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New Alternative Methods For Estimation Of Asymmetric Stochastic Volatility Model

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Two methods, the Quasi-likelihood (QL) and Asymptotic Quasi-likelihood (AQL) for finding a point estimate of unknown parameters in asymmetric stochastic volatility (ASV) model with leverage effect are proposed. The QL estimation (QLE) developing if probability distribution of (ASV) model is not available. The AQL estimation (AQLE) building on QLE technique and is obtained where variance and covariance are not available. The AQL estimation substitutes the variance and covariance by kernel estimator in QL. Application of the QLE and AQLE to analyze several data sets modeled by ASV model are considered.

keywords: Asymmetric Stochastic Volatility (ASV) Model; Quasi likelihood Estimation (QLE); Asymptotic Quasi likelihood Estimation (AQLE); Kernel estimation.

1 Introduction

The asymmetric stochastic volatility y_t is

$$y_t = e^{\frac{h_t}{2}} \upsilon_t, \qquad t = 1, 2, 3, 4, \cdots, T. \tag{1}$$

$$h_t = \mu + \phi(h_{t-1} - \mu) + \psi y_{t-1} e^{\frac{-h_{t-1}}{2}} + \tau \nu_t, \qquad t = 1, 2, 3, 4, \cdots, T. \tag{2}$$

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Where υ_t are independent and identically distributed (i.i.d). $E(\upsilon_t) = 0$ and $Var(\upsilon_t) = \sigma_\upsilon^2$, and ν_t are i.i.d. $E(\nu_t) = 0$ and $Var(\nu_t) = \sigma_\nu^2$.

In the stochastic volatility literature, the asymmetric volatility response is often studied by specifying a negative correlation between the return innovation and the volatility innovation. This classical leverage stochastic volatility was first estimated by (Harvey and Shepard, 1993). The model specification requires the correlation coefficient between the two innovations remains constant, regardless of price movements. On the other hand, (Daouk, 2007) reported evidence of strong leverage effect in down markets than in up markets. Obviously, this empirical result cannot be explained by the classical leverage stochastic volatility model with a constant leverage effect. (Asai and McAleer, 2011) defined asymmetry as the "differential impacts of positive and negative shocks on the volatility". A specific type of asymmetry is the leverage effect, which is typically modelled as a negative correlation between the innovations of the returns and volatilities. In the literature, for the application and estimation of SV and ASV models (see (Jacquire et al., 1994); (Breidt and Carriquiry, 1996); (Pitt and Shepard, 1999); (Men et al., 2017); (Men and Wirjanto, 2018); (Chan and Grant, 2015); and (Pinho et al., 2016)), (A. and Rodriguez-Yam, 2005) suggested estimation method assuming the probability distribution is known. (Sandmann and Koopman, 1998) proposed the maximum likelihood estimation. (Alzghool, 2017a) obtained the QL and AQL estimation for the classical SV model.

The purpose of this article is to extend (Alzghool, 2017a) estimation procedure by considering the estimation of unknown parameter of asymmetric stochastic volatility (ASV) model with leverage effect by the Quasi-likelihood (QL) and Asymptotic Quasi-likelihood (AQL) approaches. Distribution assumptions of ASV processes are not required by QL method. But, The QL technique assume knowing the first two moments of the process. However, The AQL estimation procedure is suggested when the variance covariance matrix of process is unknown. The AQL estimation substitutes the variance and covariance matrix by kernel estimation in QL. In Section 2, we describe the QL and AQL approaches. Section 3 explain our suggested QL and AQL estimation methods to get point estimation for the unknown parameters of ASV model, and provide simulation studies. In Section 4, we apply our estimation methods to several data sets are modeled by ASV model. Concluding remarks and summary are drawn in the last section.

2 The QLE and AQLE methods

Let the observation equation given by

$$\mathbf{y}_t = \mathbf{f}_t(\theta) + \zeta_t, \qquad t = 1, 2, 3 \cdots, T, \tag{3}$$

ζ_t is a sequence of martingale difference with respect to $\mathcal{F}_t$, $\mathcal{F}_t$ denotes the σ-field generated by $\mathbf{y}_t, \mathbf{y}_{t-1}, \cdots, \mathbf{y}_1$ for $t \geq 1$; that is, $E(\zeta_t|\mathcal{F}_{t-1})= E_{t-1}(\zeta_t) = 0$; where $\mathbf{f}_t(\theta)$ is an $\mathcal{F}_{t-1}$ measurable; and θ is parameter vector, which belongs to an open subset $\Theta \in R^d$. Note that θ is a parameter of interest.

2.1 The QL method

For the model given by (3), Assume that $E_{t-1}(\zeta_t \zeta_t{}') = \mathbf{\Sigma}_t$ is known. Now, the linear class $\mathcal{G}_T$ of the estimating function (EF) can be defined by

$$\mathcal{G}_T = \{\sum_{t=1}^{T} \mathbf{W}_t(\mathbf{y}_t - \mathbf{f}_t(\theta))\}$$

and the quasi-likelihood estimation function (QLEF) can be defined by

$$\mathbf{G}_T^*(\theta) = \sum_{t=1}^{T} \dot{\mathbf{f}}_t(\theta)\mathbf{\Sigma}_t^{-1}(\mathbf{y}_t - \mathbf{f}_t(\theta)) \tag{4}$$

where $\mathbf{W}_t$ is $\mathcal{F}_{t-1}$-measureable and $\dot{\mathbf{f}}_t(\theta) = \partial \mathbf{f}_t(\theta)/\partial\theta$. Then, the estimation of θ by the QL method is the solution of the QL equation $\mathbf{G}_T^*(\theta) = 0$ (see (Hedye, 1997)). If the sub-estimating function spaces of $\mathcal{G}_T$ are considered as follows,

