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FUZZY CONTINUOUS REVIEW INVENTORY MODEL WITHOUT BACKORDER FOR DETERIORATING ITEMS

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Abstract: *In this paper we have developed a fuzzy continuous review inventory model for deteriorating items. We discussed only the without shortage case. Demand rate is constant and the cycle time is uncertain and it is possible to describe it by triangular fuzzy number (symmetric). The results are illustrated with the help of numerical example. We have discussed the percent of increase if the uncertainties are accounted for in appropriate manner. A sensitivity analysis is carried out to demonstrate the effects of changing parameter values on the optimal solution of the system.*

***Keywords**: Inventory, Fuzzy variable, EOQ Model, Deterioration, Triangular Fuzzy number.*

1. Introduction

Inventory control plays an important role as the total investment in inventories of various kinds is quite substantial. Almost every business must carry out some inventory for smooth and efficient running of its operation. The problem is to take decisions that how much should be stocked and when should be stocked for un-interrupted production. Bellan and Zadeh [2] first introduced fuzzy set theory in fuzzy decision making process. Zadeh ([22], [23]) showed that for the new products and seasonal items it is better to use fuzzy numbers rather than probabilistic approaches. Tanaka et al. [15] applied the concepts of fuzzy sets to decision making problems by considering the objectives as fuzzy goals over the α-cuts of a fuzzy constraints set and Zimmermann [24] showed the classical algorithms can be used to solve multi-objective fuzzy linear programming problems. Liberatore [6] showed that sometimes the uncertainties can be

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captured stochastically. Chang, Yao and Lee [4] developed economic reorder point for fuzzy backorder quantity. Yao and Lee [20] established fuzzy inventory model with backorder for fuzzy order quantity. Ouyang and Wu [8] discussed about a minimax distribution free procedure for mixed inventory model with variable lead time. Ouyang and Yao [7] developed a minimax distribution free procedure for mixed inventory model involving variable lead time with fuzzy demand. Yao et. al. [21] established a fuzzy inventory of two replaceable merchandises without backorder based on the signed distance of fuzzy sets. Salameh and Jaber [11] model was modified by Chang [3] in which the author developed an application of fuzzy sets theory to the EOQ model with imperfect quality items. Yao and Chiang [19] developed inventory model without backorder with fuzzy total cost and fuzzy storing cost defuzzified by centroid and signed distance. Tütüncü et al. [16] developed new models of continuous review inventory control with or without backorder in the presence of uncertainty. They used fuzzy set concepts to treat imprecision regarding the costs of continuous review inventory control.
The effect of deterioration is very important aspect of inventory systems. Deterioration refers to decay or damage or spoilage or vaporized such that the item can not be used for its original purpose. Food items, drugs, pharmaceuticals, radioactive substances are examples of that kind of items. This kind of real life situation was first captured by Whitin [17] who considered fashion goods deteriorating at the end of a prescribed storage period. Ghare and Schrader [5] developed an inventory model with a constant rate of deterioration. An order level inventory model for items deteriorating at a constant rate was discussed by Shah and Jaiswal [14]. Aggarwal [1] reconsidered this model by rectifying the error in the work of Shah and Jaiswal [14] in calculating the average inventory holding cost. Roy and Samanta [9] discussed about an EOQ model for deteriorating items with stock-dependent time varying demand. Samanta and Roy [12] developed a deterministic inventory model of deteriorating items with two different types of rates of production. Samanta and Roy [13] explained a production inventory model with deteriorating items. Roy and Samanta [10] introduced an inventory model of deteriorating items with time-varying demand.
In this paper we are developing a fuzzy continuous inventory model without shortage for deteriorating items. We have assumed that the demand rate is constant. To capture the real life situation we are considering that the cycle time is uncertain and it is possible to describe it by triangular fuzzy number (symmetric). Below is the list of assumptions we have considered to develop the model.

2. Notations and assumptions

The following notations are used for developing the model.

(i) $f(t) = a$ is the demand rate at time t defined in the interval $(0,T)$ where a is positive constants.

(ii) h is the holding cost per unit per unit time.

(iii) A is the replenishment cost per cycle.

(iv) C is the unit cost of the item.

(v) Replenishment is instantaneous and lead-time is zero.

(vi) No shortage in inventory is allowed.

