Electronic Journal of Applied Statistical Analysis
EJASA, Electron. J. App. Stat. Anal.
http://siba-ese.unisalento.it/index.php/ejasa/index
e-ISSN: 2070-5948
DOI: 10.1285/i20705598v18n2p344

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15 October 2025

Electronic Journal of Applied Statistical Analysis
Vol. 18, Issue 02, October 2025, 344-358
DOI: 10.1285/i2070598v18n2p344

Improved group sampling plans under odd-Perks-Lomax model with limited risks

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15 October 2025

In this article, a group sampling scheme for lot sentencing is developed under time-censoring when the lifetime of a product follows the odd-Perks-Lomax distribution. The test plans are constructed by limiting a linear combination of the producer and consumer risks from frequentist and Bayesian frameworks. Integer nonlinear programming is used to designate the optimal number of groups and acceptance limit. Several tables and figures are constructed to scrutinize the performance of the proposed testing strategies. The proposed optimal test plans outperform the traditional optimal two-point plan in terms of sample size. Furthermore, using prior information for defectives proportion increases the effectiveness of the proposed plans. A numerical example is provided to demonstrate the application of the introduced scheme.

keywords: Group acceptance sampling plan; integer nonlinear programming; operating characteristics; truncated life test; weighted-average of risks.

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ISSN: 2070-5948
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1 Introduction

Acceptance sampling constitutes a broadly employed framework within the statistical quality control, especially pertinent to the context of industrial manufacturing operations. This approach precisely specifies the essential sample size required to evaluate the acceptability of a submitted lot, employing empirical data sourced from systematic experimental studies. In these schemes, the product lots are subjected to inspection protocols to determine their acceptance or rejection based on randomly selected samples extracted from the lots. Several lot inspection methodologies are documented in the academic literature, each presenting a diverse array of perspectives. Among others, we can cite Fernández et al. (2011), Wu and Huang (2014), Al-Omari et al. (2016), Zhao et al. (2019) and, more recently, Al-Omari and Alomani (2022), Pérez-González et al. (2023), Tripathi et al. (2024), Al-Omari and Alomani (2024), Naghizadeh Qomi and Fernández (2024), Naghizadeh Qomi and Tripathi (2025) and Naghizadeh Qomi et al. (2025).

Single acceptance sampling plans (*SASPs*) by attributes are the simplest plans in industrial quality, where a batch of products is accepted if no more than c failures occur during the experiment time, see Al-Husseini et al. (2023) and references therein. The advancement of *SASPs* has culminated in the development of group acceptance sampling plans (*GASPs*), wherein testers are able to assess multiple items simultaneously, thereby resulting in significant savings in both time and cost. Papers by Aslam and Jun (2009a,b), Aslam et al. (2009), Rao (2009, 2010), Aslam et al. (2011) and more recently, Tripathi et al. (2021), Tripathi and Aslam (2024), Al-Omari and Ismail (2024), Ekemezie et al. (2024) and Nwankwo et al. (2024) introduced *GASPs*.

Ekemezie et al. (2024) used odd-Perks-Lomax (*OPL*) distribution to design an optimal *GASP* by using two-point approach where the number of groups and the acceptance number are determined by considering the consumer's risk and producer's risk at the same time. Our motivation is to design a *GASP* where the optimal components of the plan are determined by limiting a weighted-average of the conventional and expected producer and consumer risks.

The remainder of the article is structured as follows. The methodology of *GASP* is proposed in Section 2. The *GASP*s with conventional limited risks are summarized and developed to find best plans with limited weighted-average risks in Section 3. The optimal *GASP*s with expected limited weighted-average risks are determined in Section 4. A real data example is provided in Section 5. Finally, concluding remarks are offered in Section 6.

2 Design of the group sampling plans for OPL distribution

According to Aslam and Jun (2009a) and Ekemezie et al. (2024), the procedure of the time truncated *GASP* is as follows:

1. Determine the number of groups g and allocate k unit to each group so that the

sample size is $n = k \times g$.

2. Opt the acceptance limit or number c $(< k)$ for a group and the termination time t_0.

3. Conduct the test for the g groups altogether and record the number of failures for every group.

4. Accept the batch provided that no more than c failures are observed across each of the groups; otherwise, terminate the experiment and dismiss the batch.

The proposed plan would be described by (g, c), when the experiment time t_0 is fixed.