$$\mathcal{G}_t = \{\mathbf{W}_t(\mathbf{y}_t - \mathbf{f}_t(\theta))\}$$

then the QLEF can be defined by

$$\mathbf{G}_{(t)}^*(\theta) = \dot{\mathbf{f}}_t(\theta)\mathbf{\Sigma}_t^{-1}(\mathbf{y}_t - \mathbf{f}_t(\theta)) \tag{5}$$

and the estimation of θ by the QL method is the solution of the QL equation $\mathbf{G}_{(t)}^*(\theta) = 0$. A limitation of the QL method is that the nature of $\mathbf{\Sigma}_t$ may not be obtainable. A misidentified $\mathbf{\Sigma}_t$ could result in a deceptive inference about parameter θ. In the next subsection, we introduce the AQL method, which is basically the QL estimation assuming that the covariance matrix $\mathbf{\Sigma}_t$ is unknown.

2.2 The AQL method

The QLEF (see (4) and (5)) relies on the information of Σ_t. Such information is not always accessible. To find the QL when $E_{t-1}(\zeta_t \zeta_t{}')$ is not accessible, (Lin, 2000) proposed the AQL method.
Definition 2.2.1: *Let* $\mathbf{G}_{T,n}^*$ *be a sequence of the EF in* $\mathcal{G}$*. For all* $\mathbf{G}_T \in \mathcal{G}$*, if*

$$(E\dot{\boldsymbol{G}}_T)^{-1}(E\boldsymbol{G}_T\boldsymbol{G}_T)'(E\dot{\boldsymbol{G}}_T')^{-1} - (E\dot{\boldsymbol{G}}_{T,n}^*)^{-1}(E\boldsymbol{G}_{T,n}^*\boldsymbol{G}_T^{*\prime})(E\dot{\boldsymbol{G}}_{T,n}^{*\prime})^{-1}$$

is asymptotically non-negative definite, $\mathbf{G}_{T,n}^*$ *can be denoted as the asymptotic quasi-likelihood estimation function (AQLEF) sequence in* $\mathcal{G}$*, and the AQL sequence estimates* $\theta_{T,n}$ *by the AQL method is the solution of the AQL equation* $\mathbf{G}_{T,n}^* = 0$.

Suppose, in probability, $\mathbf{\Sigma}_{t,n}$ is converging to $E_{t-1}(\zeta_t \zeta_t{}')$. Then,

$$\mathbf{G}_{T,n}^*(\theta) = \sum_{t=1}^{T} \dot{\mathbf{f}}_t(\theta)\mathbf{\Sigma}_{t,n}^{-1}(\mathbf{y}_t - \mathbf{f}_t(\theta)) \tag{6}$$

expresses an AQLEF sequence. The solution of $\mathbf{G}^*_{T,n}(\theta) = 0$ expresses the AQL sequence estimate $\{\theta^*_{T,n}\}$, which converges to θ under certain regular conditions. In this paper, the kernel smoothing estimator of $\mathbf{\Sigma}_t$ is suggested to find $\mathbf{\Sigma}_{t,n}$ in the AQLEF (Eq. 6). A wide-ranging appraisal of the Nadaraya–Watson (NW) estimator-type kernel estimator is available in (Härdle, 1990). By using these kernel estimators, the AQL equation becomes

$$\mathbf{G}^*_{T,n}(\theta) = \sum_{t=1}^{T} \dot{\mathbf{f}}_t(\theta)\hat{\mathbf{\Sigma}}^{-1}_{t,n}(\hat{\theta}^{(0)})(\mathbf{y}_t - \mathbf{f}_t(\theta)) = 0. \tag{7}$$

The estimation of θ by the AQL method is the solution to (Eq. 7). Iterative techniques are suggested to solve the AQL equation (Eq. 7). Such techniques start with the ordinary least squares (OLS) estimator $\hat{\theta}^{(0)}$ and use $\hat{\mathbf{\Sigma}}_{t,n}(\hat{\theta}^{(0)})$ in the AQL equation (Eq. 7) to obtain the AQL estimator $\hat{\theta}^{(1)}$. Repeat this a few times until it converges. For estimation unknown parameters in fanatical models by QL and AQL approaches (See, ((Alzghool, 2017a), (Alzghool, 2017b)), (Alzghool and Al-Zubi, 2018), ((Alzghool and Lin, 2007),(Alzghool and Lin, 2008), (Alzghool and Lin, 2010))). The next sections present the parameter estimation of ASV model using the QL and AQL methods.

3 Point estimate of ASV model

Estimation of ASV model by applying the QLE and AQLE are developed. Moreover, simulation studies conducted for different values of unknown parameters and all result reports in this section.

3.1 Parameters estimation of ASV model using the QL method

Asymmetric stochastic volatility model is given by

$$y_t = e^{\frac{h_t}{2}}\upsilon_t, \qquad t = 1, 2, 3, \cdots, T. \tag{8}$$

$$h_t = \mu + \phi(h_{t-1} - \mu) + \sigma\eta_t, \qquad t = 1, 2, 3, \cdots, T. \tag{9}$$

Where

$$\begin{pmatrix} \upsilon_t \\ \eta_t \end{pmatrix} \sim \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma^2_\upsilon & \rho \\ \rho & \sigma^2_\eta \end{pmatrix} \right],$$