(vii) T is the length of a cycle and it is uncertain.

(viii) A constant fraction θ ($0<\theta<<1$), of the on-hand inventory deteriorates per unit time.

3. The mathematical model and its analysis

First we will discuss about crisp model. Let $Q(t)$ be the on-hand inventory at time $t(0 \le t \le T)$. In this model, uniform replenishment rate starts with inventory level q. The inventory level decreases due to both demand and deterioration. Ultimately the inventory reaches 0 at the end of the cycle time T. Then the differential equation governing the instantaneous state of $Q(t)$ at any time t is given by :

$$\frac{dQ(t)}{dt} + \theta Q(t) = -a \text{ , (} a > 0\text{). Where } Q(0) = q \text{ and } Q(T) = 0 \tag{1}$$

$$\text{Hence } [Q(t)] = q\exp(-\theta t) - \exp(-\theta t)\int_0^t a\exp(\theta t)dt \tag{2}$$

Using: $Q(T) = 0$, we have $q = \left\{\frac{a}{\theta}\right\}\exp(\theta T) - \left(\frac{a}{\theta}\right)$

Now from (2):

$$[Q(t)] = \frac{1}{\theta}a(\exp(\theta(T-t)) - 1) = (T-t)[a + \frac{(T-t)}{2}\theta a + \theta\frac{(T-t)^2}{6}\theta a] \tag{3}$$

neglecting higher powers of θ. Hence:

$$Q(t) = (T-t)a[1 + \frac{1}{a}\{\frac{(T-t)}{2}\}\theta a + \frac{(T-t)^2}{6}\theta^2 a] \tag{4}$$

The inventory I_T in a cycle is given by:

$$I_T = \int_0^T Q(t)dt = a[\frac{1}{2}T^2 + \frac{1}{a}\{\frac{1}{6}T^3\theta a + \frac{1}{24}\theta^2 a T^4\}] \tag{5}$$

Total deterioration in a cycle:

$$D = q - TotalDemand = q - \int_0^T adt = (\{\frac{a}{\theta}\}\exp(\theta T) - (\frac{a}{\theta})) - aT \tag{6}$$

The average system cost:

$$C(T) = \frac{1}{T}[A + CD + hI_T] \qquad (7)$$

$$= \frac{A}{T} + C[\frac{1}{2}a\theta T^2] + h[\frac{1}{2}aT + \frac{1}{6}\theta aT^2 + \frac{1}{24}a\theta^2 T^3] \qquad (8)$$

4. Fuzzy continuous review inventory model without backorder

Let us consider that the cycle time is uncertain and it is possible to describe it with triangular fuzzy number (symmetric). Then the cycle time is $\tilde{T} = (T - \Delta, \Delta, T + \Delta)$.
So from (7) the cost function with fuzzy cycle time is:

$$\tilde{C}(\tilde{T}) = \frac{1}{\tilde{T}}[A + C\tilde{D} + h\tilde{I}_T] = \frac{A}{\tilde{T}} + C[\frac{1}{2}a\theta\ \tilde{T}^2] + h[\frac{1}{2}a\tilde{T} + \frac{1}{6}\theta a\tilde{T}^2 + \frac{1}{24}a\theta^2\tilde{T}^3]$$

To defuzzify the cost function we will introduce the signed distance. We know for any a and $0 \in$ R, the signed distance from a to 0 is $d_0(a,0) = a$. If $a < 0$, the distance from a to 0 is $-a$ = -$d_0(a,0)$. Let Ψ be the family of all fuzzy sets $\tilde{B}$ defined on R for which the α-cut $B(\alpha) = [B_L(\alpha), B_U(\alpha)]$ exists for every $\alpha \in [0, 1]$. Both $B_L(\alpha)$ and $B_U(\alpha)$ are continuous functions on $\alpha \in [0, 1]$. Then we can say for any $\tilde{B} \in \Psi$ we have $\tilde{B} = \bigcup_{0 \le \alpha \le 1}[B_L(\alpha)_\alpha, B_U(\alpha)_\alpha]$.
So for $\tilde{B} \in \Psi$ we can define the signed distance of $\tilde{B}$ to $\tilde{0}$ (y axis) as:

$$d(\tilde{B}, \tilde{0}_1) = \frac{1}{2}\int_0^1 [B_L(\alpha) + B_U(\alpha)]d\alpha$$