Ekemezie et al. (2024) proposed a new flexible model called *OPL* distribution with probability density function (pdf) as

$$f(t; \alpha, \beta, \theta, \lambda) = \frac{\theta\alpha\beta(\beta+1)(1+t/\lambda)^{\alpha-1}\exp\{\theta[(1+t/\lambda)^{\alpha}-1]\}}{\lambda\big\{1+\beta\exp\{\theta[(1+t/\lambda)^{\alpha}-1]\}\big\}}, \quad t > 0, \tag{1}$$

and the corresponding cumulative distribution function (cdf) given by

$$F(t; \alpha, \beta, \theta, \lambda) = 1 - \frac{1+\beta}{1+\beta\exp\{\theta[(1+t/\lambda)^{\alpha}-1]\}}, \quad t > 0, \tag{2}$$

where $\alpha, \beta, \theta, \lambda > 0$. They pointed out that the additional parameter(s) does not hinder the tractability of the model and its characteristics. Moreover, this model is unique in providing a better goodness of fit than some known conventional models such as the Weibull and the Exponentiated Weibull. Furthermore, the pdf of the proposed model has a reversed bathtub shape, L-shape, and strictly decreasing (positively skewed) shape.

The median lifetime can be considered as the quality characteristic of interest. The median of *OPL* distribution is given by $\mathcal{M} = m\lambda$, where m is a function of $\phi = (\alpha, \beta, \theta)$ as

$$m \equiv m(\phi) = \left[1 + \frac{1}{\theta}\ln\left(\frac{0.5+\beta}{0.5\beta}\right)\right]^{1/\alpha} - 1. \tag{3}$$

Let $\mathcal{M}_0$ be the specified median life and the termination time is given by $t_0 = q\mathcal{M}_0$, where q is a positive constant known as termination ratio.

The operating characteristic (OC) curve depicts the relationship between the probability of accepting a lot and the true proportion of defective items p. The OC function is defined by $A(p) \equiv A(p; g, c)$ and is given by

$$A(p) = \left[\sum_{i=0}^{c}\binom{k}{i}p^i(1-p)^{k-i}\right]^g, \tag{4}$$

where p denotes the probability that a product in a group fails before the time t_0 and is given by

$$p \equiv p(q, \alpha, \beta, \theta, r)$$

$$= \quad 1 - \frac{1+\beta}{1+\beta\exp\{\theta[(1+qm/r)^{\alpha}-1]\}}, \tag{5}$$

where the ratio $r = \mathcal{M}/\mathcal{M}_0$ is expressed as the quality level of a product.

assume that the producer and consumer characterize the acceptable and rejectable defective rates, defined as $p_0 \equiv p(q,\alpha,\beta,\theta,r_0)$ and $p_1 \equiv p(q,\alpha,\beta,\theta,r_1)$, respectively, where r_0 is the median ratio at the producer's risk and r_1 is the median ratio at the consumer's risk. The conventional producer and consumer risks (PR and CR) are defined respectively as $\sup_{p\le p_0}\{1-A(p)\}$ and $\sup_{p\ge p_1}\{A(p)\}$. Since $A(p)$ is a decreasing function of p, then the PR and CR are given by $PR(g,c,p_0) = 1-A(p_0)$ and $CR(g,c,p_1) = A(p_1)$, which can be expressed as

$$PR(g,c,p_0) = 1 - \left[\sum_{i=0}^{c}\binom{k}{i}p_0^i(1-p_0)^{k-i}\right]^g,$$

and

$$CR(g,c,p_1) = \left[\sum_{i=0}^{c}\binom{k}{i}p_1^i(1-p_1)^{k-i}\right]^g.$$

3 Optimal *GASP* with limited weighted average of risks

The conventional two-point method to derive optimal *GASP* controls consumer and producer risks concurrently. The consumer requests that the lot acceptance probability should be smaller than the determined consumer's risk $\beta^\star$ at a lower quality level (usually at ratio 1), whereas the producer demands that the lot rejection probability should be smaller than the specified producer's risk $\alpha^\star$ at a higher quality level. The producer desires $PR(g,c,p_0) \le \alpha^\star$, whereas the consumer tends to $CR(g,c,p_1) \le \beta^\star$. Optimal $(\alpha^\star,\beta^\star)-$(PR,CR) plans, (g_t,c_t), can be determined by solving the constrained optimization problem

