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Exponentiated Poisson-Power Lindley Distribution: Properties and Applications

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In this article, the Exponentiated Poisson-power Lindley (EPPL) distribution, an extension of the Poisson-power Lindley (PPL) distribution, is introduced. Various properties of the EPPL distribution, including a linear representation, are studied. The method of maximum likelihood is utilized for parameter estimation, and its accuracy is validated through simulation technique. The flexibility of the proposed distribution is illustrated by considering the survival time of gastric patients. Additionally, a time-truncated acceptance sampling plan, based on the lifetime of products following the EPPL distribution is proposed. Key metrics such as minimum sample sizes, operating characteristic functions and the smallest ratios of mean life to specified are analyzed and presented. To illustrate the practical application of the proposed sampling plan in survival time data, a real data set comprising the survival times of cancer patients after a particular treatment is utilized and the effect of the treatment in the lifetime of the patients is examined based on the proposed sampling plan. In this article, both real datasets considered are survival time data from the medical field, highlighting the applicability of the newly introduced distribution in this context.

keywords: Acceptance sampling plan, Consumer's risk, Exponentiated Poisson-power Lindley distribution, Operating characteristic function, Time truncated life test.

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1 Introduction

Developing new distributions from the existing ones has great importance in the statistical world. Addition of parameters is one of the prominent method of generating new flexible distributions to handle the specific real world scenarios. Among them, exponentiating the base distribution is one of the simplest form of generalization, which riches the properties of the former distribution. The expontiated distributions in the literature include, exponentiated Weibull distribution (Mudholkar and Srivastava, 1993), exponentiated exponential distribution (Gupta et al., 1998), exponentiated Fr´echet distribution,(Nadarajah and Kotz, 2003), exponentiated Gumbel distribution (Nadarajah, 2006), exponentiated Lomax distribution (Abdul-Moniem and Abdel-Hameed, 2012), exponentiated power Lindley distribution (Ashour and Eltehiwy, 2015), exponentiated Chen distribution (Dey et al., 2017), exponentiated uniform distribution (Ramires et al., 2019) and exponentiated power function distribution (Arshad et al., 2020).

Now a days, statisticians show great interest in dealing with new generalized forms of Poisson distribution for modeling the scenario which are not equidispersed and Poisson-X family is such a generalized family introduced by Tahir et al. (2016). Poisson-power Lindley distribution is a new member of Poisson-X family introduced by George and George (2023b) which is capable of handling continuous data, including data with skewed and heavy-tailed nature, which the Poisson distribution cannot accommodate. Motivated by this, we introduce exponentiated generalization of Poisson-power Lindley distribution by considering Poisson-power Lindley distribution as the baseline distribution. Given the relevance of the exponentiated class of distributions in applied sciences, the exponentiated extension of the PPL distribution can be regarded as an advanced form of the PPL distribution, offering enhanced statistical properties and broad applications across various disciplines, including reliability, quality control, biology, industry, survival time data, and others.

Acceptance sampling plan (ASP) is a core tool of statistical quality control, which offers high quality products to the consumers by facilitates a reliable accept-reject decision about the manufactured product. In the literature, there is an extensive study on acceptance sampling plan based on different lifetime distributions, see Kantam et al. (2001) for log-logistic distribution, Balakrishnan et al. (2007) and Aslam et al. (2010) for generalized Birnbaum–Saunders distribution and generalized exponential distribution respectively. Rao (2013) used Marshall–Olkin extended exponential distribution. Generalized inverted exponential distribution and new Weibull-Pareto distribution are considered by Al-Omari (2015) and Al-Omari et al. (2016) respectively. Jose et al. (2018) constructed ASP based on Harris extended Weibull distribution. Al-Omari and Alomani (2022) used two-parameter Xgamma distribution. George and George (2023a) used Poisson-exponentiated Weibull distribution. Zeghdudi distribution and Marshall-Olkin Bilal distribution are considered respectively by AlSultan and Al-Omari (2023) and İrhad et al. (2024). Al-Omari and Ismail (2024) developed ASP using gamma Lindley distribution.

The rest of the article is organized as follows. Exponentiated Poisson-power Lindley distribution and its properties are studied in Section 2. In Section 3, the flexibility of the new distribution is explored using real data. Acceptance sampling plan and its real data application using survival time data is narrated in Section 4. Section 5 concludes the article.

2 Exponentiated Poisson-Power Lindley distribution

The cumulative distribution function and corresponding probability density function of the Poisson-power Lindley distribution are given as,

$$F(x) = \left(1-e^{-1}\right)^{-1}\left[1-e^{-\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^{m}}\right];x>0\ \alpha,\ \beta,\ m>0 \tag{1}$$

and

$$f(x) = \frac{m}{(1-e^{-1})}\left[\frac{\alpha\beta^2}{\beta+1}(1+x^{\alpha})x^{\alpha-1}e^{-\beta x^{\alpha}}\right]\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^{m-1}e^{-\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^{m}};x>0,\ \alpha,\ \beta,m>0. \tag{2}$$

Considering Poisson-power Lindley distribution as baseline distribution, we obtain the cumulative distribution function of exponentiated Poisson-power Lindley distribution as,

$$G(x;\alpha,\beta,m,\gamma) = \left(1-e^{-1}\right)^{-\gamma}\left[1-e^{-\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^{m}}\right]^{\gamma}, \quad x>0,\ \alpha,\ \beta,\ m,\ \gamma>0. \tag{3}$$

Corresponding probability density function is

$$g(x;\alpha,\beta,m,\gamma) = \frac{\gamma m}{(1-e^{-1})^{\gamma}}\left[1-e^{-\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^{m}}\right]^{(\gamma-1)}\left[\frac{\alpha\beta^2}{\beta+1}(1+x^{\alpha})x^{\alpha-1}e^{-\beta x^{\alpha}}\right]\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^{m-1}e^{-\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^{m}};x>0,\ \alpha,\ \beta,\ m,\ \gamma>0. \tag{4}$$

Figure 1 present the graphs of pdf of the EPPL distribution for varying parameter values.