υ_t are independent and identically distributed (i.i.d). $E(\upsilon_t) = 0$ and $Var(\upsilon_t) = \sigma^2_\upsilon$, and η_t are i.i.d. $E(\eta_t) = 0$ and $Var(\eta_t) = \sigma^2_\eta$. The two innovations υ_t and η_t are assumed to jointly follow unknown probability distributions with a correlation coefficient given by $\rho = corr(\upsilon_t, \eta_t)$. In order to introduce a correlation structure between y_t and the log volatility, we rewrite equation 9 as

$$h_t = \mu + \phi(h_{t-1} - \mu) + \psi y_{t-1} e^{\frac{-h_{t-1}}{2}} + \tau\nu_t, \qquad t = 1, 2, 3, \cdots, T. \tag{10}$$

where $\psi = \sigma\rho$ and $\tau = \sigma\sqrt{1-\rho^2}$. The ASVM in (8) can be transformed into a linear model as follows:

$$\ln(y_t^2) = h_t + \ln {\upsilon_t}^2, \qquad t = 1, 2, 3 \cdots, T. \tag{11}$$

(Abramovitz and Stegun, 1970) showed that if $\upsilon_t \sim N(0,1)$, then $E(\ln \upsilon_t^2) = -1.2704$ and $Var(\ln \upsilon_t^2) = \pi^2/2$. Now, assume that $\varepsilon_t = \ln \upsilon_t^2 + 1.2704$. Thus, $E(\varepsilon_t) = 0$. However, if υ_t has an unknown distribution, let $\epsilon_t = \ln \upsilon_t^2 - \lambda$, then $E(\ln \upsilon_t^2) = \lambda$ and $Var(\ln \upsilon_t^2) = \sigma_\epsilon^2$. Therefore, For this scenario, the martingale difference is

$$\begin{pmatrix} \epsilon_t \\ \tau\nu_t \end{pmatrix} = \begin{pmatrix} \ln(y_t^2) - h_t - \lambda \\ h_t - \mu - \phi(h_{t-1} - \mu) - \psi y_{t-1} e^{\frac{-h_{t-1}}{2}} \end{pmatrix}.$$

First, to estimate h_t, the QLEF is given by

$$\begin{aligned} G_{(t)}(h_t) = (-1, 1) & \begin{pmatrix} \sigma_\epsilon^2 & \rho \\ \rho & \tau^2\sigma_\nu^2 \end{pmatrix}^{-1} \times \\ & \begin{pmatrix} \ln(y_t^2) - h_t - \lambda \\ h_t - \mu - \phi(h_{t-1} - \mu) - \psi y_{t-1} e^{\frac{-h_{t-1}}{2}} \end{pmatrix} \\ = & \frac{(\rho + \sigma_\epsilon^2)(h_t - \mu - \phi(h_{t-1} - \mu) - \psi y_{t-1} e^{\frac{-h_{t-1}}{2}})}{\tau^2\sigma_\nu^2\sigma_\epsilon^2 - \rho^2} \\ & - \frac{(\tau^2\sigma_\nu^2 + \rho)(\ln(y_t^2) - h_t - \lambda)}{\tau^2\sigma_\nu^2\sigma_\epsilon^2 - \rho^2}. \end{aligned} \tag{12}$$

Given that $\hat{h}_0 = 0$, the initial values $\omega_0 = (\lambda_0, \mu_0, \phi_0, \psi_0, \tau_0, \rho_0, \sigma_{\nu_0}^2, \sigma_{\epsilon_0}^2)$, and $\hat{h}_{t-1}$ is the QL estimation of h_{t-1}, the QL estimation of h_t is the solution of $G_{(t)}(h_t) = 0$,

$$\begin{aligned} \hat{h}_t = & \frac{(\rho_0 + \sigma_{\epsilon_0}^2)(\mu_0 + \phi_0(\hat{h}_{t-1} - \mu_0) + \psi_0 y_{t-1} e^{\frac{-\hat{h}_{t-1}}{2}})}{\tau_0^2\sigma_{\nu_0}^2 + 2\rho_0 + \sigma_{\epsilon_0}^2} \\ & + \frac{(\tau_0^2\sigma_{\nu_0}^2 + \rho_0)(\ln(y_t^2) - \lambda_0)}{\tau_0^2\sigma_{\nu_0}^2 + 2\rho_0 + \sigma_{\epsilon_0}^2}, \qquad t = 1, 2, 3 \cdots, T. \end{aligned} \tag{13}$$

Second, using $\{\hat{h}_t\}$ and $\{y_t\}$, and considering λ, μ, ϕ, and ψ as unknown parameters, the QLEF can be given by

$$\begin{aligned} G_T(\lambda, \mu, \phi, \psi) = \sum_{t=1}^{T} & \begin{pmatrix} -1 & 0 \\ 0 & -1 \\ 0 & -h_{t-1}^* \\ 0 & -y_{t-1} e^{\frac{-\hat{h}_{t-1}}{2}} \end{pmatrix} \begin{pmatrix} \sigma_{\epsilon_0}^2 & \rho_0 \\ \rho_0 & \tau^2\sigma_{\nu_0}^2 \end{pmatrix}^{-1} \times \\ & \begin{pmatrix} \ln(y_t^2) - h_t - \lambda \\ h_t - \mu - \phi(h_{t-1} - \mu) - \psi y_{t-1} e^{\frac{-h_{t-1}}{2}} \end{pmatrix}. \end{aligned}$$