$$\begin{aligned} &\text{Minimize} && g \\ &\text{Subject to} && PR(g,c,p_0) \le \alpha^\star, \\ &&& CR(g,c,p_1) \le \beta^\star, \\ &&& g\in\mathbb{N},\ c\in\mathbb{N}\cup\{0\}, \\ &&& c<k, \end{aligned} \tag{6}$$

where $\mathbb{N} = \{1,2,3,...\}$ is the set of positive integers. Ekemezie et al. (2024) have determined the optimal *GASP* by satisfying the minimization problem (6) and provided some tables for the results. We present the optimal number of groups and acceptance numbers of *GASP*, (g_t,c_t), and the associated PR and CR for $q = 0.5, 1$, $k = 5, 10$ and two sets of parameters $\phi = (1.75, 2, 3)$ and $\phi = (0.15, 1.25, 1.5)$ in Tables 1 and 2, respectively. The cells with a dash (-) indicate the g_t cannot be obtained to satisfy the conditions.

Fernández et al. (2020) designed optimal acceptance test plans by limiting a weighted-average of producer and consumer risks. Our aim is to expand this method to design of

Table 1: Optimal $(\alpha^\star, \beta^\star)-$(PR,CR) plans, (g_t, c_t), and the associated risks(%) for $\phi = (1.75, 2, 3)$.

		$k=5$								$k=10$							
		$q=0.5$				$q=1.0$				$q=0.5$				$q=1.0$			
$\beta^\star$	r_0	g_t	c_t	PR	CR	g_t	c_t	PR	CR	g_t	c_t	PR	CR	g_t	c_t	PR	CR
0.25	2	930	4	4.96	24.98	-	-	-	-	46	5	4.36	24.87	8	6	4.61	22.12
	6	11	2	1.10	22.13	3	2	2.17	12.50	2	2	2.00	21.21	1	3	1.05	17.19
	10	3	1	2.31	20.16	1	1	2.90	18.75	1	1	3.17	19.79	1	2	1.65	5.47
	14	3	1	1.21	20.16	1	1	1.54	18.75	1	1	1.70	19.79	1	2	0.66	5.47
0.10	2	-	-	-	-	-	-	-	-	391	6	3.36	9.95	41	7	3.14	9.97
	6	17	2	1.70	9.72	4	2	2.88	6.25	3	2	2.99	9.77	2	3	2.09	2.95
	10	5	1	3.82	6.93	4	2	0.68	6.25	3	2	0.73	9.77	1	2	1.65	5.47
	14	5	1	2.00	6.93	2	1	3.05	3.52	2	1	3.36	3.92	1	2	0.66	5.47
0.05	2	-	-	-	-	-	-	-	-	508	6	4.35	4.99	54	7	4.11	4.80
	6	22	2	2.19	4.90	5	2	3.59	3.13	4	2	3.97	4.50	2	3	2.09	2.95
	10	6	1	4.56	4.06	5	2	0.85	3.13	4	2	0.97	4.50	2	2	3.27	0.30
	14	6	1	2.40	4.06	2	1	3.05	3.52	2	1	3.36	3.92	2	2	1.32	0.30
0.01	2	-	-	-	-	-	-	-	-	-	-	-	-	427	8	2.60	0.99
	6	34	2	3.36	0.94	7	2	4.99	0.78	15	3	1.28	0.76	3	3	3.11	0.51
	10	34	2	0.76	0.94	7	2	1.19	0.78	6	2	1.45	0.95	2	2	3.27	0.30
	14	9	1	3.57	0.82	3	1	4.54	0.66	6	2	0.56	0.95	2	2	1.32	0.30

Table 2: Optimal $(\alpha^\star, \beta^\star)-$(PR,CR) plans, (g_t, c_t), and the associated risks(%) for $\phi = (0.15, 1.25, 1.5)$.