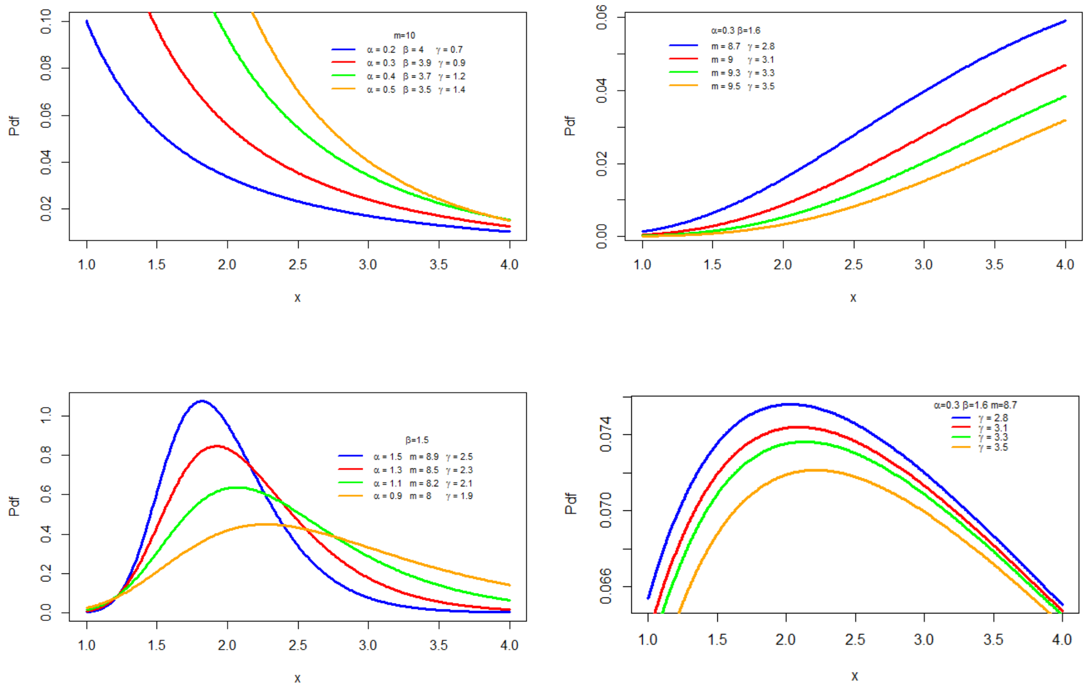

Figure 1: pdf plots of exponentiated Poisson-power Lindley distribution for different values of the parameters

A comparison of the pdf curves of EPPL distribution with PPL distribution is given in Figure 2.

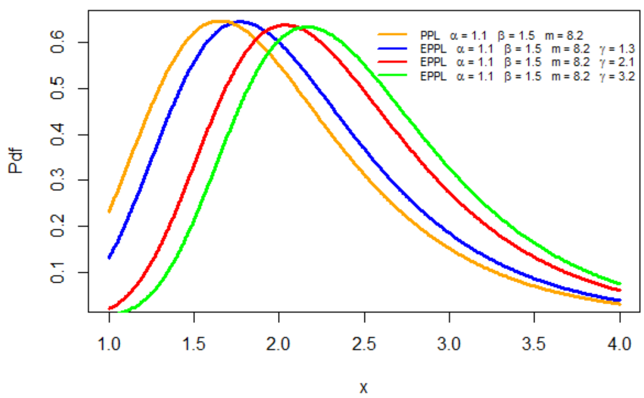

Figure 2: PPL vs EPPL

As the additional parameter γ increases, the EPPL distribution becomes more negatively skewed than the PPL distribution while keeping the other parameters constant ($\alpha = 1.1, \beta = 1.5, m = 8.2$). Therefore, the EPPL distribution can be considered as a good model for the negatively skewed data.

2.1 Properties of Exponentiated Poisson-Power Lindley Distribution

The survival function of EPPL distribution is given as,

$$\overline{Q(x)} = 1 - \left(1 - e^{-1}\right)^{-\gamma} \left[1 - e^{-[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}]^m}\right]^{\gamma}. \tag{5}$$

Hazard rate function and cumulative hazard rate function of the EPPL distribution are respectively,

$$h(x) = \frac{\dfrac{\gamma m\left[\frac{\alpha\beta^2}{\beta+1}(1+x^{\alpha})x^{\alpha-1}e^{-\beta x^{\alpha}}\right]e^{-\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^m}}{\left[1-e^{-\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^m}\right]^{-(\gamma-1)}\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^{-(m-1)}}}{(1-e^{-1})^{\gamma} - \left[1 - e^{-[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}]^m}\right]^{\gamma}} \tag{6}$$

and

$$H(x) = -log\left[1 - \left(1 - e^{-1}\right)^{-\gamma}\left[1 - e^{-[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}]^m}\right]^{\gamma}\right]. \tag{7}$$

For the EPPL distribution, the reversed hazard rate function is

$$n(x) = \frac{\dfrac{\gamma m\left[\frac{\alpha\beta^2}{\beta+1}(1+x^{\alpha})x^{\alpha-1}e^{-\beta x^{\alpha}}\right]e^{-\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^m}}{\left[1-e^{-\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^m}\right]^{-(\gamma-1)}\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^{-(m-1)}}}{\left[1 - e^{-[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}]^m}\right]^{\gamma}}. \tag{8}$$

Residual life time at time t and the survival function corresponding to it are given by,

$$r_{x_t}(x) = \frac{\dfrac{\gamma m\left[\frac{\alpha\beta^2}{\beta+1}(1+x^{\alpha})x^{\alpha-1}e^{-\beta x^{\alpha}}\right]e^{-\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^m}}{\left[1-e^{-\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^m}\right]^{-(\gamma-1)}\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^{-(m-1)}}}{(1-e^{-1})^{\gamma} - \left[1 - e^{-[1-(1+\frac{\beta}{\beta+1}t^{\alpha})e^{-\beta t^{\alpha}}]^m}\right]^{\gamma}} \tag{9}$$

and

$$R_{x_t}(x) = \frac{\left(1-e^{-1}\right)^{\gamma} - \left[1 - e^{-[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}]^m}\right]^{\gamma}}{(1-e^{-1})^{\gamma} - \left[1 - e^{-[1-(1+\frac{\beta}{\beta+1}t^{\alpha})e^{-\beta t^{\alpha}}]^m}\right]^{\gamma}}. \tag{10}$$

The past life time and corresponding distribution function of the EPPL distribution are respectively,

$$d_{t_x}(x) = \frac{\dfrac{\gamma c\left[\frac{\alpha\beta^2}{\beta+1}(1+x^\alpha)x^{\alpha-1}e^{-\beta x^\alpha}\right]e^{-\left[1-(1+\frac{\beta}{\beta+1}x^\alpha)e^{-\beta x^\alpha}\right]^m}}{\left[1-e^{-\left[1-(1+\frac{\beta}{\beta+1}x^\alpha)e^{-\beta x^\alpha}\right]^m}\right]^{-(\gamma-1)}\left[1-(1+\frac{\beta}{\beta+1}x^\alpha)e^{-\beta x^\alpha}\right]^{-(m-1)}}}{\left[1-e^{-[1-(1+\frac{\beta}{\beta+1}t^\alpha)e^{-\beta t^\alpha}]^m}\right]^\gamma} \tag{11}$$

and

$$D_{t_x}(x) = \frac{(1-e^{-1})^\gamma - \left[1-e^{-[1-(1+\frac{\beta}{\beta+1}x^\alpha)e^{-\beta x^\alpha}]^m}\right]^\gamma}{(1-e^{-1})^\gamma - \left[1-e^{-[1-(1+\frac{\beta}{\beta+1}t^\alpha)e^{-\beta t^\alpha}]^m}\right]^\gamma}. \tag{12}$$

The hrf plot of EPPL distribution for different parameter values are present in Figure 3