The QL estimation of λ, μ, ϕ, and ψ is the solution of $G_T(\lambda,\mu,\phi,\psi)=0$. Therefore,

$$\hat{\lambda}=\frac{\sum_{t=1}^{T}\ln(y_t^2)-\sum_{t=1}^{T}\hat{h}_t}{T}, \tag{14}$$

$$\hat{\psi}=\frac{\rho(S_{y_t,y_{t-1}}S^2_{h^*_{t-1}}-S_{y_{t-1},h^*_{t-1}}S_{y_t,h^*_{t-1}})}{\sigma^2_\epsilon(S^2_{y_{t-1},h^*_{t-1}}-S^2_{y_{t-1}}S^2_{h^*_{t-1}})}+ \frac{(\rho+\sigma^2_\epsilon)(S_{y_{t-1},h^*_{t-1}}S_{h_t,h^*_{t-1}}-S_{y_{t-1},h_t}S^2_{h^*_{t-1}})}{\sigma^2_\epsilon(S^2_{y_{t-1},h^*_{t-1}}-S^2_{y_{t-1}}S^2_{h^*_{t-1}})} \tag{15}$$

$$\hat{\phi}=\frac{\rho(S_{h_t,h^*_{t-1}}-S_{y_t,h^*_{t-1}})+\sigma^2_\epsilon(S_{h_t,h^*_{t-1}}-\hat{\psi}S_{y_{t-1},h^*_{t-1}})}{\sigma^2_\epsilon S^2_{h^*_{t-1}}}, \tag{16}$$

$$\hat{\mu}=\frac{\sum_{t=1}^{T}\hat{h}_t-\hat{\phi}\sum_{t=1}^{T}\hat{h}^*_{t-1}-\hat{\psi}\sum_{t=1}^{T}y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}}}{T}, \tag{17}$$

Further,

$$\hat{\sigma^2_\epsilon}=\frac{\sum_{t=1}^{T}(\hat{\epsilon_t}-\bar{\hat{\epsilon}})^2}{T-1}, \tag{18}$$

$$\hat{\sigma^2_\eta}=\frac{\sum_{t=1}^{T}(\hat{\eta_t}-\bar{\hat{\eta}})^2}{T-1} \tag{19}$$

$$\hat{\sigma^2_\upsilon}=\frac{\sum_{t=1}^{T}(\hat{\upsilon_t}-\bar{\hat{\upsilon}})^2}{T-1} \tag{20}$$

$$\hat{\rho}=\frac{\sum_{t=1}^{T}(\hat{\eta_t}-\bar{\hat{\eta}})(\hat{\upsilon_t}-\bar{\hat{\upsilon}})}{\hat{\sigma_\eta}\hat{\sigma_\upsilon}} \tag{21}$$

$$\hat{\tau}=\hat{\sigma_\eta}\sqrt{1-\hat{\rho}^2} \tag{22}$$

where

- $\hat{\epsilon_t}=\ln(y_t^2)-\hat{h}_t-\hat{\lambda}$, and $\hat{\upsilon_t}=y_t e^{\frac{-\hat{h}_t}{2}}$, $\quad t=1,2,3,\cdots,T$
- $\hat{\eta_t}=\hat{h_t}-\hat{\mu}-\hat{\phi}\hat{h}^*_{t-1}$, and $\hat{h}^*_{t-1}=\hat{h}_{t-1}-\hat{\mu}$, $\quad t=1,2,3,\cdots,T$.
- $\tau\hat{\nu_t}=\hat{h}_t-\hat{\mu}-\hat{\phi}(\hat{h}_{t-1}-\hat{\mu})-\hat{\psi}y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}}$, $\quad t=1,2,3,\cdots,T$.
- $S_{y_t,y_{t-1}}=T\sum_{t=1}^{T}\ln y_t^2 y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}}-\sum_{t=1}^{T}\ln y_t^2\sum_{t=1}^{T}y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}}$
- $S^2_{h^*_{t-1}}=T\sum_{t=1}^{T}(\hat{h}^*_{t-1})^2-(\sum_{t=1}^{T}\hat{h}^*_{t-1})^2$
- $S_{y_{t-1},h^*_{t-1}}=T\sum_{t=1}^{T}\hat{h}^*_{t-1}y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}}-\sum_{t=1}^{T}\hat{h}^*_{t-1}\sum_{t=1}^{T}y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}}$
- $S_{y_t,h^*_{t-1}}=T\sum_{t=1}^{T}\ln y_t^2\hat{h}^*_{t-1}-\sum_{t=1}^{T}\hat{h}^*_{t-1}\sum_{t=1}^{T}\ln y_t^2$

- $S_{y_{t-1},h_t} = T\sum_{t=1}^{T}\hat{h}_t y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}} - \sum_{t=1}^{T}\hat{h}_t\sum_{t=1}^{T}y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}}$
- $S_{h_t,h^*_{t-1}} = T\sum_{t=1}^{T}\hat{h}_t\hat{h}^*_{t-1} - \sum_{t=1}^{T}\hat{h}_t\sum_{t=1}^{T}\hat{h}^*_{t-1}$
- $S^2_{y_{t-1}} = T\sum_{t=1}^{T}\left(y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}}\right)^2 - \left(\sum_{t=1}^{T}y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}}\right)^2$

$\hat{\omega} = (\hat{\lambda},\hat{\mu},\hat{\phi},\hat{\psi},\hat{\tau},\hat{\rho},\hat{\sigma}^2_\nu,\hat{\sigma}^2_\epsilon)$ is an updated initial value in the iterative procedure. The initial values h_0 and $\boldsymbol{\theta}_0$ might be affected the estimation results of ASVM. For an extensive discussion on assigning initial values in the QL estimation procedures, see (Alzghool and Lin, 2011).