		$k=5$								$k=10$							
		$q=0.5$				$q=1.0$				$q=0.5$				$q=1.0$			
$\beta^\star$	r_0	g_t	c_t	PR	CR	g_t	c_t	PR	CR	g_t	c_t	PR	CR	g_t	c_t	PR	CR
0.25	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	6	134	4	2.08	24.85	44	4	4.43	24.74	8	5	2.38	23.07	8	6	2.95	22.12
	10	16	3	1.75	23.06	7	3	4.02	23.38	4	4	1.78	15.87	3	5	1.57	24.19
	14	4	2	3.26	21.54	7	3	1.88	23.38	2	3	2.43	14.47	2	4	2.44	14.21
0.10	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	6	222	4	3.41	9.96	-	-	-	-	13	5	3.84	9.22	13	6	4.75	8.61
	10	26	3	2.83	9.22	73	4	1.92	9.85	6	4	2.66	6.32	5	5	2.6	9.39
	14	6	2	4.85	10	12	3	3.2	8.28	3	3	3.62	5.5	3	4	3.63	5.36
0.05	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	6	289	4	4.42	4.96	-	-	-	-	17	5	4.99	4.43	54	7	2.4	4.8
	10	33	3	3.58	4.85	95	4	2.5	4.9	7	4	3.09	3.99	7	5	3.62	3.64
	14	33	3	1.43	4.85	15	3	3.99	4.44	4	3	4.8	2.09	4	4	4.82	2.02
0.01	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
	6	-	-	-	-	-	-	-	-	81	6	2.76	0.99	82	7	3.62	0.99
	10	444	4	1.35	0.99	146	4	3.81	0.97	11	4	4.82	0.63	25	6	1.69	0.90
	14	51	3	2.21	0.93	146	4	1.42	0.97	11	4	1.68	0.63	10	5	1.8	0.88

time-truncated *GASP* for *OPL* distribution. To do this, consider the following weighted-average risks (WR) as

$$WR(g, c, p_0, p_1) = w_0 PR(g, c, p_0, p_1) + w_1 CR(g, c, p_0, p_1), \tag{7}$$

where the positive constants w_0 and w_1 are the producer and consumer weights, respectively and $w_0 + w_1 = 1$.

Suppose that the analyst wants to control the risk incurred by the selected *GASP* by considering $\gamma \in (0, 1)$ as the maximum risk tolerated, where $\gamma \leq \min\{w_0, w_1\}$. Our aim is to determine the minimum number of groups and acceptance number that satisfy the inequality $WR(g, c, p_0, p_1) \leq \gamma$. Optimal $\gamma-$WR plans, $(g^\star, c^\star)$, can be determined by solving the constrained optimization problem

$$\begin{array}{ll} \text{Minimize} & g \\ \text{Subject to} & WR(g, c, p_0, p_1) \leq \gamma, \\ & g \in \mathbb{N},\ c \in \mathbb{N} \cup \{0\}, \\ & c < k. \end{array} \tag{8}$$

Optimal $\gamma-$WR plan, $(g^\star, c^\star)$, and the associated risks(%) are reported in Tables 3 and 4 for selected values of $\gamma = 0.01, 0.05,\ r_0 = 2(4)14,\ r_1 = 1,\ k = 5,\ q = 0.5, 1.0$ and $w_0 = 0.2, 0.5, 0.8$ when $\phi = (1.75, 2, 3)$ and $\phi = (0.15, 1.25, 1.5)$, respectively. It is observed that optimal number of groups and acceptance number tend to decrease as γ increases. For instance, if $r_0 = 6$, $q = 0.5$ and $w_0 = 0.2$, from Table 3, the optimal $0.01-$WR and $0.05-$WR plans are $(208, 3)$ and $(7, 1)$, respectively. Moreover, optimal number of groups decrease when r_0 increases. For example, if $w_0 = 0.2$, and $q = 0.5$, then the optimal $0.01-$WR group numbers are 208 and 34 when $r_0 = 6$ and $r_0 = 10$, respectively.

For a graphical comparison of traditional optimal two-point $(\gamma, \gamma)-$(PR,CR) [hereafter $\gamma-$(PR,CR)] plan and $\gamma-$WR plan, Figure 1 shows the optimal $0.05-$(PR,CR) and $0.05-$WR group numbers versus w_0 for $q = 0.5,\ k = 5$ and $r_0 = 6$ when $\phi = (1.75, 2, 3)$ and $\phi = (0.15, 1.25, 1.5)$. It can be seen that the optimal $0.05-$(PR,CR) group numbers are 22 and 289 when $\phi = (1.75, 2, 3)$ and $\phi = (0.15, 1.25, 1.5)$, respectively, whereas the values of optimal $0.05-$WR group numbers are less than 22 and 289. This shows that the optimal $\gamma - WR$ plan outperforms the traditional optimal $\gamma - (PR, CR)$ plan in terms of the number of groups (sample size), when the PR and CR are at most γ.