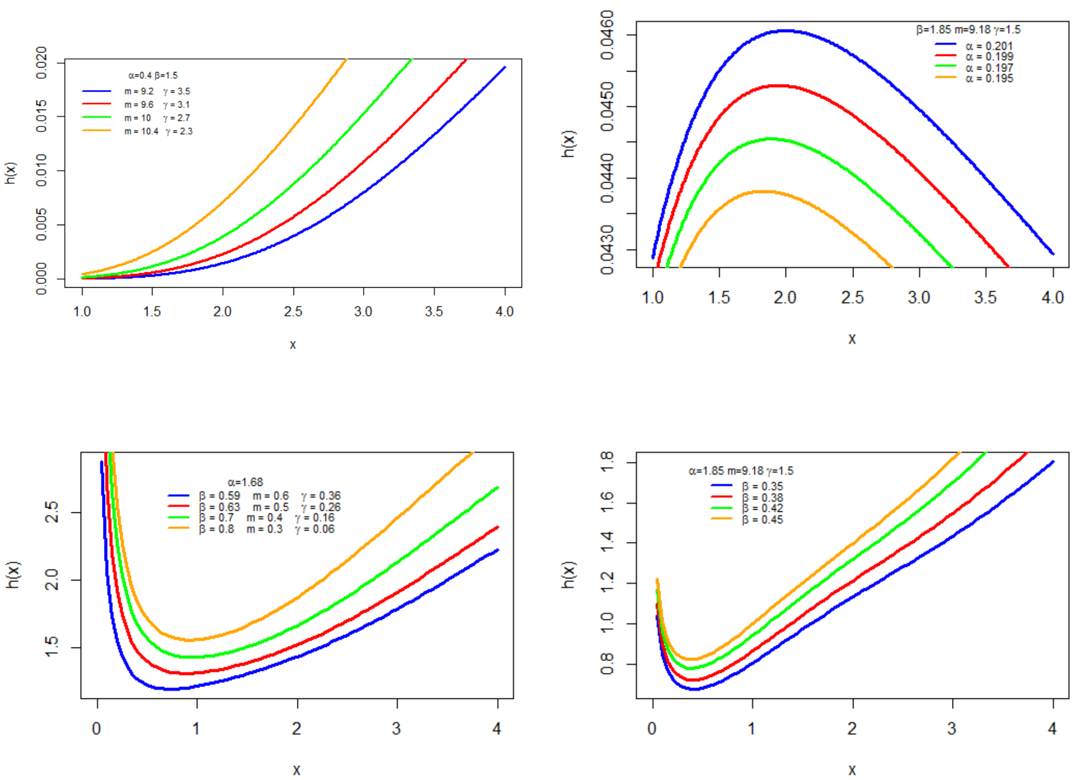

Figure 3: hrf plots of exponentiated Poisson-power Lindley distribution for different values of parameters

From the hrf graphs we can conclude that, For a fixed value of α and β with different m and γ value we obtain increasing failure rate. On decreasing the values of α and on fixing other parameter values we obtain reverse U shaped curves. The U shaped curves are found with constant α value in addition to varying remaining parameter values. If we keep the value of α, m, γ fixed, and allow the values of β to change, we obtain J shaped curves.

2.2 Lemma 1

The cumulative distribution function of the exponentiated Poisson-power Lindley distribution can be represented as the distribution of the failure time of a series system with independent units.

Proof

Let $X_1, X_2 ... X_\gamma$ be the failure times of independent units of a series system having Poisson-power Lindley distribution with cdf,

$$F(x) = \left(1 - e^{-1}\right)^{-1} \left[1 - e^{-\left[1-(1+\frac{\beta}{\beta+1}x^\alpha)e^{-\beta x^\alpha}\right]^m}\right], \quad x > 0, \alpha, \beta, m > 0. \tag{13}$$

Now, the probability that the system will fail before time x is given by,

$$\begin{aligned} P\left[max\left(X_1, X_2 ... X_\gamma\right) \leq x\right] &= P\left(X_1 \leq x\right) P\left(X_1 \leq x\right) ... P\left(X_1 \leq x\right) \\ &= \left[F(x)\right]^\gamma \\ &= \left(1 - e^{-1}\right)^{-\gamma} \left[1 - e^{-\left[1-(1+\frac{\beta}{\beta+1}x^\alpha)e^{-\beta x^\alpha}\right]^m}\right]^\gamma, \end{aligned} \tag{14}$$

which is the cdf of exponentiated Poisson-power Lindley distribution.

2.3 Linear Representation

Consider the cdf of EPPL distribution given in (3). Let $U(x) = 1 - (1 + \frac{\beta}{\beta+1}x^\alpha)e^{-\beta x^\alpha}$. On giving expansion to the exponential term of (3) and by simplification, the cdf of EPPL distribution can be re written as,

$$G(x) = \sum_{j=0}^{\infty} m(j, \gamma) \left[U(x)\right]^{(j+1)m\gamma} \tag{15}$$

where $m(j, \gamma) = \left[\frac{(-1)^j}{(j+1)!(1-e^{-1})}\right]^\gamma$ and $\left[U(x)\right]^{(j+1)m\gamma}$ is the cdf of exponentiated power Lindley distribution with power parameter $(j+1)m\gamma$. Thus, the exponentiated Poisson-power Lindley distribution is a linear form of exponentiated power Lindley distribution.

In addition to exponential expansion, if we consider the series expansion of $(1 - z)^{\tau-1}$ for $\mid z \mid< 1$ and $\tau > 0$, we obtain another form of the pdf (4) of EPPL distribution as

$$\begin{aligned} g(x) &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\left[\frac{(-1)^i}{(i+1)!}\right]^{\gamma-1}\left[\frac{(-1)^j}{j!}\right]\frac{m\gamma\alpha\beta^2}{(1+\beta)(1-e^{-1})^{\gamma}}x^{\alpha-1}(1+x^{\alpha})e^{-\beta x^{\alpha}} \\ & \left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^{mj+m[\gamma(i+1)-i]-1} \qquad (16) \\ &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\binom{mj+m(\gamma(i+1)-i)-1}{k}\left[\frac{(-1)^i}{(i+1)!}\right]^{\gamma-1}\left[\frac{(-1)^j}{j!}\right](-1)^k x^{\alpha-1} \\ & \frac{m\gamma\alpha\beta^2}{(1+\beta)^{k+1}(1-e^{-1})^{\gamma}}(1+x^{\alpha})e^{-(k\beta x^{\alpha}+\beta x^{\alpha})}\left[1+\beta(1+x^{\alpha})\right]^k \qquad (17) \\ &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{q=0}^{k}\binom{mj+m(\gamma(i+1)-i)-1}{k}\binom{k}{q}\left[\frac{(-1)^i}{(i+1)!}\right]^{\gamma-1}\left[\frac{(-1)^j}{j!}\right] \\ & \frac{m\gamma\alpha\beta^{2+q}}{(1+\beta)^{k+1}(1-e^{-1})^{\gamma}}(-1)^k x^{\alpha-1}e^{-(k\beta x^{\alpha}+\beta x^{\alpha})}(1+x^{\alpha})^{q+1} \qquad (18) \\ &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{q=0}^{k}\sum_{n=0}^{q+1}\binom{mj+m(\gamma(i+1)-i)-1}{k}\binom{k}{q}\binom{q+1}{n}\left[\frac{(-1)^i}{(i+1)!}\right]^{\gamma-1} \\ & \left[\frac{(-1)^j}{j!}\right]\frac{m\gamma\alpha\beta^{2+q}}{(1+\beta)^{k+1}(1-e^{-1})^{\gamma}}(-1)^k x^{\alpha n+\alpha-1}e^{-(k\beta x^{\alpha}+\beta x^{\alpha})}. \qquad (19) \end{aligned}$$

