258	15.84978	567.677
B-Structure 4	53.32869	21.78842	604.750
C- Structure 1-5-6	57.86064	40.074848	854.038
D- Structure 8-7-2	269.526335	67.56796	2603.205

2.4 Input oriented model results

The table of efficiency is shown below, the table of weights of the input and the table of weights of the output, in an input-oriented model.

Table 11: Efficiency of Structure with First Aid in input-oriented model.

	Efficiency
A-Structure 3	0.9775
B-Structure 4	1
C-Structure 1-5-6	1
D-Structure 8-7-2	1

Even in this case, only Structure 3 results as inefficient.

Table 12: Weight of Output 1-2-3 of examined Healtcare Structures with First Aid.

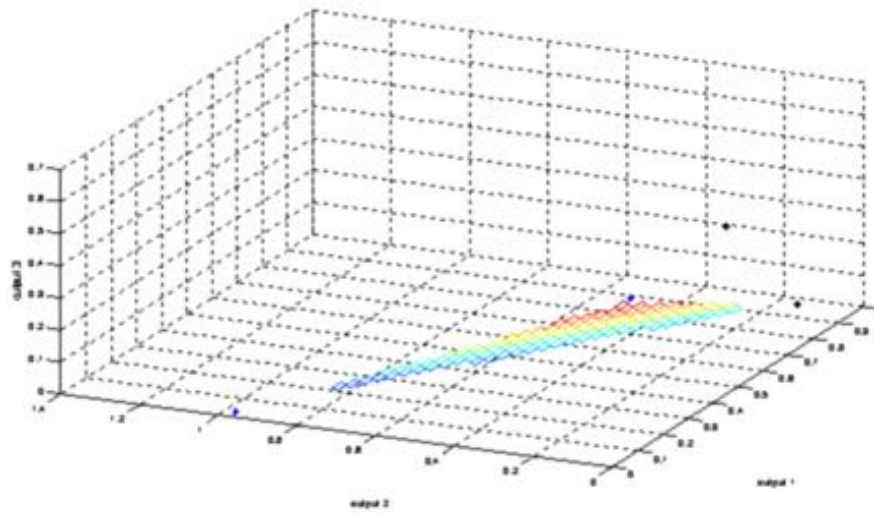|	WEIGHT OUTPUT 1	WEIGHT OUTPUT 2	WEIGHT OUTPUT 3
A-Structure 3	0.009	0	0.0009
B-Structure 4	0.0093	0.0124	0.0004
C-Structure 1-5-6	0.0004	0.0243	0
D-Structure 8-7-2	0.0032	0.0011	0

Note that, even in the presence of apparently efficient structures, they represent cases in which output relative to the performance of first aid demonstrates a weight equal to 0: for a policy maker the efficiency of the last two structures in the table should be considered as unsatisfactory because focus resources on ordinary admissions and day hospital admissions but totally neglect the performance of first aid.

Table 13: Weight of Input 1-2-3-4 of examinated Healtcare Structures with First Aid.

	WEIGHT INPUT 1	WEIGHT INPUT 2	WEIGHT INPUT 3	WEIGHT INPUT 4
A-Structure 3	0.0123	0	0	0
B-Structure 4	0.0007	0.001	0.0006	0.004
C-Structure 1-5-6	0.0042	0.0008	0	0.0006
D-Structure 8-7-2	0.0003	0.0001	0.0024	0.0001
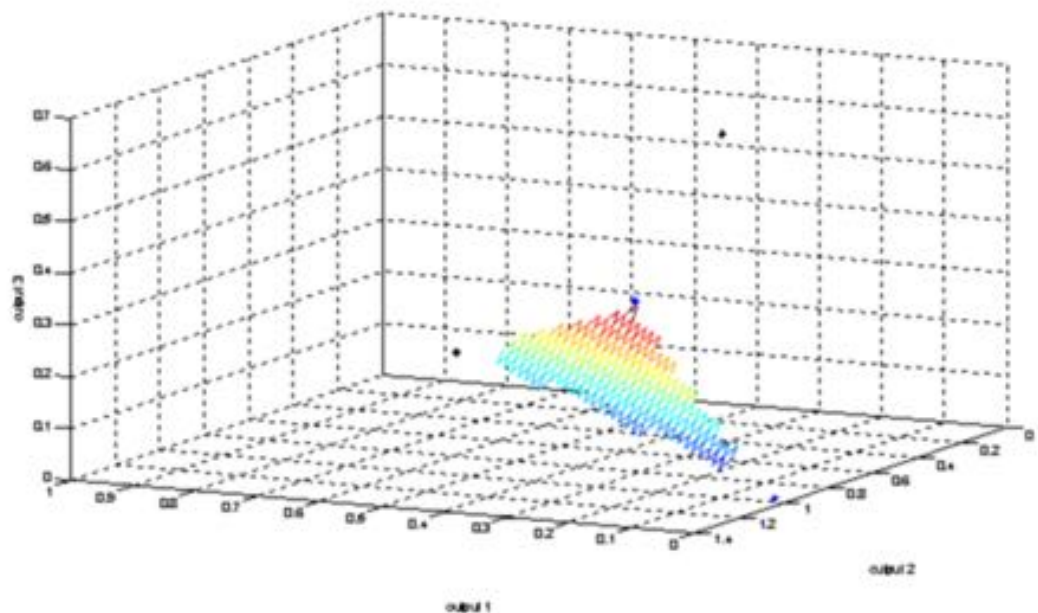
3.jpg