4 Optimal *GASP* with limited expected weighted average of risks

In general, traditional conventional risks are typically used in cases where the prior distribution of p is either unavailable or has been discarded. Suppose that there are some information about the failure rate, p. This information can be used for evaluation of producer's and consumer's risks.

Table 3: Optimal $\gamma-$WR plans, $(g^\star, c^\star)$, and the associated risks(%) for $k = 5$ and $\phi = (1.75, 2, 3)$.

			$(w_0, w_1) = (0.2, 0.8)$					$(w_0, w_1) = (0.5, 0.5)$					$(w_0, w_1) = (0.8, 0.2)$				
q	γ	r_0	$g^\star$	$c^\star$	WR	PR	CR	$g^\star$	$c^\star$	WR	PR	CR	$g^\star$	$c^\star$	WR	PR	CR
0.5	0.01	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		6	208	3	0.99	0.51	1.10	194	3	0.99	0.48	1.50	156	3	0.99	0.39	3.41
		10	34	2	0.91	0.76	0.94	32	2	0.98	0.72	1.24	27	2	0.98	0.61	2.47
		14	33	2	0.92	0.28	1.08	30	2	0.94	0.25	1.63	24	2	0.90	0.20	3.72
	0.05	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		6	7	1	4.62	13.56	2.38	19	2	4.64	1.89	7.39	12	2	4.82	1.20	19.29
		10	6	1	4.16	4.56	4.06	6	1	4.31	4.56	4.06	4	1	4.82	3.07	11.82
		14	6	1	3.73	2.40	4.06	5	1	4.47	2.00	6.93	3	1	5.00	1.21	20.16
1.0	0.01	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		6	22	3	0.99	0.81	1.04	22	3	0.92	0.81	1.04	19	3	0.95	0.7	1.93
		10	7	2	0.86	1.19	0.78	7	2	0.98	1.19	0.78	15	3	0.95	0.08	4.44
		14	7	2	0.71	0.45	0.78	6	2	0.97	0.38	1.56	5	2	0.88	0.32	3.13
	0.05	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		6	3	1	4.63	20.49	0.66	4	2	4.57	2.88	6.25	3	2	4.24	2.17	12.5
		10	2	1	3.96	5.72	3.52	2	1	4.62	5.72	3.52	3	2	2.91	0.51	12.5
		14	2	1	3.42	3.05	3.52	2	1	3.28	3.05	3.52	1	1	4.98	1.54	18.75

Table 4: Optimal $\gamma-$WR plans, $(g^\star, c^\star)$, and the associated risks(%) for $k = 5$ and $\phi = (0.15, 1.25, 1.5)$.

			$(w_0, w_1) = (0.2, 0.8)$					$(w_0, w_1) = (0.5, 0.5)$					$(w_0, w_1) = (0.8, 0.2)$				
q	γ	r_0	$g^\star$	$c^\star$	WR	PR	CR	$g^\star$	$c^\star$	WR	PR	CR	$g^\star$	$c^\star$	WR	PR	CR
0.5	0.01	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		10	453	4	0.99	1.38	0.90	-	-	-	-	-	-	-	-	-	-
		14	55	3	0.99	2.38	0.65	397	4	0.99	0.37	1.62	315	4	0.99	0.29	3.79
	0.05	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		6	40	3	4.93	14.43	2.55	274	4	4.99	4.20	5.80	198	4	4.99	3.05	12.78
		10	10	2	4.72	14.99	2.15	30	3	4.82	3.26	6.39	20	3	4.94	2.18	15.98
		14	8	2	4.99	6.41	4.64	27	3	4.79	1.17	8.41	17	3	4.8	0.74	21.04
1.0	0.01	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		6	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		10	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		14	149	4	0.99	1.45	0.88	-	-	-	-	-	-	-	-	-	-
	0.05	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		6	104	4	4.98	10.16	3.68	-	-	-	-	-	-	-	-	-	-
		10	16	3	4.67	8.94	3.61	80	4	4.99	2.11	7.89	52	4	4.94	1.37	19.19
		14	6	2	4.65	17.00	1.56	14	3	4.60	3.73	5.46	10	3	4.65	2.68	12.54

Figure 1: Optimal 0.05−WR and 0.05−(PR,CR) group numbers versus w_0 for $\phi = (1.75, 2, 3)$ [left] and $\phi = (0.15, 1.25, 1.5)$ [right] when $r_0 = 6$, $q = 0.5$ and $k = 5$.