2.4 Moment

The r^{th} moment of EPPL distribution, from (19) is obtained as

$$\begin{aligned} E(X^r) &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{q=0}^{k}\sum_{n=0}^{q+1}\binom{mj+m(\gamma(i+1)-i)-1}{k}\binom{k}{q}\binom{q+1}{n}\left[\frac{(-1)^i}{(i+1)!}\right]^{\gamma-1} \\ & \left[\frac{(-1)^j}{j!}\right]\frac{m\gamma\beta^{2+q}(-1)^k}{(1+\beta)^{k+1}(1-e^{-1})^{\gamma}}\frac{\Gamma(n+r\alpha^{-1}+1)}{[\beta(k+1)]^{n+r\alpha^{-1}+1}}. \qquad (20) \end{aligned}$$

Proof:

$$E(X^r) = \int_0^{\infty} x^r g(x)dx. \qquad (21)$$

Using the expression for g(x) given in (19),

$$\begin{aligned} E(X^r) &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{q=0}^{k}\sum_{n=0}^{q+1}\binom{mj+m(\gamma(i+1)-i)-1}{k}\binom{k}{q}\binom{q+1}{n}\left[\frac{(-1)^i}{(i+1)!}\right]^{\gamma-1} \\ & \left[\frac{(-1)^j}{j!}\right]\frac{m\gamma\alpha\beta^{2+q}}{(1+\beta)^{k+1}(1-e^{-1})^{\gamma}}(-1)^k\int_0^{\infty}x^{\alpha n+\alpha+r-1}e^{-\beta x^{\alpha}(k+1)}dx. \qquad (22) \end{aligned}$$

Let $v = \beta x^{\alpha}(k+1)$, and on simplification,

$$\begin{aligned}
E(X^r) &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{q=0}^{k}\sum_{n=0}^{q+1}\binom{mj+m(\gamma(i+1)-i)-1}{k}\binom{k}{q}\binom{q+1}{n}\left[\frac{(-1)^i}{(i+1)!}\right]^{\gamma-1} \\
&\left[\frac{(-1)^j}{j!}\right]\frac{m\gamma\alpha\beta^{2+q}}{(1+\beta)^{k+1}(1-e^{-1})^{\gamma}}(-1)^k\int_0^{\infty}\frac{v^{(n+r\alpha^{-1})}e^{-v}}{\alpha[\beta(k+1)]^{n+r\alpha^{-1}+1}}dv \\
&= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{q=0}^{k}\sum_{n=0}^{q+1}\binom{mj+m(\gamma(i+1)-i)-1}{k}\binom{k}{q}\binom{q+1}{n}\left[\frac{(-1)^i}{(i+1)!}\right]^{\gamma-1} \\
&\left[\frac{(-1)^j}{j!}\right]\frac{m\gamma\beta^{2+q}}{(1+\beta)^{k+1}(1-e^{-1})^{\gamma}}(-1)^k\frac{\Gamma(n+r\alpha^{-1}+1)}{[\beta(k+1)]^{n+r\alpha^{-1}+1}}. \qquad (23)
\end{aligned}$$

2.5 Moment Generating Function

For the EPPL distribution, the moment generating function is

$$\begin{aligned}
M_X(t) &= \sum_{i=0}^{\infty}\sum_{j=0}^{\infty}\sum_{k=0}^{\infty}\sum_{q=0}^{k}\sum_{n=0}^{q+1}\sum_{p=0}^{\infty}\binom{mj+m(\gamma(i+1)-i)-1}{k}\binom{k}{q}\binom{q+1}{n}\left[\frac{(-1)^i}{(i+1)!}\right]^{\gamma-1} \\
&\left[\frac{(-1)^j}{j!}\right]\frac{m\gamma\alpha\beta^{2+q}(-1)^k}{(1+\beta)^{k+1}(1-e^{-1})^{\gamma}}\frac{t^p}{p!}\frac{\Gamma(n+p\alpha^{-1}+1)}{[\beta(k+1)]^{n+p\alpha^{-1}+1}}. \qquad (24)
\end{aligned}$$

2.6 Quantile Function

The quantile function of EPPL distribution is given by

$$X = \left[-1-\frac{1}{\beta}-\frac{1}{\beta}W_{-1}\left(-\frac{\beta+1}{e^{\beta+1}}\left[1-\left\{-log\left(1-u^{\frac{1}{\gamma}}\left(1-e^{-1}\right)\right)\right\}^{\frac{1}{m}}\right]\right)\right]^{\frac{1}{\alpha}}. \qquad (25)$$

where $W_{-1}(.)$ denotes the negative branch of the Lambert W function.

2.7 Order Statistics

For the EPPL distribution, the pdf of r^{th} order statistics, $X_{(r)}$ is given as

$$\begin{aligned}
g_r(x) &= \frac{r!}{(i-1)!\,(r-i)!}\sum_{j=0}^{r-i}(-1)^j\binom{r-i}{j}\left(1-e^{-1}\right)^{-\gamma(i+j)} \\
&\left[1-e^{-\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^m}\right]^{\gamma(i+j)-1} m\gamma\left[\frac{\alpha\beta^2}{\beta+1}(1+x^{\alpha})x^{\alpha-1}e^{-\beta x^{\alpha}}\right] \\
&\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^{m-1} e^{-\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^m}. \qquad (26)
\end{aligned}$$

Now, the cdf for the largest order statistic $X_{(n)}$ and smallest order statistic $X_{(1)}$ are respectively,

$$G_n(x) = \left(1-e^{-1}\right)^{-n\gamma}\left[1-e^{-\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^m}\right]^{n\gamma} \tag{27}$$

and

$$G_1(x) = 1-\left[1-\left(1-e^{-1}\right)^{-\gamma}\left[1-e^{-[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}]^m}\right]^{\gamma}\right]^n \tag{28}$$