3.2 Parameters estimation of ASV model using the AQL method

Consider the ASVM given by (8) and (9) and the same argument listed under (9). First, to estimate h_t, the AQLEF sequence is given by

$$G_{(t)}(h_t) = (-1,1)\boldsymbol{\Sigma}^{-1}_{t,n}\begin{pmatrix} \ln(y_t^2) - h_t - \lambda \\ h_t - \mu - \phi(h_{t-1}-\mu) - \psi y_{t-1}e^{\frac{-h_{t-1}}{2}} \end{pmatrix}$$

Given $\hat{h}_0 = 0$, $\theta_0 = (\lambda_0,\mu_0,\phi_0,\psi_0,\tau_0)$, $\boldsymbol{\Sigma}^{(0)}_{t,n} = \boldsymbol{I}_2$, and $\hat{h}_{t-1}$ is the AQL estimation of h_{t-1}, the AQL estimation of h_t is the solation of $G_{(t)}(h_t) = 0$; that is,

$$\hat{h}_t = \frac{\ln(y_t^2) - \lambda_0 + \mu_0 + \phi_0(\hat{h}_{t-1}-\mu_0) + \psi_0 y_{t-1}e^{\frac{-h_{t-1}}{2}}}{2}, t = 1,2,3\cdots,T. \tag{23}$$

Second, using the kernel estimation method, we find

$$\hat{\boldsymbol{\Sigma}}_{t,n}(\theta^{(0)}) = \begin{pmatrix} \hat{\sigma}_n(y_t) & \hat{\sigma}_n(y_t,h_t) \\ \hat{\sigma}_n(h_t,y_t) & \hat{\sigma}_n(h_t) \end{pmatrix}.$$

Third, using $\{\hat{h}_t\}$ and $\{y_t\}$, and considering λ, μ, ϕ, and ψ as unknown parameters, the AQLEF can be given by

$$G_T(\lambda,\mu,\phi,\psi) = \sum_{t=1}^{T}\begin{pmatrix} -1 & 0 \\ 0 & -1 \\ 0 & -\hat{h}^*_{t-1} \\ 0 & -y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}} \end{pmatrix}\hat{\boldsymbol{\Sigma}}^{-1}_{t,n}$$

$$\times\begin{pmatrix} \ln(y_t^2) - h_t - \lambda \\ h_t - \mu - \phi(h_{t-1}-\mu) - \psi y_{t-1}e^{\frac{-h_{t-1}}{2}} \end{pmatrix}.$$

The AQL estimation of λ, μ, ϕ, and ψ is the solution of $G_T(\lambda,\mu,\phi,\psi) = 0$. Therefore,

$$\hat{\psi} = \frac{(S_{y_{t-1}}S_{h^*_{t-1}} - S_{y_{t-1},h^*_{t-1}}S_\Delta)(S_{y_t,h^*_{t-1}} - S_{h_t,h^*_{t-1}} - S_{h^*_{t-1},h_t}) -}{S_{y_{t-1}}(S_{y_{t-1}}S^2_{h^*_{t-1}} - S_{h^*_{t-1}}S_{y_{t-1},h^*_{t-1}}) +}$$

$$\frac{(S_{y_{t-1}}S^2_{h^*_{t-1}} + S_{h^*_{t-1}}S_{y_{t-1},h^*_{t-1}})(S_{y_t} - S_{h_{t,1}} - S_{h_{t,2}})+}{S_{y_{t-1},h^*_{t-1}}(S_{y_{t-1},h^*_{t-1}}S_\Delta - S_{y_{t-1}}S_{h^*_{t-1}})+}$$

$$\frac{(S_{h^*_{t-1}}S_{h^*_{t-1}} + S_{h^*_{t-1}}S^2_{h^*_{t-1}})(S_{y_{t-1},y_t} - S_{h_t,y_{t-1}} - S_{y_{t-1},h_t})}{S^2_{y_{t-1}}(S^2_{h^*_{t-1}}S_\Delta - S_{h^*_{t-1}}S_{h^*_{t-1}})}, \tag{24}$$

$$\hat{\phi} = \frac{S_\Delta(S_{y_t,h^*_{t-1}} - S_{h^*_{t-1},h_t} - S_{h_t,h^*_{t-1}}) - S_{h^*_{t-1}}(S_{y_t} - S_{h_{t,1}} - S_{h_{t,2}})+}{S_{h^*_{t-1}}S_{h^*_{t-1}} - S_\Delta S^2_{h_{t-1^*}}},$$

$$\frac{(S_{y_{t-1},h^*_{t-1}}S_\Delta - S_{h^*_{t-1}}S_{y_{t-1}})\hat{\psi}}{S_{h^*_{t-1}}S_{h^*_{t-1}} - S_\Delta S^2_{h_{t-1^*}}}, \tag{25}$$

$$\hat{\mu} = \frac{S_{h_t,1} + S_{h_t,2} - S_{y_t} - S_{h^*_{t-1}}\hat{\phi} - S_{y_{t-1}}\hat{\psi}}{S_\Delta}, \tag{26}$$

$$\hat{\lambda} = \frac{\sum_{t=1}^T \frac{\hat{\sigma}_n(h_t)}{\Delta_t}\ln y_t^2 - \sum_{t=1}^T \frac{\hat{\sigma}_n(h_t)}{\Delta_t}\hat{h}_t - \sum_{t=1}^T \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t}\hat{h}_t + \hat{\mu}\sum_{t=1}^T \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t}}{\sum_{t=1}^T \frac{\hat{\sigma}_n(h_t)}{\Delta_t}}$$

$$+\frac{\hat{\phi}\sum_{t=1}^T \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t}\hat{h}^*_{t-1} + \hat{\psi}\sum_{t=1}^T \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t} y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}}}{\sum_{t=1}^T \frac{\hat{\sigma}_n(h_t)}{\Delta_t}}, \tag{27}$$