Assume that the failure rate p follows a Beta distribution with parameters $a, b > 0$ and its pdf is given by

$$h(p) = \frac{p^{a-1}(1-p)^{b-1}}{\mathcal{B}(1, a, b)},$$

where $\mathcal{B}(x, a, b) = \int_0^x p^{a-1}(1-p)^{b-1}$. Following Fernández and Pérez-González (2012a,b), the expected producer risk (EPR) is the conditional expectation of rejecting an acceptable lot, whereas the expected consumer risk (ECR) is the conditional expectation of accepting a rejectable lot. For a given number of groups, g_e, and acceptance number, c_e, the expected producer and consumer risks are defined, respectively by

$$\begin{aligned} EPR(g_e, c_e, p_0) &= E[1 - A(p)|p \le p_0] \\ &= 1 - \int_0^{p_0} \frac{A(p)h(p)}{Pr(p \le p_0)} dp,, \end{aligned}$$

and

$$\begin{aligned} ECR(g_e, c_e, p_1) &= E[A(p)|p \ge p_1] \\ &= \int_{p_1}^1 \frac{A(p)h(p)}{Pr(p \ge p_1)} dp. \end{aligned}$$

Now, consider expected WR (EWR) of the form

$$EWR(g_e, c_e, p_0, p_1) = w_0 EPR(g_e, c_e, p_0) + w_1 ECR(g_e, c_e, p_1). \tag{9}$$

We would like to obtain the optimal $\gamma-$EWR plans, g_e^* and c_e^*, where $EWR \leq \gamma$. The constrained minimization problem can be stated as follows:

$$\begin{array}{ll} \text{Minimize} & g_e \\ \text{Subject to} & EWR(g_e, c_e, p_0, p_1) \leq \gamma, \\ & g_e \in \mathbb{N},\ c_e \in \mathbb{N} \cup \{0\}, \\ & c_e < k. \end{array} \tag{10}$$

Optimal $\gamma-$EWR plans, $(g_e^\star, c_e^\star)$, and the associated risks (EWR, EPR, ECR) are summarized in Table 5 for different values of $\gamma = 0.01, 0.05,\ r_0 = 2(4)14,\ q = 0.5, 1.0,\ k = 5$, $w_0 = 0.2, 0.5, 0.8$ and $\phi = (0.15, 1.25, 1.5)$ when the prior distribution is $Beta(a, b)$ with mode $5(p_0 + p_1)/6$ and $a + b = 5$. A similar trend to the Table 3 holds for the results of Table 5. In comparison of Tables 3 and 5, it is clear to observe that $g_e^\star \leq g^\star$ and $c_e^\star \leq c^\star$.

Figure 2 shows the optimal $\gamma-$WR and $\gamma-$EWR number of groups and the corresponding optimal acceptance numbers versus w_0 for $r_0 = 6, k = 5,\ q = 0.5$ and $\phi = (1.75, 2, 3)$ when the maximum risk level is 0.05. We can conclude that the EWR number of groups and acceptance numbers are less than the corresponding minimum-WR number of groups and acceptance numbers.

The graphs of optimal $\gamma-$WR and $\gamma-$EWR group numbers and the corresponding optimal acceptance numbers, versus the maximum risk level γ, are displayed in Figure 3, when $r_0 = 6, k = 5,\ q = 0.5,\ w_0 = 0.5$ and $\phi = (1.75, 2, 3)$. Clearly, $g_e^\star$ tends to decrease when γ increases. Moreover, $g_c^\star \leq g^\star$ and $c_e^\star \leq c^\star$.

5 Real data application

The following data set reported in Smith and Naylor (1987) represents the strengths of 1.5 cm glass fibers. The corresponding observations are shown in Table 6. Ekemezie et al. (2024) obtained the maximum likelihood estimates of the parameters for this data set as $\hat{\alpha} = 5.5043,\ \hat{\beta} = 0.0327,\ \hat{\theta} = 0.0944$ and $\hat{\lambda} = 1.6677$. The suitability of data for the *OPL* distribution was assessed using the log-likelihood (LL) function, the Akaike information criterion (AIC), the consistent Akaike Information criterion (CAIC), Bayesian information criterion (BIC), Kolmogorov-Smirnov (K-S) statistic and p-value. Table 7 represents the model fitting summary of the chosen data set. From Table 7, it is clear that the *OPL* distribution has a good fit to considered data set as compared to Weibull and exponentiated Weibull distributions.