2.8 Parameter Estimation

We estimate the parameters using method of maximum likelihood. Let X1, X2,...,Xn be independent and identically distributed exponentiated Poisson-power Lindley random variables, then the log likelihood function is given by,

$$\begin{aligned} logL(\alpha,\beta,m,\gamma;x) = {} & nlog\left(\frac{m}{(1-e^{-1})^{\gamma}}\right)+nlog\left(\frac{\alpha\beta^2}{\beta+1}\right)+\sum_{i=1}^{n}log(1+x_i^{\alpha})+ \\ & (\alpha-1)\sum_{i=1}^{n}log(x_i)-\beta\sum_{i=1}^{n}x_i^{\alpha}-\sum_{i=1}^{n}\left[1-\left(1+\frac{\beta x_i^{\alpha}}{\beta+1}\right)e^{-\beta x_i^{\alpha}}\right]+ \\ & (m-1)\sum_{i=1}^{n}log\left[1-\left(1+\frac{\beta x_i^{\alpha}}{\beta+1}\right)e^{-\beta x_i^{\alpha}}\right]+(\gamma-1) \\ & \sum_{i=1}^{n}log\left[1-e^{-\left[1-(1+\frac{\beta}{\beta+1}x^{\alpha})e^{-\beta x^{\alpha}}\right]^m}\right]. \end{aligned} \tag{29}$$

On solving $\frac{\partial logL}{\partial\alpha}=0$, $\frac{\partial logL}{\partial\beta}=0$, $\frac{\partial logL}{\partial m}=0$, $\frac{\partial logL}{\partial\gamma}=0$ we get the maximum likelihood estimators of α,β,m,γ respectively. Since we cannot solve it analytically, we use the nlm package of R software for the further computations.

2.9 Simulation

To check the performance of the maximum likelihood estimators of the EPPL distribution, Monte Carlo simulation study is conducted. Here we consider two sets of parameter values, Set I: $\alpha=0.152,\beta=2.6,m=0.85,\gamma=1.7$ and Set II: $\alpha=1.2,\beta=0.68,m=1.4,\gamma=0.95$. Using the quantile function of EPPL distribution we simulate data for different sample sizes n=20, 30, 50 and 100 and obtain the maximum likelihood estimate (MLE) of the parameters for a repeated times of 10000. Table 1 provide the results.

Table 1: Simulation result

	Set I			Set II		
n	MLE	Bias	MSE	MLE	Bias	MSE
	0.3754	0.2234	0.1272	1.3121	0.1121	0.1251
	2.3971	-0.2028	0.1547	0.7528	0.0728	0.0840
20	0.9607	0.1107	0.0949	1.5499	0.1499	0.0915
	1.9138	0.2138	0.1017	0.7992	-0.1507	0.0898
	0.3546	0.2026	0.0661	1.2835	0.0835	0.0458
	2.4374	-0.1625	0.0993	0.7451	0.0651	0.0295
30	0.8910	0.0410	0.0250	1.5081	0.1081	0.0283
	1.8544	0.1544	0.0409	0.8348	-0.1151	0.0227
	0.2132	0.0612	0.0075	1.2402	0.0402	0.0062
	2.5282	-0.0717	0.0196	0.6977	0.0177	0.0051
50	0.8685	0.0185	0.0053	1.4685	0.0685	0.0074
	1.7688	0.0688	0.0068	0.9118	-0.0381	0.0028
	0.1546	0.0026	0.0003	1.2077	0.0077	0.0001
	2.5924	-0.0075	0.0007	0.6864	0.0064	0.0001
100	0.8535	0.0035	0.0002	1.4154	0.0154	0.0002
	1.7149	0.0149	0.0003	0.9467	-0.0032	0.0001

Above study shows that, on increasing the sample size, there is notable decrease in bias and the MSE which indicates the exactness of the maximum likelihood estimates of EPPL distribution.

3 Application

Here, the flexibility of the EPPL distribution is illustrated using the survival time data of gastric patients who taken chemotherapy and radiation given by Bekker et al. (2000) 0.047, 0.115, 0.121, 0.132, 0.164, 0.197, 0.203, 0.260, 0.282, 0.296, 0.334, 0.395, 0.458, 0.466, 0.501, 0.507, 0.529, 0.534, 0.540, 0.641, 0.644, 0.696, 0.841, 0.863, 1.099, 1.219, 1.271, 1.326, 1.447, 1.485, 1.553, 1.581, 1.589, 2.178, 2.343, 2.416, 2.444, 2.825, 2.830, 3.578, 3.658, 3.743, 3.9781.341 , 4.003, 4.033.
The descriptive statistics of the given data are; minimum=0.047, first quartile=0.395, median=0.841, mean=1.341, third quartile=2.178, maximum=4.033.

To find the MLEs of the parameters, we use the nlm function of R software. The significant statistics are calculated for establishing the goodness of fit of the distribution. For comparison study, histogram of the data with embedded pdf plots of exponentiated Poisson-power Lindley (EPPL) distribution along with Poisson-power Lindley (PPL) (George and George (2023b)), exponentiated power Lindley Poisson (EPLP) (Pararai et al. (2017)) and exponentiated generalized power Lindley (EG-PL) (MirMostafaee et al. (2019)) distributions are depicted in Figure 4. It is seen that the EPPL distribution yields a better fit for the considered data than the compared models.

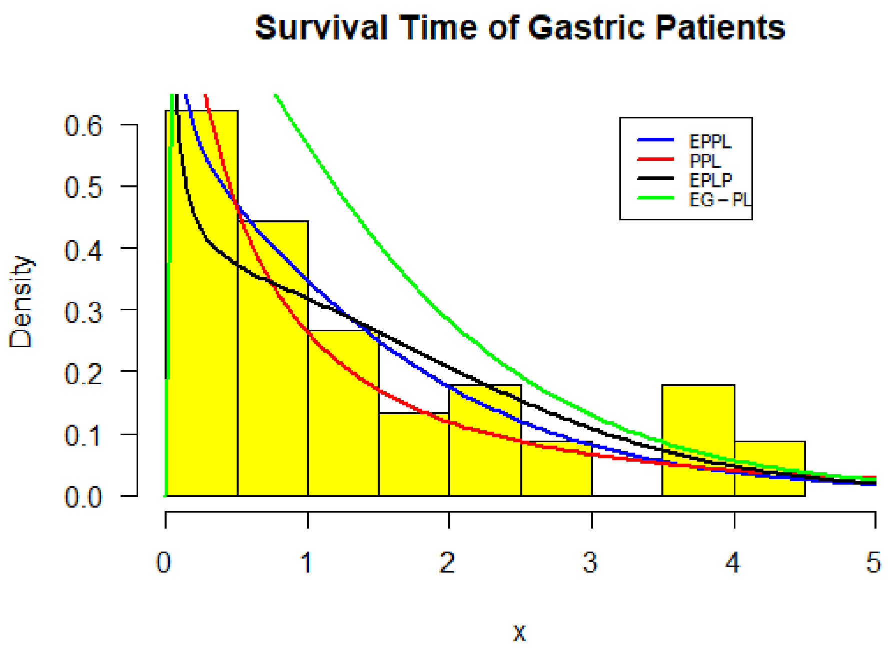

Figure 4: Fitted pdf plot for gastric patients data

Now, from Table 2 we can see that, EPPL distribution has the smallest value of -logL, Akaike information criterion (AIC), Bayesian information criterion (BIC), Kolmogorov-Smirov (K-S) statistic, and highest p-value as compared to Poisson-power Lindley, exponentiated power Lindley Poisson and exponentiated generalized power Lindley distributions. Therefore EPPL distribution seems to be a better model for the given survival time data.