For the triangular fuzzy number $\tilde{A} = (a, b, c)$, the α-cut of $\tilde{A}$ is $A(\alpha)$=[$A_L(\alpha), A_U(\alpha)$], for $\alpha \in [0,1]$, where $A_L(\alpha) = a + (b - c)\alpha$ and $A_U(\alpha) = c - (c - b)\alpha$, the signed distance of $\tilde{A}$ to $\tilde{0}$ (y axis) is:

$$d(\tilde{A}, \tilde{0}_1) = \frac{1}{4}(a + 2b + c)$$

The signed distance of $C(T)$ and 0 is:

$$d(\tilde{C}, \tilde{0}) = Ad(1/\tilde{T}, \tilde{0}) + C[\frac{1}{2}a\theta(d(\tilde{T}, \tilde{0}))^2] + h[\frac{1}{2}ad(\tilde{T}, \tilde{0}) + \frac{1}{6}\theta a(d(\tilde{T}, \tilde{0}))^2 + \frac{1}{24}a\theta^2(d(\tilde{T}, \tilde{0}))^3]$$

From the definition of signed distance we can write:

$d(\tilde{T},\tilde{0}) = T$

and as $\tilde{T} \in \Psi$, where Ψ be the family of all fuzzy sets $\tilde{T}$ defined on R for which the α -cut $T(\alpha)$ =[$T_L(\alpha), T_U(\alpha)$] exists for every $\alpha \in [0,1]$ and both $T_L(\alpha), T_U(\alpha)$ are continuous functions on $\alpha \in [0,1]$. Then for $\tilde{T} \in \Psi$, the signed distance is:

$$d(\tilde{C},\tilde{0}) = Ad(1/\tilde{T},\tilde{0}) + C[\frac{1}{2}a\theta(d(\tilde{T},\tilde{0}))^2] + h[\frac{1}{2}ad(\tilde{T},\tilde{0}) + \frac{1}{6}\theta a(d(\tilde{T},\tilde{0}))^2 + \frac{1}{24}a\theta^2(d(\tilde{T},\tilde{0}))^3] \quad (9)$$

From the definition of signed distance we can write:

$$d(\tilde{T},\tilde{0}) = T \quad (10)$$

and as $\tilde{T} \in \Psi$, where Ψ be the family of all fuzzy sets $\tilde{T}$ defined on R for which the α -cut $T(\alpha)$ =[$T_L(\alpha), T_U(\alpha)$] exists for every $\alpha \in [0,1]$ and both $T_L(\alpha), T_U(\alpha)$ are continuous functions on $\alpha \in [0,1]$. Then for $\tilde{T} \in \Psi$, the signed distance is:

$$d(1/\tilde{T},\tilde{0}) = \frac{1}{2}\int_0^1 [(1/T)_L(\alpha) + (1/T)_U(\alpha)]d\alpha = \frac{1}{2}\int_0^1 [\frac{1}{T+\Delta-\Delta\alpha} + \frac{1}{T-\Delta+\Delta\alpha}]d\alpha = = \frac{1}{2\Delta}\ln(\frac{T+\Delta}{T-\Delta}) \quad (11)$$

From (8), (9), (10), (11) we can write the defuzzified total cost:

$$C(T) = d(\tilde{C}(\tilde{T}),\tilde{0}) = \frac{1}{2\Delta}A\ln(\frac{T+\Delta}{T-\Delta}) + C[\frac{1}{2}a\theta\ T^2] + h[\frac{1}{2}aT + \frac{1}{6}\theta aT^2 + \frac{1}{24}\theta^2 aT^3] \quad (12)$$

Theorem 1: The average system cost function $C(T)$, given by (12), is strictly convex.

Proof. Here $\frac{dC(T)}{dT} = \frac{A}{2\Delta}[\frac{1}{T+\Delta} - \frac{1}{T-\Delta}] + Ca\theta\ T + h[\frac{1}{2}a + \frac{1}{3}\theta aT + \frac{1}{8}a\theta^2T^2]$

And $$\frac{d^2C(T)}{dT^2} = 2A\frac{T}{(T^2-\Delta^2)^2} + Ca\theta + h[\frac{1}{3}\theta a + \frac{1}{4}\theta^2 aT] > 0 \quad (13)$$

Hence $C(T)$ is strictly convex.