Optimal $0.05-$WR plans, $(g^\star, c^\star)$, and $0.05-$EWR plans, $(g_e^\star, c_e^\star)$ with associated risks(%) are summarized in Table 8 for $r_0 = 2(4)14,\ q = 0.5,\ k = 5$ and $w_0 = 0.5$ when $\hat{\phi} = (5.5043, 0.0327, 0.0944)$.

Suppose a manufacturer asserts that the median life of the products is greater than $\mathcal{M}_0 = 1000$ hours, and the lifetime of the products conforms with an *OPL* distribution

Table 5: Optimal $\gamma-$EWR plans, $(g_e^\star, c_e^\star)$, and the associated risks(%) for $k = 5$ and $\phi = (1.75, 2, 3)$.

			$(w_0, w_1) = (0.2, 0.8)$					$(w_0, w_1) = (0.5, 0.5)$					$(w_0, w_1) = (0.8, 0.2)$				
q	γ	r_0	$g_e^\star$	$c_e^\star$	EWR	EPR	ECR	$g_e^\star$	$c_e^\star$	EWR	EPR	ECR	$g_e^\star$	$c_e^\star$	EWR	EPR	ECR
0.5	0.01	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		6	17	2	0.96	0.63	1.04	16	2	0.92	0.60	1.25	12	2	0.91	0.45	2.74
		10	6	1	0.84	2.15	0.51	15	2	0.85	0.12	1.57	10	2	0.93	0.08	4.30
		14	6	1	0.64	1.12	0.52	5	1	0.98	0.93	1.03	5	1	0.95	0.93	1.03
	0.05	2	10	2	4.48	8.33	3.51	31	3	4.96	1.71	8.22	13	3	4.84	0.72	21.33
		6	3	1	4.06	2.90	4.35	3	1	3.63	2.90	4.35	2	1	3.56	1.95	10.03
		10	2	0	3.95	16.52	0.81	3	1	2.78	1.08	4.48	2	1	2.64	0.72	10.29
		14	2	0	3.08	12.10	0.82	3	1	2.55	0.56	4.53	2	1	2.38	0.37	10.40
1.0	0.01	2	-	-	-	-	-	-	-	-	-	-	-	-	-	-	-
		6	5	2	0.71	1.64	0.48	11	3	0.88	0.15	1.61	8	3	0.86	0.11	3.84
		10	5	2	0.48	0.38	0.51	4	2	0.76	0.3	1.21	3	2	0.79	0.23	3.03
		14	2	1	0.97	1.67	0.79	4	2	0.68	0.11	1.24	3	2	0.69	0.08	3.09
	0.05	2	7	3	4.34	6.02	3.92	7	3	4.97	6.02	3.92	11	4	4.94	0.57	22.44
		6	2	1	2.21	8.11	0.74	2	1	4.43	8.11	0.74	1	1	4.65	4.16	6.57
		10	1	0	4.37	18.32	0.88	1	1	4.22	1.60	6.84	1	1	2.65	1.60	6.84
		14	1	0	3.40	13.41	0.90	1	1	3.89	0.84	6.95	1	1	2.06	0.84	6.95

Figure 2: Optimal group and acceptance numbers with minimum γ-WR and γ-EWR versus w_0 when $\gamma = 0.05, r_0 = 6, k = 5$, $q = 0.5$ and $\phi = (1.75, 2, 3)$.

Figure 3: Optimal group and acceptance numbers with minimum γ-WR and γ-EWR versus γ when $w_0 = 0.5, r_0 = 6, k = 5$, $q = 0.5$ and $\phi = (1.75, 2, 3)$.

Table 6: Strengths of 1.5 cm glass fibers.

0.55	0.93	1.25	1.36	1.49	1.52	1.58	1.61	1.64	1.68	1.73	1.81	2.00	0.74	1.04	1.27
1.39	1.49	1.53	1.59	1.61	1.66	1.68	1.76	1.82	2.01	0.77	1.11	1.28	1.42	1.5	1.54
1.6	1.62	1.66	1.69	1.76	1.84	2.24	0.81	1.13	1.29	1.48	1.5	1.55	1.61	1.62	1.66
1.7	1.77	1.84	0.84	1.24	1.3	1.48	1.51	1.55	1.61	1.63	1.67	1.7	1.78	1.89	

Table 7: Distribution fit test results.