Table 2: MLE, -logL, AIC, BIC, K-S and p-value of the fitted models

Distribution	Parameters	-logl	AIC	BIC	K-S	p-value
EPPL	$\alpha = 1.1308$ $\beta = 0.682$ $m = 1.784$ $\gamma = 0.3612$	183.6929	375.3858	382.6124	0.085228	0.8718
PPL	$\alpha = 0.548$ $\beta = 1.359$ $m = 1.581$	207.1517	420.3034	425.7234	0.1193	0.5059
EPLP	$\alpha = 1.22$ $\beta = 0.725$ $\omega = 0.228$ $\theta = 3.12$	192.2496	392.4991	399.7258	0.13093	0.3896
EG-PL	$\lambda = 4.503$ $\beta = 1.55$ $a = 0.177$ $b = 1.05$	254.6221	517.2442	524.4709	0.13134	0.3858

4 Acceptance Sampling Plan

An acceptance sampling plan, makes the selection procedure of a lot of product more reliable and easier. Let the lot have N products. If N is a large quantity, it is not convenint to inspect each unit, as it make the test procedure so hard in terms of time and money. In a time truncated life test, we consider a sample of size n and allow the test to terminate at a prefixed time t. If $(a+1)^{th}$ failure happened before or at t for a prefixed acceptance number a, the lot is decided as rejected. If the number of failures is not more than a, at t, the lot is termed as acceptable. The life time of the product have EPPL distribution with mean life time τ is given by

$$G(x;\alpha,\beta,m,\gamma,\tau) = \left(1-e^{-1}\right)^{-\gamma}\left[1-e^{-\left[1-(1+\frac{\beta}{\beta+1}\left(\frac{x}{\tau}\right)^{\alpha})e^{-\beta\left(\frac{x}{\tau}\right)^{\alpha}}\right]^{m}}\right]^{\gamma} \tag{30}$$
$$x>0,\alpha,\beta,m,\gamma,\tau>0$$

If τ_0 be the specified average life time, then $G(\alpha,\beta,m,\gamma,\tau) \leq G(\alpha,\beta,m,\gamma,\tau_0) \Leftrightarrow \tau \geq \tau_0$. As the selection of the lot is based on the sample units, there may arise risks. Probability of selecting a bad lot by the consumer is the consumer's risk and producer's risk is the probability associated with rejecting a good lot. Thus, the life testing based on EPPL distribution is characterized by $\left(n, a, \frac{t}{\tau_0}\right)$. We have to select the smallest value of n for which (31) holds. As N is large enough, we consider the binomial distribution.

$$L\left(p_0\right) = \sum_{i=0}^{b} \binom{n}{i} p_0^i \left(1-p_0\right)^{n-i} \leq 1-p^*. \tag{31}$$

By choosing $\alpha = 1, \beta = 2, m = 2$ and $\gamma = 0.98$ with p^* set to 0.75, 0.90, 0.95 and 0.99 and $\frac{t}{\tau_0}$ assigned the values 0.628, 0.942, 1.257, 1.571, 2.356, 3.141, 3.927 and 4.712, the obtained values of n is illustrated in Table 3. If we make a comparison, it can be seen that these obtained n values is less than the n values obtained by two parameter Quasi Lindley distribution by Al-Omari and Al-Nasser (2019) and three parameter Lindley distribution by Al-Omari et al. (2019) under same p^* and $\frac{t}{\zeta_0}$. Again, for the same sampling scenario, the n values are almost same or less than the n values obtained by Poisson-power Lindley distribution by George and George (2023b). The Poisson approach, given in (32) is opted instead of binomial, if p_0 is small with large n. Here $\lambda = np_0$.

$$L\left(p_0\right) = \sum_{i=0}^{b} \frac{\lambda^i}{i!} e^{-\lambda} \leq 1-p^*. \tag{32}$$

The values of operating characteristic function of the sampling plan $(n, a, \frac{t}{\tau_0})$ with $p = G\left(\frac{t}{\tau_0} / \frac{\tau}{\tau_0}\right)$ is given in (33) and Table 4 yields the corresponding results. From the table it is seen that, the OC values increases as $\frac{\tau}{\tau_0}$ increases which revels that, the consumer's risk is decreases on the increase of $\frac{\tau}{\tau_0}$.

$$L(p) = \sum_{i=0}^{a} \binom{n}{i} p^i \left(1-p\right)^{n-i}. \tag{33}$$

$$\sum_{i=0}^{b} \binom{n}{i} p_0^i \left(1-p_0\right)^{n-i} \geq 0.95 \tag{34}$$

Table 5 gives the minimum ratio of $\frac{\tau}{\tau_0}$ which holds (34). That is, we find the values of $\frac{\tau}{\tau_0}$ which promises the producer's risk (α) less than or equal to 0.05. For example, when $a = 3$, $\frac{t}{\tau_0} = 4.712$, $p^* = 0.75$ then $\frac{\tau}{\tau_0} = 2.3144$ (Table 5). That means, being satisfied with (34), the average life of the product is 2.3144 times the specified.

Table 3: Minimum sample sizes using binomial probabilities

		t/τ_0							
p^*	a	0.628	0.942	1.257	1.571	2.356	3.141	3.927	4.712
	0	3	2	1	1	1	1	1	1
	1	5	3	3	2	2	2	2	2
	2	8	5	4	4	3	3	3	3
	3	10	7	5	5	4	4	4	4
	4	12	8	7	6	5	5	5	5
0.75	5	15	10	8	7	6	6	6	6
	6	17	11	9	8	7	7	7	7
	7	19	13	11	9	8	8	8	8
	8	22	15	12	11	9	9	9	9
	9	24	16	13	12	10	10	10	10
	10	26	18	14	13	12	11	11	11
	0	4	2	2	2	1	1	1	1
	1	7	4	3	3	2	2	2	2
	2	10	6	5	4	3	3	3	3
	3	12	8	6	5	5	4	4	4
	4	15	10	8	7	6	5	5	5
0.90	5	17	11	9	8	7	6	6	6
	6	20	13	10	9	8	7	7	7
	7	22	15	12	10	9	8	8	8
	8	25	16	13	12	10	9	9	9
	9	27	18	14	13	11	10	10	10
	10	30	20	16	14	12	11	11	11
	0	5	3	2	2	1	1	1	1
	1	8	5	4	3	3	2	2	2
	2	11	7	5	5	4	3	3	3
	3	14	9	7	6	5	4	4	4
	4	17	11	8	7	6	5	5	5
0.95	5	19	12	10	8	7	6	6	6
	6	22	14	11	10	8	7	7	7
	7	24	16	13	11	9	9	8	8
	8	27	17	14	12	10	10	9	9
	9	29	19	15	13	11	11	10	10
	10	32	21	17	15	12	12	11	11
	0	8	4	3	3	2	1	1	1
	1	11	7	5	4	3	2	2	2
	2	14	9	7	5	4	3	3	3
	3	18	11	8	7	5	4	4	4
	4	21	13	10	8	6	5	5	5
0.99	5	24	15	11	9	8	6	6	6
	6	26	16	13	11	9	7	7	7
	7	29	18	14	12	10	9	8	8
	8	32	20	16	13	11	10	9	9
	9	34	22	17	15	12	11	10	10
	10	37	23	18	16	13	12	11	11