Since $C(T)$ is strictly convex in T, there exists an unique optimal cycle time T^* that minimizes $C(T)$. This optimal cycle time T^* is the solution of the equation dC/dT =0.

From (8) and (12) we can say that if there is no uncertainty (Δ=0), then the fuzzy model will convert into the crisp model because $(1/2\Delta)\ln[(T+\Delta)/(T-\Delta)]$= $(1/T)[1+(\Delta/T)^2+\ldots]$ and if we consider the limit as $\theta \to 0$ then $T^* = \sqrt{(2A/ah)}$ which is standard EOQ cycle time. Again as $\theta \to 0$, $C(T^*) \to (ahT^*/2) + (A/T^*)$, which is well known result for the Wilson [18] EOQ model. If we put b =0 and β =0 in equation (7) of Roy and Samanta [9], then we get the same expression of average system cost function as we got in this paper for average system cost (8). Defuzzified total cost (12) is also same, only we got extra term $(1/2\Delta)\ln[(T+\Delta)/(T-\Delta)]$ as we have accounted uncertainties.

5. Numerical

Given A=40, a =5, h =12, θ=0.01, Δ=0.04, C =10.
We obtain for crisp model total cost= 69.74295, cycle time =1.139630 and for fuzzy model total cost = 69.75736, cycle time =1.140322
So the result shows that if the uncertainties are accounted for in appropriate manner the cycle time would increase 0.06%.

6. Sensitivity analysis

The sensitivity analysis is performed by changing the value of each of the parameters by –50%, -20%, 20% and 50%, taking one parameter at time and keeping the remaining parameters unchanged. We now study sensitivity of the optimal solution to changes in the values of the different parameters associated with the system based on the above example.

A careful study of Table 1: sensitivity analysis reveals the following points:

(i) Cycle time T is slightly sensitive to changes in the values of the parameters θ, and Δ, and it is moderately sensitive to changes in C and highly sensitive to changes in h, a and A.
(ii) Crisp system cost is slightly sensitive to changes in the values of the parameters Δ and moderately sensitive to changes in C, θ and highly sensitive to changes in h, a and A.
(iii) Fuzzy system cost is moderately sensitive to changes in C, θ, Δ and highly sensitive to changes in h, a and A.

Here we have assumed that insensitive, moderately sensitive and highly sensitive imply % changes are +10 to –10, +50 to –50 and more respectively.

Table1. Sensitivity Analysis

Parameter	% Change	% Change in T	% Change in crisp cost	% Change in fuzzy cost
C	-50	1,45586	69,57987	69,59407
	-20	1,42413	69,6779	69,69222
	20	1,13825	69,80778	69,82226
	50	1,13518	69,90457	69,91916
h	-50	1,58405	49,76574	49,77111
	-20	1,26964	62,50872	62,51915
	20	1,04372	76,30178	76,32056
	50	0,93591	85,20578	85,23183
α	-50	1,60372	49,44841	49,45359
	-20	1,27284	62,42799	62,43835
	20	1,04226	76,35609	76,37495
	50	0,93355	85,3148	85,34105
θ	-50	1,14774	69,51392	69,52804
	-20	1,14326	69,65168	69,66597
	20	1,13742	69,8338	69,84831
	50	1,13313	69,96926	69,98393
Λ	-50	1,13981	69,74295	69,74655
	-20	1,14007	69,74295	69,75217
	20	1,14063	69,74297	69,7637
	50	1,14118	69,74301	69,77536
A	-50	0,80987	49,22112	49,24123
	-20	1,02147	62,33696	62,35299
	20	1,24751	76,44713	76,46032
	50	1,39233	85,54231	85,55417

6. Conclusions

In this paper we have developed an inventory model for deteriorating items. To capture the real life situation we have considered that the cycle time is uncertain and it is possible to describe it by triangular fuzzy number (symmetric). Numerically we tried to compare the crisp model with fuzzy model and we concluded that if the uncertainties are accounted for in appropriate manner the cycle time would increase. Sensitivity analysis is studied to see how far the output of the model is affected by changes or errors in its input parameters based on the numerical example.

We only considered the without shortage case in this paper. In future we will study the fuzzy continuous inventory model with shortage.