Further,

$$\hat{\sigma^2_\epsilon} = \frac{\sum_{t=1}^T(\hat{\epsilon_t} - \bar{\hat{\epsilon}})^2}{T-1}, \tag{28}$$

$$\hat{\sigma^2_\eta} = \frac{\sum_{t=1}^T(\hat{\eta_t} - \bar{\hat{\eta}})^2}{T-1} \tag{29}$$

$$\hat{\sigma^2_\nu} = \frac{\sum_{t=1}^T(\hat{\nu_t} - \bar{\hat{\nu}})^2}{T-1} \tag{30}$$

$$\hat{\sigma^2_\upsilon} = \frac{\sum_{t=1}^T(\hat{\upsilon_t} - \bar{\hat{\upsilon}})^2}{T-1} \tag{31}$$

$$\hat{\rho} = \frac{\sum_{t=1}^T(\hat{\eta_t} - \bar{\hat{\eta}})(\hat{\upsilon_t} - \bar{\hat{\upsilon}})}{\hat{\sigma_\eta}\hat{\sigma_\upsilon}} \tag{32}$$

$$\hat{\tau} = \hat{\sigma_\eta}\sqrt{1 - \hat{\rho}^2} \tag{33}$$

where

- $\hat{\epsilon_t} = \ln(y_t^2) - \hat{h}_t - \hat{\mu}$, and $\hat{\upsilon_t} = y_t e^{\frac{-\hat{h}_t}{2}}, t = 1, 2, 3, \cdots, T$
- $\hat{\eta_t} = \hat{h_t} - \hat{\mu} - \hat{\phi}\hat{h}^*_{t-1}$, and $\hat{h}^*_{t-1} = \hat{h}_{t-1} - \hat{\mu}, t = 1, 2, 3, \cdots, T$.
- $\tau\hat{\nu}_t = (\hat{h}_t - \hat{\mu} - \hat{\phi}(\hat{h}_{t-1} - \hat{\mu}) - \hat{\psi}y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}})$, $t = 1, 2, 3, \cdots, T$.
- $\Delta_t = \hat{\sigma}_n(h_t)\hat{\sigma}_n(y_t) - \hat{\sigma^2}_n(y_t, h_t), t = 1, 2, 3, \cdots, T$.

- $S_{y_t} = (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(h_t)}{\Delta_t})(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t} \ln{(y_t^2)}) - (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t})(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(h_t)}{\Delta_t} \ln{(y_t^2)})$

- $S_{h_t,1} = (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t}\hat{h}_t)(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(h_t)}{\Delta_t}) - (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(h_t)}{\Delta_t}\hat{h}_t)(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t})$

- $S_{h_t,2} = (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t)}{\Delta_t}\hat{h}_t)(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(h_t)}{\Delta_t}) - (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t}\hat{h}_t)(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t})$

- $S_{\Delta} = (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(h_t)}{\Delta_t})(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t)}{\Delta_t}) - (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t})^2$

- $S_{h^*_{t-1}} = (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t)}{\Delta_t}h^*_{t-1})(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(h_t)}{\Delta_t}) - (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t}h^*_{t-1})(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t})$

- $S_{y_{t-1}} = (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t)}{\Delta_t}y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}})(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(h_t)}{\Delta_t}) - (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t})(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t}y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}})$

- $S_{y_t,h^*_{t-1}} = (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t} \ln y_t^2 \hat{h}^*_{t-1})(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(h_t)}{\Delta_t}) - (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(h_t)}{\Delta_t} \ln y_t^2)(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t}\hat{h}^*_{t-1})$

- $S_{h_t,h^*_{t-1,1}} = (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t}\hat{h}_t\hat{h}^*_{t-1})(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(h_t)}{\Delta_t}) - (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t}\hat{h}^*_{t-1})(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(h_t)}{\Delta_t}\hat{h}_t)$

- $S_{h_t,h^*_{t-1,2}} = (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t)}{\Delta_t}\hat{h}_t\hat{h}^*_{t-1})(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(h_t)}{\Delta_t}) - (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t}\hat{h}^*_{t-1})(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t}\hat{h}_t)$

- $S^2_{h^*_{t-1}} = (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t)\hat{h^*}^2_{t-1}}{\Delta_t})(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(h_t)}{\Delta_t}) - (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t}\hat{h^*}_{t-1})^2$

- $S_{y_{t-1},h^*_{t-1}} = (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t)}{\Delta_t}\hat{h}^*_{t-1}y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}})(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(h_t)}{\Delta_t}) - (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)\hat{h}^*_{t-1}}{\Delta_t})(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}}}{\Delta_t})$

- $S_{y_{t-1},y_t} = (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t} \ln{(y_t^2)}y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}})(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(h_t)}{\Delta_t}) - (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(h_t)}{\Delta_t} \ln{(y_t^2)})(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t}y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}})$

- $S_{y_{t-1},h_t,1} = (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t}\hat{h}_t y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}})(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(h_t)}{\Delta_t}) - (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(h_t)}{\Delta_t}\hat{h}_t)(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t}y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}})$

- $S_{y_{t-1},h_t,2} = (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t)}{\Delta_t}\hat{h}_t y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}})(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(h_t)}{\Delta_t}) - (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t}\hat{h}_t)(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t}y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}})$

- $S^2_{y_{t-1}} = (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t)}{\Delta_t}(y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}})^2)(\sum_{t=1}^{T} \frac{\hat{\sigma}_n(h_t)}{\Delta_t}) - (\sum_{t=1}^{T} \frac{\hat{\sigma}_n(y_t,h_t)}{\Delta_t} y_{t-1}e^{\frac{-\hat{h}_{t-1}}{2}})^2$

Table I: The QL and AQL estimates; the RMSE of each estimate is given.