Model	LL	AIC	CAIC	BIC	K-S statistic	p-value
OPL	−13.28	32.23	32.96	40.80	0.0961	0.6052
Weibull	−15.21	34.57	34.77	38.86	0.1708	0.0507
Exponentiated Weibull	−14.68	35.35	35.76	41.78	0.1462	0.1351

with parameters $\hat{\phi} = (5.5043, 0.0327, 0.0944)$. Considering $r_0 = 14$, $k = 10$, $q = 0.5$ and the true median life is $\mathcal{M} = 14000$ hours, optimal $(0.05, 0.05)$-(PR,CR) plans are obtained $(84, 1)$ from Table 7 of Ekemezie et al. (2024), while assuming $w_0 = 0.5$, the optimal 0.05-WR group plans are $(11, 0)$ with corresponding risks $WR = 4.88\%$, $PR = 6.37\%$ and $CR = 3.40\%$ obtained from Table 8 of our work.

Using (5), we obtain $p_0 = 0.0019$ and $p_1 = 0.9973$ when $r_1 = 1, r_0 = 14, q = 0.5$.

Table 8: Optimal 0.05−CWR and 0.05−EWR plans, and the associated risks(%) for selected values of $w_0 = 0.5$, $q = 0.5$ and $\hat{\phi} = (5.5043, 0.0327, 0.0944)$.

		0.05−WR					0.05−EWR				
k	r_0	$g^\star$	$c^\star$	WR	PR	CR	$g_e^\star$	$c_e^\star$	EWR	EPR	ECR
5	2	-	-	-	-	-	28	1	4.93	0.50	9.37
	6	-	-	-	-	-	4	0	4.88	1.61	8.15
	10	-	-	-	-	-	4	0	4.53	0.89	8.17
	14	22	0	4.88	6.37	3.40	4	0	4.40	0.61	8.18
10	2	-	-	-	-	-	8	1	4.67	0.63	8.71
	6	67	1	4.93	0.73	9.14	2	0	4.88	1.61	8.15
	10	66	1	4.85	0.22	9.47	2	0	4.53	0.89	8.17
	14	11	0	4.88	6.37	3.40	2	0	4.40	0.61	8.18

Assume that the failure probability, p, has a $Beta(a, b)$ distribution with $a + b = 5$ and mode $(a - 1)/(a + b - 2) = 0.8327$, which implies that $a = 3.4981$ and $b = 1.5018$. From Table 8, the optimal γ-EWR plans are $(g_e^\star, c_e^\star) = (2, 0)$. The corresponding risks are $EWR = 4.40\%$, $EPR = 0.61\%$ and $ECR = 8.18\%$. Therefore, the sample size is $n = 20$ (10×2). According to these specifications, a total of 20 products are needed and that ten items will be allocated to each of the two testers. We will accept the lot if no failure occurs before $t_0 = 500$h in each of the ten groups.

6 Concluding remarks

Ekemezie et al. (2024) have designed *GASPs* for *OPL* distribution by using two-point approach where the group number and acceptance number will be determined by considering the producer's and consumer's risks simultaneously. In this work, we presented a method in developing an optimal *GASP* for *OPL* distribution using limited conventional and expected weighted-average of risks.

γ-WR group sampling plans, $(g^\star, c^\star)$, are determined by solving constrained optimization problems. It is observed that the optimal γ-WR plans outperform the traditional optimal γ-(PR,CR) plans in terms of sample size. Moreover, γ-EWR plans, $(g_e^\star, c_e^\star)$, are determined. It is seen that $g_e^\star \leq g^\star$ and $c_e^\star \leq c^\star$. A real data analysis is provided to illustrate the results by comparison of the existence method and the proposed method.

The method introduced in this article for *OPL* distribution can be applied to other lifetime distributions. Moreover, other sampling plans such as double and repetitive plans will be determined by a weighted-average of the classical/expected producer and consumer risks (WR/EWR).

Acknowledgements

The authors thank the Editor and Reviewers for providing valuable comments and suggestions.

Data availability statement

The authors confirm that the data supporting the findings of this study are available within the article.

Disclosure statement

No potential conflict of interest was reported by the authors.

Funding

Not applicable.

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