Table 4: OC values for the plan $(n, a, t/\tau_0)$

$p^\star$	n	a	$\frac{t}{\tau_0}$	τ/τ_0					
				2	4	6	8	10	12
	8	2	0.628	0.8342	0.9938	0.9989	0.9998	0.9999	0.9999
	5	2	0.942	0.7897	0.9887	0.9989	0.9997	0.9999	0.9999
	4	2	1.257	0.7983	0.9874	0.9986	0.9995	0.9998	0.9999
	4	2	1.571	0.5856	0.9657	0.9932	0.9979	0.9995	0.9997
	3	2	2.356	0.5495	0.9585	0.9856	0.9971	0.9985	0.9996
0.75	3	2	3.141	0.2895	0.7985	0.9478	0.9845	0.9957	0.9978
	3	2	3.927	0.1558	0.6532	0.8798	0.95699	0.9853	0.9941
	3	2	4.712	0.0991	0.5398	0.7969	0.9175	0.9683	0.9842
	10	2	0.628	0.7253	0.9849	0.9981	0.9997	0.9998	0.9999
	6	2	0.942	0.6862	0.9768	0.9978	0.9994	0.9998	0.9999
	5	2	1.257	0.5591	0.9584	0.9951	0.9986	0.9996	0.9999
	4	2	1.571	0.5856	0.9657	0.9932	0.9979	0.9995	0.9997
	3	2	2.356	0.5495	0.9585	0.9856	0.9971	0.9985	0.9996
0.90	3	2	3.141	0.2895	0.7985	0.9478	0.9845	0.9957	0.9978
	3	2	3.927	0.1558	0.6532	0.8798	0.95699	0.9853	0.9941
	3	2	4.712	0.0991	0.5398	0.7969	0.9175	0.9683	0.9842
	11	2	0.628	0.6656	0.9795	0.9975	0.9995	0.9998	0.9999
	7	2	0.942	0.5689	0.9659	0.9963	0.9989	0.9997	0.9998
	5	2	1.257	0.5591	0.9584	0.9951	0.9986	0.9996	0.9999
	5	2	1.571	0.3375	0.8863	0.9790	0.9964	0.9985	0.9995
	4	2	2.356	0.1998	0.7761	0.9457	0.9864	0.9958	0.9998
0.95	3	2	3.141	0.2895	0.7985	0.9478	0.9845	0.9957	0.9978
	3	2	3.927	0.1558	0.6532	0.8798	0.95699	0.9853	0.9941
	3	2	4.712	0.0991	0.5398	0.7969	0.9175	0.9683	0.9842
	14	2	0.628	0.4498	0.9489	0.9946	0.9988	0.9998	0.9999
	9	2	0.942	0.3758	0.9281	0.9885	0.9982	0.9994	0.9997
	7	2	1.257	0.2758	0.8765	0.9782	0.9962	0.9989	0.9996
	5	2	1.571	0.3375	0.8863	0.9790	0.9964	0.9985	0.9995
	4	2	2.356	0.1998	0.7761	0.9457	0.9864	0.9958	0.9998
0.99	3	2	3.141	0.2895	0.7985	0.9478	0.9845	0.9957	0.9978
	3	2	3.927	0.1558	0.6532	0.8798	0.95699	0.9853	0.9941
	3	2	4.712	0.0991	0.5398	0.7969	0.9175	0.9683	0.9842

Table 5: Minimum ratio of true and specified mean life for the acceptability of a lot with $\alpha = 0.05$

		t/τ_0							
p^*	a	0.628	0.942	01.257	1.571	2.356	3.141	3.927	4.712
	0	0.31411	0.46989	0.6293	0.7865	1.1795	1.5724	1.9659	2.3589
	1	0.3113	0.4709	0.6284	0.7832	1.1746	1.5659	1.9578	2.3492
	2	0.3100	0.4672	0.6220	0.7774	1.1701	1.5600	1.9504	2.3403
	3	0.3085	0.4584	0.6151	0.7687	1.1572	1.5427	1.9288	2.3144
	4	0.2782	0.4598	0.6176	0.7717	1.1543	1.5389	1.9241	2.3087
0.75	5	0.3046	0.4603	0.6175	0.7709	1.1500	1.5332	1.9169	2.300
	6	0.3005	0.4559	0.6105	0.7576	1.1377	1.5167	1.8963	2.2754
	7	0.2987	0.4547	0.6029	0.7706	1.1314	1.5083	1.886	2.2627
	8	0.2977	0.4546	0.6022	0.7511	1.1229	1.4971	1.8717	2.2459
	9	0.2960	0.4508	0.5973	0.7498	1.1178	1.4903	1.8633	2.2358
	10	0.2959	0.4494	0.5963	0.7484	1.1132	1.4881	1.8605	2.2324
	0	0.3144	0.4703	0.6276	0.7844	1.1795	1.5724	1.9659	2.3589
	1	0.3135	0.4700	0.6288	0.7859	1.1746	1.5659	1.9578	2.3492
	2	0.3111	0.4666	0.6235	0.7799	1.1701	1.5600	1.9504	2.3403
	3	0.30850	0.4649	0.6216	0.7756	1.1632	1.5427	1.9288	2.3144
	4	0.3081	0.4611	0.6161	0.7719	1.1573	1.5389	1.9241	2.3087
0.90	5	0.3022	0.4598	0.6122	0.7663	1.1453	1.5332	1.9169	2.300
	6	0.3042	0.4571	0.6087	0.7597	1.1399	1.5167	1.8963	2.2754
	7	0.2817	0.4545	0.6055	0.7545	1.1319	1.5083	1.886	2.2627
	8	0.3012	0.4513	0.6012	0.7526	1.1296	1.4971	1.8717	2.2459
	9	0.2982	0.4473	0.5964	0.7465	1.1193	1.4903	1.8633	2.2358
	10	0.2965	0.4454	0.5951	0.7421	1.1132	1.4881	1.8605	2.2324
	0	0.3152	0.4719	0.6298	0.7872	1.1795	1.5724	1.9659	2.3589
	1	0.3148	0.4701	0.6292	0.7863	1.1746	1.5659	1.9578	2.3492
	2	0.3146	0.4689	0.6286	0.7856	1.1772	1.5600	1.9504	2.3403
	3	0.3143	0.4686	0.6284	0.7826	1.1738	1.5427	1.9288	2.3144
	4	0.3123	0.4669	0.6248	0.7788	1.1679	1.5389	1.9241	2.3087
0.95	5	0.3106	0.4665	0.6223	0.7786	1.1672	1.5332	1.9169	2.300
	6	0.3106	0.4650	0.6219	0.7759	1.1627	1.5167	1.8963	2.2754
	7	0.3103	0.4647	0.6211	0.7750	1.1596	1.5460	1.886	2.2627
	8	0.3097	0.4158	0.6164	0.7683	1.1545	1.5391	1.8717	2.2459
	9	0.3077	0.4618	0.6158	0.7665	1.1495	1.5326	1.8633	2.2358
	10	0.3070	0.4600	0.6145	0.7663	1.1132	1.5325	1.8605	2.2324
	0	0.3158	0.4743	0.6328	0.7909	1.1818	1.5724	1.9659	2.3589
	1	0.3153	0.4722	0.6317	0.7903	1.1795	1.5725	1.9578	2.349
	2	0.3147	0.4717	0.6302	0.6783	1.1794	1.5724	1.9504	2.3403
	3	0.3126	0.4716	0.6298	0.7854	1.1763	1.5682	1.9288	2.3144
	4	0.3126	0.4714	0.6278	0.7825	1.1733	1.5643	1.92411	2.3087
0.99	5	0.3124	0.4689	0.6255	0.7774	1.1698	1.5032	1.9169	2.3002
	6	0.3119	0.4667	0.6228	0.7803	1.1694	1.5552	1.9444	2.2754
	7	0.3118	0.4658	0.6220	0.7801	1.1658	1.5568	1.9464	2.2627
	8	0.3104	0.4636	0.6198	0.7731	1.1616	1.5504	1.9384	2.2459
	9	0.2929	0.4629	0.6194	0.7715	1.1611	1.5415	1.9272	2.2358
	10	0.3079	0.4682	0.6254	0.7709	1.1554	1.5325	1.9238	2.2324