	λ	μ	ϕ	ψ	ρ	τ	σ_η	σ_ϵ
true	-1.271	-0.141	0.980	-0.037	-0.610	0.048	0.061	2.22
QL	-1.276	-0.140	0.882	-0.035	-0.830	0.027	0.049	2.236
RMSE	0.109	0.011	0.111	0.009	0.234	0.023	0.013	0.086
AQL	-1.191	-0.150	-0.883	-0.029	-0.155	0.190	0.192	2.137
RMSE	0.097	0.085	0.102	0.010	0.456	0.143	0.116	0.133
true	-1.271	-0.353	0.950	-0.072	-0.750	0.063	0.096	2.22
QL	-1.274	-0.349	0.823	-0.056	-0.434	0.138	0.149	2.135
RMSE	0.037	0.0134	0.133	0.022	0.317	0.089	0.054	0.117
AQL	-1.204	-0.208	0.878	-0.061	-0.179	0.258	0.262	2.099
RMSE	0.093	0.183	0.079	0.133	0.572	0.211	0.143	0.168
true	-1.271	-0.147	0.980	-0.083	-0.500	0.143	0.166	2.22
QL	-1.271	-0.149	0.924	-.064	-0.299	0.228	0.237	2.081
RMSE	0.031	0.018	0.060	0.023	0.203	0.085	0.072	0.160
AQL	-1.214	-0.051	0.940	0.071	-0.221	0.269	0.277	2.167
RMSE	0.115	0.248	0.046	0.154	0.286	0.129	0.095	0.114
true	-1.271	-0.706	0.900	-0.108	-0.800	0.081	0.135	2.22
QL	-1.272	-0.708	0.780	-0.079	-0.628	0.126	0.150	2.152
RMSE	0.052	0.010	0.118	0.032	0.173	0.046	0.016	0.106
AQL	-1.201	-0.289	0.853	0.086	-0.253	0.258	0.267	2.101
RMSE	0.104	0.460	0.059	0.194	0.550	0.179	0.141	0.135
true	-1.271	-0.629	0.962	-0.122	-0.642	0.146	0.191	2.22
QL	-1.275	-0.629	0.895	-0.096	-0.536	0.187	0.215	2.141
RMSE	0.046	0.015	0.069	0.030	0.108	0.042	0.021	0.113
AQL	-1.224	-0.212	0.924	0.104	-0.306	0.272	0.287	2.149
RMSE	0.124	0.521	0.048	0.226	0.338	0.129	0.105	0.099

The AQL estimation of λ, μ, ϕ and ψ is the solation of $G_T(\lambda, \mu, \phi, \psi) = 0$. Then $\theta^{(0)} = (\lambda_0, \mu_0, \phi_0, \psi_0)$ is updated and replaced by the $\hat{\theta} = (\hat{\lambda}, \hat{\mu}, \hat{\phi}, \hat{\psi})$, the estimate of $\theta = (\lambda, \mu, \phi, \psi)$. The estimation procedure will be iteratively repeated until it converges. $\omega_0 = (\lambda_0, \mu_0, \phi_0, \psi_0, \tau_0, \rho_0, \sigma^2_{\nu_0}, \sigma^2_{\epsilon_0})$ is an updated initial value in the iterative procedure. The initial values h_0 and ω_0 might be affected by the estimation results of ASVM. For an extensive discussion on assigning initial values in the QL estimation procedures, see (Alzghool and Lin, 2011). Our simulation was carried out as follows. Firstly, we independently simulate 1,000 samples with size T=1000 from ASVM (1) and (2) based on a true parameter ω. After the series y_t are generated, we pretend that h_t are unobserved and ω are unknown. Then we apply the above estimation procedure to y_t only to obtain the estimation of h_t and ω. For this simulation study, We consider different parameter settings for $\omega = (\lambda, \mu, \phi, \psi, \rho, \tau, \sigma_\delta, \sigma_\epsilon)$ and compute mean and root mean squared errors (RMSE) for ω where N = 1000 independent samples.

In Table (I), QL represents the QL estimate and AQL represents the AQL estimate. The results in Table I confirm that QL and AQL have succeeded in SVM parameter estimation. The effect of sample size on parameter estimation is considered. Samples of sizes T = 500, 1000, 2000, 3000, and 6000 were generated. The results in Table II show that the RMSE decreases when the sample size increases. In Table II, The QL and AQL estimation methods show the property of consistency, the RMSE decrease as sample size increase.

Table II: The QL and AQL estimates; the RMSE of each estimate is given below that estimate.

	λ	μ	ϕ	ψ	ρ	τ	σ_η	σ_ϵ
true	-1.271	-0.141	0.98	-0.037	-0.61	0.048	0.061	2.22
QL	-1.283	-0.139	0.872	-0.035	-.825	0.027	0.049	2.229
(T=500)	0.154	0.016	0.134	0.013	0.236	0.024	0.014	0.121
AQL	-1.196	-0.139	0.876	-0.029	-0.155	0.191	0.193	2.129
	0.097	0.106	0.113	0.012	0.458	0.146	0.148	0.135
QL	-1.276	-0.140	0.882	-0.035	-0.830	0.027	0.049	2.236
(T=1000)	0.109	0.011	0.111	0.009	0.234	0.023	0.013	0.086
AQL	-1.191	-0.150	0.883	-0.029	-0.155	0.190	0.192	2.137
	0.097	0.085	0.102	0.010	0.456	0.143	0.116	0.133
QL	-1.273	-0.141	0.887	-0.035	-0.833	0.027	0.049	2.240
(T=2000)	0.079	0.069	0.100	0.006	0.231	0.022	0.013	0.061
AQL	-1.188	-0.153	0.886	-0.030	-0.156	0.189	0.191	2.137
	0.096	0.072	0.097	0.009	0.455	0.142	0.100	0.131
QL	-1.272	-0.140	0.890	-0.035	-0.836	0.027	0.049	2.235
(T=3000)	0.065	0.005	0.095	0.0053	0.231	0.022	0.013	0.051
AQL	-1.191	-0.159	0.887	-0.029	-0.155	0.188	0.191	2.137
	0.093	0.065	0.095	0.008	0.455	0.141	0.095	0.130
QL	-1.272	-0.141	0.891	-0.035	-0.838	0.027	0.049	2.237
(T=6000)	0.046	0.037	0.091	0.004	0.230	0.022	0.013	0.039
AQL	-1.189	-0.156	0.888	-0.030	-0.156	0.188	0.190	2.139
	0.093	0.063	0.093	0.008	0.455	0.140	0.088	0.129