4.jpg

Two points should be noted in the figures located on a Cartesian plane, while being efficient, output 3 is almost equal to 0 in structures 3 and 4.

2.5 Output-oriented model results

The table of efficiency is shown below, the table of weights of the input and the table of weights of the output, in an output-oriented model.

Table 14: Efficiency of Structure with First Aid in onput-oriented model.

	Efficiency
A-Structure 3	0.9775
B-Structure 4	1
C-Structure 1-5-6	1
D-Structure 8-7-2	1

In this case only Structure 3 results as being inefficient.

Table 15: Weight of Output 1-2-3 of examined Healtcare Structures with First Aid.

	WEIGHT OUTPUT 1	WEIGHT OUTPUT 2	WEIGHT OUTPUT 3
A-Structure 3	0.0099	0	0.0009
B-Structure 4	0.0093	0.0124	0.0004
C-Structure 1-5-6	0.0004	0.0243	0
D-Structure 8-7-2	0.0032	0.0011	0

Note that the value of the weights of the output is the same for both models used. Therefore, reference should be made to the preceding considerations.

Table 16: Weight of Input 1-2-3-4 of examined Healtcare Structures with First Aid.

	WEIGHT INPUT 1	WEIGHT INPUT 2	WEIGHT INPUT 3	WEIGHT INPUT 4
A-Structure 3	0.0123	0	0	0
B-Structure 4	0.0007	0.001	0.0006	0.004
C-Structure 1-5-6	0.0042	0.0008	0	0.0006
D-Structure 8-7-2	0.0003	0.0001	0.0024	0.0001

Even the weights of the inputs are identical to those obtained with the input-oriented model and, consequently, the images relating to the efficiency frontier are identical.

3 Conclusions

The organizational dimension of the National Health System is the result of a series of reforms initiated in the early 1990s, which introduced market mechanisms to the healthcare system through the process of Business management, as illustrated in Decreto legislativo of 30 December 1992, No. 502. Regional governments thus became decision makers in the planning of health care, organization and the supply of health services in relation to the demands of the population of the local area. The function of controlling and monitoring suitability belongs, moreover, to regional governments as well as the quality and efficiency of the services provided. The model created in the present study proved both fast and particularly effective in identifying less efficient structures in the sample examined, both in terms of the input oriented model and the output-oriented model. Sustainability of universal health provision, the goal of any healthcare system, requires a certain level of rationalization in terms of resources and the consolidation of health facilities in order to ensure that public resources are revealed as inefficient in meeting the demand for services. The model created in this study could facilitate, due to its immediacy and ease of interpretation, the process of rationalization, identifying, without identifying all components of a production process, the relative efficiency or

inefficiency of a structure from a sample examined: this rapid identification limits any possible dissolution of public resources in inefficient structures and avoids the coexistence of structures that duplicate a service with different investment costs to public spending, raising, inevitably, the cost of the service. Even with the methodological attention required by the model during selection, aggregation and specification of the variables of the sample, the DEA could be a valuable tool for decision makers within public authorities. Indeed, while a relative efficiency, the administrator, through such analysis, gains significant insight into their local situation: a smart model for developing solutions, as flexible as possible, for the sustainability of healthcare services in the local area. A model with the ability to facilitate the transition from one "single" social responsibility, that is to say of a single structure, towards a "collective social responsibility" along a pathway for growth shared between structures and public authorities with the shared aim of sustainable development.

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