4.1 Real Data Application

In this section, we consider a real data set to demonstrate the application of acceptance sampling plan. The data which represents the survival time of head and neck cancer patients after radiation and chemotherapy, studied by Sule et al. (2020) is considered and it is given below. 12.20, 23.56, 23.74, 25.87, 31.98, 37, 41.35, 47.38, 55.46, 58.36, 63.47, 68.46, 74.47, 78.26, 81.43, 84, 92, 94, 110, 112, 119, 127, 130, 133, 140, 146, 155, 159, 173, 179, 194, 195, 209, 249, 281, 319, 339, 432, 469, 519, 633, 725, 817, 1776.
The descriptive statistics of the given data are; minimum=12.2, first quartile=67.21, median=128.5, mean=223.48, third quartile=219, maximum=1776.

The ML estimates of the parameters of the EPPL distribution obtained from the above data are $\alpha = 1.148$, $\beta = 0.002$, $m = 0.442$ and $\gamma = 0.98$. The corresponding p-value and K-S statistics are respectively 0.5201 and 0.11927, which ensures the fitness of EPPL distribution to the given data. Figure 5 shows the embedded pdf curve of the data set.

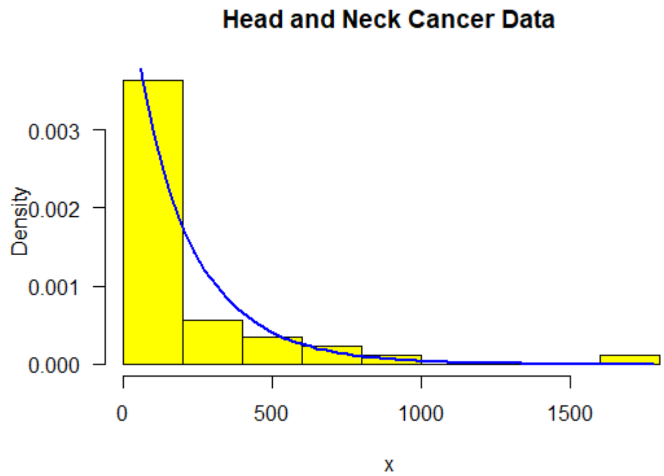

Figure 5: Fitted pdf plot for the survival time of head and neck cancer patients data

Here, for the data the mean survival time τ_0 is 223 and in order to develop ASP, we consider three truncation periods 875, 1050 and 1600 so that the respective values obtained for $\frac{t}{\tau_0}$ are 3.924, 4.71, 7.175.
Using the obtained ML estimates of EPPL distribution for the given data, we construct an ASP for $p^* = 0.75$ and Table 6 gives the minimum values of n satisfies (31).

Table 6: Minimum sample sizes using Binomial probabilities

		t/τ_0		
p^*	a	3.924	4.71	7.175
	0	43	37	26
	1	83	71	51
	2	122	105	74
	3	158	137	97
	4	194	168	119
0.75	5	230	199	141
	6	265	229	163
	7	300	259	184
	8	334	290	206
	9	369	319	228
	10	402	349	249

The inference obtained from the table is that the selected sampling plan is $\left(n = 37, a = 0, \frac{t}{\tau_0} = 0.4.71\right)$ and the treatment is effective if among the first 37 observations no failures occur, which means all the 37 survive in an opted truncation period of 1050. Since for the selected ASP, there is only 1 survival within the truncation period, we cannot support the treatment as an effective one. In order to support the performance of Table 6, we find the OC values and is provided in Table 7.

Table 7: OC values for the plan $(n, a, t/\tau_0)$

$p^\star$	n	a	$\frac{t}{\tau_0}$	τ/τ_0					
				2	4	6	8	10	12
	43	0	3.924	0.4319	0.5873	0.6579	0.7000	0.7288	0.7501
	37	0	4.71	0.4392	0.5986	0.6700	0.7122	0.7407	0.7617
	24	0	7.175	0.4519	0.6208	0.6947	0.7373	0.7656	0.7861
	122	2	3.924	0.5797	0.8095	0.8842	0.9190	0.9387	0.9512
	105	2	4.71	0.5927	0.8232	0.8952	0.9279	0.9461	0.9574
	74	2	7.175	0.6148	0.8481	0.9155	0.9443	0.9595	0.9687

Thus here we try to shows the adaptability of the constructed sampling plan based on EPPL distribtion in providing information regarding the effectiveness of the treatment in the survival time of the patients.

5 Conclusion

Exponentiated Poisson-power Lindley distribution is introduced here by generalizing the Poisson-power Lindley distribution and discussed its properties. The accuracy of the maximum likelihood estimators are studied by simulation technique. A survival time data of gastric patients, is taken to compare the performance of the EPPL distribution with some other well related distributions like Poisson-power Lindley, exponentiated power Lindley Poisson and exponentiated generalized power Lindley distributions. For the data, EPPL distribution seems to be a better model than the others. A time truncated sampling scheme is also developed and its application using the survival time data set following the newly proposed distribution is also illustrated.

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