4 Empirical applications

The QL and AQL estimation procedures were used to estimate the ASV model for several data sets:

$$y_t = e^{\frac{h_t}{2}} \upsilon_t, \qquad t = 1, 2, 3, 4, \cdots, T.$$

$$h_t = \mu + \phi(h_{t-1} - \mu) + \psi y_{t-1} e^{\frac{-h_{t-1}}{2}} + \tau \nu_t, \qquad t = 1, 2, 3, 4, \cdots, T.$$

Where

$$\begin{pmatrix} \upsilon_t \\ \eta_t \end{pmatrix} \sim \left[\begin{pmatrix} 0 \\ 0 \end{pmatrix}, \begin{pmatrix} \sigma_\upsilon^2 & \rho \\ \rho & \sigma_\eta^2 \end{pmatrix} \right].$$

4.1 Daily exchange rate of (Australian Dollar/US Dollar)

The first data set we analyze is the daily exchange rate of $r_t = AUD/USD$ (Australian Dollar/US Dollar) for period from 5/6/2010 to 5/5/2016, 1590 observations in total. The ASVM used to model $y_t = log(r_t) - log(r_{t-1})$, where $\omega = (\lambda, \mu, \phi, \psi, \rho, \tau, \sigma_\delta, \sigma_\epsilon)$ is the parameter. Table III present the estimates for parameters ω obtained using QL and AQL methods and residual sum of squares (RSS). QL represents the estimate obtained using the QL method, AQL represents the AQL estimate and $RSS = \sum_{t=1}^{T} (\ln(y_t^2) - \widehat{\ln(y_t^2)}(\hat{\theta}))^2$, where $\widehat{\ln(y_t^2)}(\hat{\theta})$ be the predicted value of $\ln(y_t^2)$ based on the observation equation.

Table III: Estimation of ω for pound/dollar exchange rate data.

	λ	μ	ϕ	ψ	ρ	τ	σ_η	σ_ϵ	RSS
QL	-11.12	-0.0002	0.90	-0.015	-0.050	0.3	0.3	1.79	5064
AQL	-11.09	0.018	0.90	-0.013	-0.012	1.08	1.08	1.22	2350

We can see from the last column in Table III that AQL gives smaller SSR. The AQL method tends to be more efficient than QL method. The QL and AQL estimates are carried out in diverse model sceneries. We carried out the QL method under the assumption that the form of covariance matrix is known and The AQL method is applied to the data by assuming of no knowledge on the covariance matrix.

4.2 Weekly price changes of Crude oil prices

The second set of data is weekly price changes of Crude oil prices P_t. The P_t of Cushing, OK West Texas Intermediate (US Dollars per Barrel) for period from 7/1/2000 to 10/6/2016, 858 observations in total. The data are obtained from the US Energy Information Administration (see, http://www.eia.gov/dnav/pet). The data are transformed into rates of change by taking the first difference of the logs. Thus, $y_t = log(P_t) - log(P_{t-1})$ and fit $\{y_t\}$ by using ASV model (1.1) and (1.2). Table IV present the estimates for parameters ω obtained using QL and AQL methods. QL represents the estimate obtained using the QL method, AQL represents the AQL estimate.

Table IV: Estimation of ω for the rates of change prices data.

	λ	μ	ϕ	ψ	ρ	τ	σ_η	σ_ϵ	RSS
QL	-7.73	-0.022	0.88	-0.006	-0.02	0.31	0.31	1.85	2924
AQL	-7.72	-0.015	0.93	-0.086	-0.04	1.117	1.119	1.230	1302

We can see from the last column in Table IV that AQL gives smaller RSS. The AQL method tends to be more efficient than QL method.

4.3 Daily $(S\&P)$500 index

The third set of data is daily log-returns of the Standard & Poors $(S\&P)$500 index from January 2005 to December 2008. the returns are defined as $y_t = log(P_t) - log(P_{t-1})$, where P_t is the closing price on day t. Thus, fit $\{y_t\}$ by using ASV model (1.1) and (1.2). Table V present the estimates for parameters ω obtained using QL and AQL methods. QL represents the estimate obtained using the QL method, AQL represents the AQL estimate.

Table V: Estimation of ω for the returns data.

	λ	μ	ϕ	ψ	ρ	τ	σ_η	σ_ϵ	RSS
QL	-10.73	-0.025	0.95	-0.109	-0.007	0.35	0.35	2.08	4301
AQL	-10.71	-0.006	0.91	-0.183	-0.007	1.27	1.23	1.38	1891

We can see from the last column in Table IV that AQL gives smaller RSS. The AQL method tends to be more efficient than QL method.

5 Summary

In this paper, two alternative methods, the QL and AQL , for estimating the parameters in asymmetric stochastic volatility (ASV) models with unspecified correlations have been presented. Results from the simulation study indicate that the AQL method is an efficient estimation procedure. The study has also shown that the QL and AQL estimating procedures are easy to implement, especially when the system's probability structure cannot be fully specified. By utilising the nonparametric kernel estimator of variance covariances matrix Σ_t to replace the true Σ_t in the standard quasi-likelihood, the AQL method avoids the risk of potential mis-specification of Σ_t and thus makes the parameter estimation more efficient.

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