References

[1]. Aggarwal, S.P. (1978). *A note on an order-level inventory model for a system with constant rate of deterioration.* Opsearch. 15. 184-187.

[2]. Bellan, R. E. and Zadeh, L.A. (1970). *Decision-making in a fuzzy environment*. Manag. Sci. 17 (4). 141-164.

[3]. Chang, H-C. (2004). *An application of fuzzy sets theory to the EOQ model with imperfect quality items*. Computers & Operations Research. 31. 2079-2092.

[4]. Chang, S.C., Yao, J.S. and Lee, H.M. (1998). *Economic reorder point for fuzzy backorder quantity*. European Journal of Operational Research. 109. 183-202.

[5]. Ghare, P.M. and Schrader, G.P. (1963). *A model for exponentially decaying inventories*. Journal of Industrial Engineering. 14. 238-243.

[6]. Liberatore, M.J. (1979). *The EOQ model under stochastic lead time*. Operations Research. 27. 391-396.

[7]. Ouyang, L.Y. and Yao, J.S. (2002). *A minimax distribution free procedure for mixed inventory model involving variable lead time with fuzzy demand*. Computers & Operations Research. 29. 471-487.

[8]. Ouyang, L.Y. and Wu, K.S. (1998). *A minimax distribution free procedure for mixed inventory model with variable lead time*. Int. Journal of Production Economics. 56-57. 511-516.

[9]. Roy, A. and Samanta, G.P. (2004). *An EOQ model for deteriorating items with stock-dependent time varying demand*. Tamsui Oxford Journal of Management Sciences. 20(2). 37-46.

[10]. Roy, A. and Samanta, G.P. (2006). An inventory model of deteriorating items with time-varying demand and shortages. AMSE periodical. 27(1). 1-19.

[11]. Salameh, M.K. and Jaber, M.Y., (2000). *Economic production quantity model for items with imperfect quality*. Int. Journal of Production Economics, 64, 59-64.

[12]. Samanta, G.P. and Roy, A. (2004). *A deterministic inventory model of deteriorating items with two rates of production and shortages*. Tamsui Oxford Journal of Mathematical Sciences. 20(2). 205-218.

[13]. Samanta, G.P. and Roy, A. (2004). *A production inventory model with deteriorating items and shortages*. Yugoslav Journal of Operation Research. 14(2). 219-230.

[14]. Shah, Y.K. and Jaiswal, M.C. (1977). A*n order-level inventory model for a system with constant rate of deterioration*. Opsearch. 14. 174-184.

[15]. Tanaka, H., Oxuda, T. and Asai, K. (1974). *On fuzzy mathematical programming. J. Cybernetics*. 3(4). 37-46.

[16]. Tütüncü, G.Y., Aköz, O., Apaydın, A. and Petrovic, D. (2008). *Continuous review inventory control in the presence of fuzzy costs*. International Journal of Production Economics. 113(2). 775-784

[17]. Whitin, T.M. (1957). *Theory of Inventory Management*. Princeton University Press, New Jersey.

[18]. Wilson, R.H. (1934). *A Scientific Routine for Stock Control*. Harvard Business Review. 13. 116-128.

[19]. Yao, J.S. and Chiang, J. (2003). *Inventory without backorder with fuzzy total cost and fuzzy storing cost defuzzified by centroid and signed distance*. European Journal of Operational Research. 148. 401-409.

[20]. Yao, J.S. and Lee, H.M. (1996). *Fuzzy inventory with backorder for fuzzy order quantity*. Information Sciences. 93. 283-319.

[21]. Yao, J.S., Ouyang, L.Y. and Chiang, J. (2003). *Models for a fuzzy inventory of two replaceable merchandises without backorders based on the signed distance of fuzzy sets*. European Journal of Operational Research. 150. 601-616.

[22]. Zadeh, L.A. (1965). Fuzzy Sets. Information and Control. 8. 338-353

[23]. Zadeh, L.A. (1973). *Outline of a new Approach to the Analysis of Complex Systems and Decision Processes*. IEEE Transactions on Systems. Man and Cybernetics. SMC-3, 28-44.

[24]. Zimmermann, H.J. (1976). *Description and optimization of fuzzy mathematical programming*. Int. J. General Syst. 2(4). 209-215.